연 속 계 진 동 해 석　학 기 말 시 험

[30 점]

대학원 기계공학과　　　　　　　　　　　　　　　　2013. 12. 14.

< closed-book exam >

1.[8점] 연속계의 진동을 해석하는 방법들을 크게 세 가지 유형으로 분류하면

(1) 엄밀해법(exact solution method),

(2) 근사해법(approximate solution method),

(3) 유한요소법(finite-element method)

들이 있다. 다음 사항을 서술하시오.

(a) 세 가지 접근방법 각각의 용도

(b) 근사해법 중 lumped-parameter method에 속하는 방법들의 종류와 절차

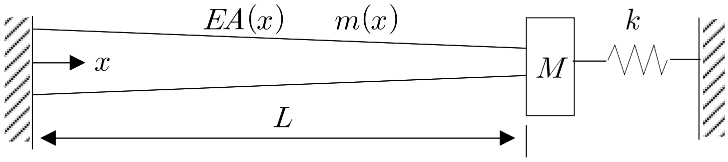
(c) 근사해법 중 series discretization method에 속하는 방법들의 종류와 절차

(d) 유한요소법의 절차

2.[3점] According to the definitions of the following functions, what does each function satisfy among the *equation of motion*, *geometric boundary conditions*, and *natural boundary conditions* of an eigenvalue problem?

(a) admissible function, (b) comparison function, (c) eigenfunction

3.[5점] A lumped mass M and a spring of the stiffness k are attached at the end of a nonuniform rod (length L, mass per unit length is $m(x)$, axial stiffness $EA(x)$) which is under axial vibration as shown in the following figure. Write (do not derive) the expressions for the potential energy and kinetic energy of this system in terms of the axial displacement $u(x,t)$ in the rod.



4.[8점] Consider the torsional vibration of a nonuniform circular shaft fixed at one end (x=0) and free at the other end (x=L). The distributions of the torsional stiffness and mass moment of inertia are as follows:

$$GJ(x) = \frac{10}{9}GJ\left[1-\frac{1}{2}\left(\frac{x}{L}\right)^2\right], \qquad I(x) = \frac{10}{9}I\left[1-\frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Obtain the lowest natural frequencies and mode shapes by the Rayleigh-Ritz method using an approximate solution in the form

$$\Theta(x) = a_1\frac{x}{L} + a_2\left(\frac{x}{L}\right)^2$$

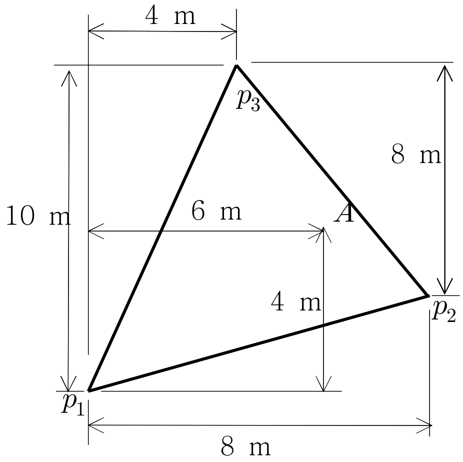
(뒷면에 계속)

5.[3점] Explain the procedure, advantages and disadvantages of the following approximate methods to solve the vibration problems of continuous systems.

(a) Galerkin's method, (b) collocation method, (c) assumed-modes method

6.[3점] 그림과 같이 삼각형의 꼭지점에서의 음압(sound pressure)을 알 때, 내부 A지점에서의 음압을 삼각형 선형 요소를 사용하여 구하시오.

p_1 = 100 Pa, p_2 = 150 Pa, p_3 = 200 Pa.



(끝)

연속계진동해석　　학기말시험　해 답　　　　2013년 2학기

[1] (a) 엄밀해법의 용도 : 균일하게 분포된 시스템에 대한 해석
근사해법의 용도 : 불균일하지만 불균일성이 크지 않은 시스템에 대한 해석
유한요소법의 용도 : 임의의 불균일 시스템에 대한 해석

(b) lumping method : 시스템을 여러 개의 segment로 분할하여 각 segment에 mass와 stiffness를 집중시키고, 시스템의 mass matrix와 stiffness influence matrix를 구성하여 algebraic eigenvalue problem을 형성함.

influence coefficients method : 시스템을 여러 개의 segment로 분할하여, 각 segment에 mass를 집중시켜서 시스템의 mass matrix를 구성하고, flexibility influence coefficient를 계산하여 구성한 matrix의 역행렬로서 stiffness influence matrix를 구성하여 algebraic eigenvalue problem을 형성함.

