기계진동학 연 습 문 제 정 답 (제3장) (2022. 4. 11.)

교 재 : D. J. Inman, Engineering Vibration, **4th** edition, Pearson, **2014**.

3.1절 <3.1> 생략(나쁜 문제) <3.2> $x(t)$ = (1.291 m) $e^{-0.5t}\sin 38.7t$

<3.3> $0 \le t < \pi$; $x(t)$ = 0.577 e^{-t} sin(1.732 t) m

$t \ge \pi$; $x(t)$ = 0.577 e^{-t} sin(1.732 t) + 0.577 $e^{-(t-\pi)}\sin 1.732(t-\pi)$ m

<3.4> $0 \le t < \pi$; $x(t)$ = 0.750 e^{-t} sin(1.414 t + 0.340) + 0.354 sin(t - 0.785) m

$t \ge \pi$; $x(t)$ = 0.750 e^{-t} sin(1.414 t + 0.340) + 0.354 sin(t - 0.785)
+ 0.707 $e^{-(t-\pi)}\sin 1.414(t-\pi)$ m

<3.5> $x(t) = \dfrac{1}{m}\, t\, e^{-\omega_n t}$

<3.6> $x(t) = \dfrac{1}{2m\omega_n\sqrt{\zeta^2-1}} e^{-\zeta\omega_n t}\left[e^{\omega_n\sqrt{\zeta^2-1}\,t} - e^{-\omega_n\sqrt{\zeta^2-1}\,t}\right]$ <3.7> 노트 참조

<3.8> $x(t)$ = 50.4×10^{-6} $e^{-39.7t}$ sin(1,986 t) m

<3.9> $|x(1)|$ = 0.0050 m > 0.0001 m, 닫히지 않는다.

<3.10> $x(t) = \dfrac{\sqrt{2gh}}{\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t$ <3.11> $x(t)$ = (1 m/s) $t\, e^{-2t}$

<3.12> $x(t)$ = (0.01 + 1.02 t) e^{-2t} m

<3.13> $x(t)$ = 1.166 e^{-t} sin(1.732t+0.0172) + 0.577 e^{-t} sin(1.732 t)
- 0.192 $e^{-(t-1)}$ sin1.732(t-1) $H(t-1)$ m
$\approx$ 1.743 e^{-t} sin(1.732 t) - 0.192 $e^{-(t-1)}$ sin1.732(t-1) $H(t-1)$ m

<3.14> $x(t)$ = (0.01914 m) $e^{-54.8t}\sin 174.2t$

$x(t_p)_{damped}$ = 0.01226 m , $x(t_p)_{undamped}$ = 0.01826 m

<3.15> 생략 <3.16> 생략 <3.17> 생략

3.2절 <3.18> 생략 <3.19> 생략 <3.20> 생략

<3.21> $0 \le t \le 4$s 일 때, $x(t)$ = 5 (t - sin t) m

t > 4s 일 때, $x(t)$ = 20 + 5 [sin(t-4) - sin t] m

<3.22> $0 \le t \le t_0$ 일 때, $x(t) = \dfrac{F_0}{k}(1-\cos\omega_n t) + \dfrac{F_0}{kt_0}\left(\dfrac{1}{\omega_n}\sin\omega_n t - t\right)$

$t > t_0$ 일 때, $x(t) = -\dfrac{F_0}{k}\cos\omega_n t + \dfrac{F_0}{kt_0\omega_n}[\sin\omega_n t - \sin\omega_n(t-t_0)]$

<3.23> $x(t)$ = (0.500 m/s) t - (0.0500 m) sin(10.0 t)]

<3.24> $0 \le t \le t_1$; $x(t)$ = 2.50 t - 4.56 sin0.548t mm

$t_1 \le t \le t_2$; $x(t)$ = 0.750 - 1.250t - 4.56 sin0.548t + 6.84 sin0.548(t-0.2) mm

$t \ge t_2$; $x(t)$ = -4.56sin0.548t + 6.85sin0.548(t-0.2) - 2.28sin0.548(t-0.6) mm

<3.25> $t_p = \dfrac{\pi}{\omega_d}$ = 1.00 s <3.26> O.S. = $\dfrac{F_0}{k} e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$ = 21.9 mm

<3.27> ω_n = 3.26 rad/s, ζ = 0.268

<3.28> $0 \le t \le 0.994$s ; $x(t)$ = 0.0300 [1 - 1.005 $e^{-0.316t}$ cos(3.14 t - 0.1002)] m

t>0.994s ; $x(t)$ = 0.0302 $e^{-0.316t}$ [1.369 cos(3.14 t - 3.23)
- cos(3.14 t - 0.1002)]

<3.29> $x(t)$ = (10.00 mm) cos4.472 t + (2.50 mm) (2 sin10t - sin5.528t - sin14.472t)

3.3절 <3.30> 증명, 유도 (노트)　　<3.31> $a_0 = 0$, $a_n = 0$ (n 짝수), $-\frac{8}{n^2\pi^2}$(n 홀수), $b_n = 0$

<3.32> $F(t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4}{n\pi}\sin nt = \sum_{m=1}^{\infty} \frac{4}{(2m-1)\pi}\sin(2m-1)t$

<3.33> $f(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{-1}{n\pi}\right)\sin nt = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin nt$

$= \frac{1}{2} - \frac{1}{\pi}(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \cdots)$　　<3.34> 생략

<3.35> $x_p(t) = \sum_{n=1,3,\ldots}^{\infty} \frac{-0.01013}{n^2\sqrt{n^4 - 24.5n^2 + 156.25}} \cos\left[nt - \tan^{-1}\frac{0.707n}{12.5 - n^2}\right]$ m

<3.36> $x_p(t) = 0.000708\ \cos(1.414\ t - 0.014) + 0.0501\ \sin(1.414\ t - 0.014)$

$= 0.0501\ \cos(1.414\ t - 1.57)$ m $= 0.0501\ \sin(1.414\ t - 0.014)$ m

<3.37> 실습　　<3.38> 실습

3.4절 <3.39> $x(t) = \frac{F_0}{k}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\cos(\omega_d t - \phi)\right]$

<3.40> $x(t) = (44.7\ \text{mm})\ e^{-15t}\ (e^{11.18t} - e^{-11.18t})$

<3.41> $x(t) = \frac{F_0}{m(\omega_n^2 - 2\zeta\omega_n a + a^2)}\left[e^{-at} - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta\omega_n - a}{\omega_d} e^{-\zeta\omega_n t}\sin\omega_d t\right]$

<3.42> $x(t) = \frac{1}{5\sqrt{5}}\ \sin\sqrt{20}\,t$ m $= 0.0894\ \sin 4.47t$ m

<3.43> $x(t) = \frac{5}{3}\ \sin t - \frac{1}{3}\ \sin 2t$ (m)