진 동 공 학　학 기 말 시 험

(40 %)

1998. 6. 23　　　　　　　　　　　　　　　　　　대학원 기계공학과

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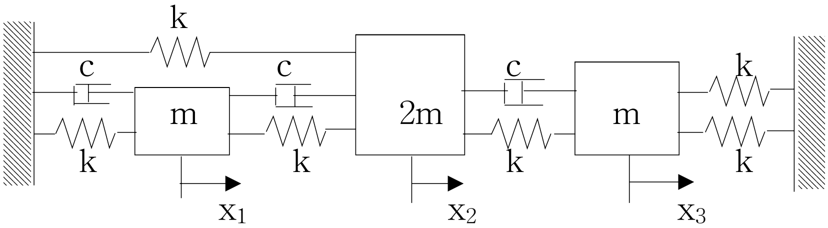
1. (4%) Derive the equation of motion of the system shown below, and express the differential equation in a matrix form.



2. (5%) Given the equation of motion for a three-degree-of-freedom system as follows, obtain the natural frequencies and mode shapes (normalize by setting the absolute maximum component to 1).

   $$\mathrm{M}\ \ddot{\mathbf{x}} + \mathrm{K}\ \mathbf{x} = 0,$$

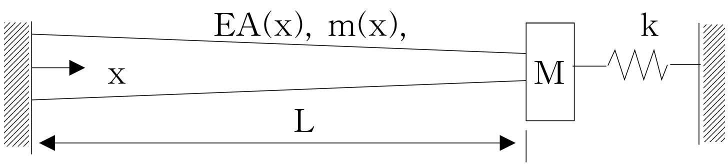
   where $\mathrm{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (kg), $\mathrm{K} = \begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix}$ (N/m), $\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$

3. (5%) The natural frequencies and corresponding mode shapes of a two degree of freedom system are known as follows:

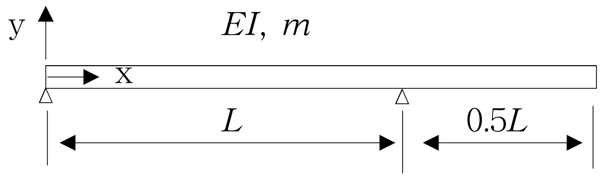
   $\omega_1 = 1.642$ rad/s, $\mathbf{u}_1 = \begin{Bmatrix} 1 \\ 0.909 \end{Bmatrix}$, $\omega_2 = 2.511$ rad/s, $\mathbf{u}_2 = \begin{Bmatrix} -0.101 \\ 1 \end{Bmatrix}$

   Obtain the solution $\mathbf{x}(t)$ for the initial conditions $\mathbf{x}(0) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$, $\dot{\mathbf{x}}(0) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

4. (5%) A lumped mass M and a spring of the stiffness k are attached at the end of a nonuniform rod (length L, mass per unit length is $m(x)$, axial stiffness $EA(x)$) which is under axial vibration as shown in the following figure. Write (do not derive) the expressions for the potential energy and kinetic energy of this system in terms of the axial displacement u(x,t) in the rod.



5. (6%) A uniform bar (length $1.5L$, mass per unit length m, flexural rigidity EI) is hinged as follows.



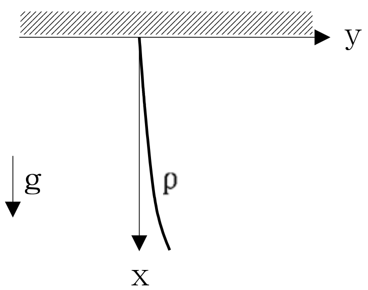
(a) write the equation of motion and boundary conditions for the bending motion in terms of the displacement $y(x,t)$.

(b) obtain the characteristic equation.

6. (3%) According to the definitions of the following functions, what does each function satisfy among the *equation of motion*, *geometric boundary conditions*, and *natural boundary conditions* of an eigenvalue problem?

(a) admissible function, (b) comparison function, (c) eigenfunction

7. (6%) A uniform heavy rope of length L hangs from the ceiling with the lower end loose in the gravity field of the earth. The mass per unit length of the rope is ρ. Use the Rayleigh-Ritz method and two-term approximation and obtain the eigenvalue problem of the lateral vibration, i.e. express [k] and [m] of $[k]\{a\} = \omega^2[m]\{a\}$



8. (6%) A uniform rectangular membrane (tension T, mass per unit area ρ) occupies the area $0 \le x \le a$, $0 \le y \le b$. Two edges at $x=0$ and $x=a$ are clamped and other two edges at $y=0$ and $y=b$ are free.

(a) write the differential equation of the transverse vibration and boundary conditions

(b) obtain the natural frequencies and mode shapes.