Holzer's method : shaft의 torsional vibration, rod의 axial vibration, string의 transverse vibration에 적용할 수 있는 방법으로서, 시스템을 분할하여 station의 mass와 field의 stiffness로부터 transfer matrix를 구성하고, 경계조건을 적용하여 frequency equation을 형성하여 natural frequency와 modal vector를 구함.

Myklestad's method : beam의 bending vibration에 적용할 수 있는 방법으로서, 시스템을 분할하여 station의 mass와 field의 stiffness로부터 transfer matrix를 구성하고, 경계조건을 적용하여 frequency equation을 형성하여 natural frequency와 modal vector를 구함.

(c) Rayleigh's quotient : maximum potential energy와 reference kinetic energy에 trial function을 대입하여 그 비율로부터 1차 고유진동수의 상한값을 구함.

Rayleigh-Ritz method : trial function의 선형 결합으로 해의 형태를 구성하고, Rayleigh quotient의 도함수가 0이 되도록 하여 mass matrix와 stiffness matrix를 형성하고, eigenvalue problem의 해를 구함.

assumed mode method : Rayleigh-Ritz method에 추가하여 external force를 포함시킨 방정식에 modal analysis를 적용하여 시스템의 response를 구함.

Galerkin's method : eigenvalue problem에 trial function을 적용할 때의 나머지(residual)에 trial function을 가중 함수(weighted function)로 곱하여 영역에서 적분한 결과가 0이 되도록 함으로써 평균적으로 방정식을 만족하는 algebraic eigenvalue problem을 구성하여 해를 구함.

collocation method : eigenvalue problem에 trial function을 적용할 때 특정 지점의 경계조건 또는 방정식을 만족하도록 함으로써 algebraic eigenvalue problem을 구성하여 해를 구함.

(d) a complex structure (system)
　　↓　discretization
finite elements
　　↓　variational approach　　← *Rayleigh-Ritz method*
equations of motion for individual elements　⇒　element matrix K_j, M_j
　　↓　assembly
equations of motion for the system　⇒　global matrix K, M
　　↓　solution
motion at the nodes
　　↓　interpolation
motion inside the elements

[2] (a) An admissible function satisfies geometric boundary conditions.

(b) A comparison function satisfies geometric and natural boundary conditions.

(c) An eigenfunction satisfies the equation of motion and geometric and natural boundary conditions.

[3] potential energy

$$V = \frac{1}{2}\int_0^L EA(x)\left[\frac{\partial u(x,t)}{\partial x}\right]^2 dx + \frac{1}{2}k\,[u(L,t)]^2$$

kinetic energy

$$T = \frac{1}{2}\int_0^L m(x)\left[\frac{\partial u(x,t)}{\partial t}\right]^2 dx + \frac{1}{2}M\left[\left.\frac{\partial u(x,t)}{\partial t}\right|_{x=L}\right]^2$$

[4] torsional vibration

differential equation of motion

$$-\frac{d}{dx}\left[GJ(x)\frac{d\Theta(x)}{dx}\right] = \omega^2 I(x)\Theta(x) \qquad 0<x<L$$

$$L = -\frac{d}{dx}\left[GJ(x)\frac{d}{dx}\right], \qquad M = I(x)$$

boundary conditions

$$\Theta(0) = 0\ , \qquad \left.GJ(x)\frac{d\Theta(x)}{dx}\right|_{x=L} = 0$$

$$N = \frac{1}{2}\int_0^L \Theta(x)\frac{d}{dx}\left[-GJ(x)\frac{d\Theta(x)}{dx}\right]dx$$

$$= \frac{1}{2}\left[-\Theta(x)\,GJ(x)\left.\frac{d\Theta(x)}{dx}\right|_0^L + \int_0^L \frac{d\Theta(x)}{dx}\,GJ(x)\frac{d\Theta(x)}{dx}dx\right]$$

$$= \frac{1}{2}\int_0^L GJ(x)\left[\frac{d\Theta(x)}{dx}\right]^2 dx = \frac{1}{2}\int_0^L GJ(x)\left[\sum_{i=1}^n a_i\frac{d\phi_i}{dx}\right]\left[\sum_{j=1}^n a_j\frac{d\phi_j}{dx}\right]dx$$

$$= \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n a_i\,a_j\int_0^L GJ(x)\frac{d\phi_i}{dx}\frac{d\phi_j}{dx}dx$$

$$\Rightarrow \qquad k_{ij} = \int_0^L GJ(x)\frac{d\phi_i}{dx}\frac{d\phi_j}{dx}dx$$

