연 속 계 진 동 해 석　　중 간 시 험

(30 %)

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1. (4%) Explain the procedures, advantages and disadvantages of the following approximate methods to solve the vibration problems of continuous systems.
   (a) Rayleigh's energy method　　(b) Rayleigh-Ritz method
   (c) assumed-modes method　　(d) Galerkin's method

2. (6%) A bar (mass per unit length $m(x)$, flexural stiffness $EI(x)$, length L) in flexural vibration is fixed at x=0 and supported by a linear spring at x=L. The stiffness of the spring is k. Prove the orthogonality of the natural modes with respect to $m(x)$, and derive the modified orthogonality with respect to the stiffness.

3. (6%) A lumped mass M is attached at the end of a uniform rod (length L, mass per unit length m, axial stiffness EA) which is under axial vibration.
   (a) Write the equation of motion and boundary conditions in terms of the displacement $u(x,t)$.
   (b) Obtain the characteristic equation.
   (c) Which situations are the following cases similar to?
   (i) $M << mL$　　(ii) $M >> mL$

4. (8%) Consider the torsional vibration of a nonuniform circular shaft fixed at one end (x=0) and free at the other end (x=L). The distributions of the torsional stiffness and mass moment of inertia are as follows:

$$GJ(x) = \frac{10}{9} GJ\left[1 - \frac{1}{2}\left(\frac{x}{L}\right)^2\right] \qquad I(x) = \frac{10}{9} I\left[1 - \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

   Obtain the lowest natural frequencies and mode shapes by the Rayleigh-Ritz method using an approximate solution in the form

$$\Theta(x) = a_1 \frac{x}{L} + a_2 \left(\frac{x}{L}\right)^2$$

5. (6%) A uniform circular membrane (tension T, mass per unit area ρ) occupies the area $0 \le r \le a$. The edge at r=a and the center at r=0 are fixed.
   (a) Write the differential equation of the transverse vibration and boundary conditions.
   (b,c) Express the natural frequencies and mode shapes at your best.