$$D = \frac{1}{2}\int_0^L \Theta(x)\,I(x)\,\Theta(x)\,dx = \frac{1}{2}\int_0^L I(x)\left[\sum_{i=1}^n a_i\phi_i\right]\left[\sum_{i=1}^n a_j\phi_j\right]dx$$

$$= \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n a_i a_j\int_0^L I(x)\,\phi_i\,\phi_j\,dx$$

$$\Rightarrow \qquad m_{ij} = \int_0^L I(x)\,\phi_i\,\phi_j\,dx$$

$$\xi = \frac{x}{L}, \quad \phi_1 = \xi, \quad \phi_2 = \xi^2, \quad \Theta(\xi) = a_1\,\xi + a_2\,\xi^2$$

$$GJ(\xi) = \frac{10}{9}GJ\left(1-\frac{1}{2}\xi^2\right)\ , \qquad I(\xi) = \frac{10}{9}I\left(1-\frac{1}{2}\xi^2\right)$$

$$k_{ij} = \int_0^L GJ(x)\frac{d\phi_i}{dx}\frac{d\phi_j}{dx}dx = \int_0^1 \frac{GJ(\xi)}{L}\frac{d\phi_i}{d\xi}\frac{d\phi_j}{d\xi}d\xi$$

$$k_{11} = \int_0^1 \frac{10}{9}\frac{GJ}{L}\left(1-\frac{1}{2}\xi^2\right)(1)^2 d\xi = \frac{10}{9}\frac{GJ}{L}\left[\xi-\frac{1}{6}\xi^3\right]_0^1 = \frac{25}{27}\frac{GJ}{L}$$

$$k_{22} = \int_0^1 \frac{10}{9}\frac{GJ}{L}\left(1-\frac{1}{2}\xi^2\right)(2\xi)^2 d\xi = \frac{10}{9}\frac{GJ}{L}\left[\frac{4}{3}\xi^3-\frac{2}{5}\xi^5\right]_0^1 = \frac{28}{27}\frac{GJ}{L}$$

$$k_{12} = k_{21} = \int_0^1 \frac{10}{9}\frac{GJ}{L}\left(1-\frac{1}{2}\xi^2\right)(1)(2\xi)d\xi = \frac{10}{9}\frac{GJ}{L}\left[\xi^2-\frac{1}{4}\xi^4\right]_0^1 = \frac{5}{6}\frac{GJ}{L}$$

$$m_{ij} = \int_0^1 I(\xi)L\,\phi_i\,\phi_j\,d\xi$$

$$m_{11} = \int_0^1 \frac{10}{9}IL\left(1-\frac{1}{2}\xi^2\right)(\xi)^2 d\xi = \frac{10}{9}IL\left[\frac{1}{3}\xi^3-\frac{1}{10}\xi^5\right]_0^1 = \frac{7}{27}IL$$

$$m_{22} = \int_0^1 \frac{10}{9}IL\left(1-\frac{1}{2}\xi^2\right)\left(\xi^2\right)^2 d\xi = \frac{10}{9}IL\left[\frac{1}{5}\xi^5-\frac{1}{14}\xi^7\right]_0^1 = \frac{1}{7}IL$$

$$m_{12} = m_{21} = \int_0^1 \frac{10}{9}IL\left(1-\frac{1}{2}\xi^2\right)(\xi)\left(\xi^2\right)d\xi = \frac{10}{9}IL\left[\frac{1}{4}\xi^4-\frac{1}{12}\xi^6\right]_0^1 = \frac{5}{27}IL$$

$$\left[\,k^{(2)}\,\right] = \frac{GJ}{54L}\begin{bmatrix}50 & 45\\ 45 & 56\end{bmatrix} \qquad \left[\,m^{(2)}\,\right] = \frac{IL}{189}\begin{bmatrix}49 & 35\\ 35 & 27\end{bmatrix}$$

$$[k^{(2)}]\left\{a^{(2)}\right\} = \lambda^{(2)}\,[m^{(2)}]\left\{a^{(2)}\right\}$$

$$\lambda_1^{(2)} = *.** \;\frac{GJ}{IL^2} \qquad \rightarrow \qquad \omega_1^{(2)} = \sqrt{\lambda_1^{(2)}} = *.**\sqrt{\frac{GJ}{IL^2}}$$

$$\left\{a^{(2)}\right\}_1 = \begin{Bmatrix}1\\ 0.***\end{Bmatrix} \qquad \rightarrow \qquad \left\{a^{(2)}\right\}_1 = \frac{1}{\sqrt{IL}}\begin{Bmatrix}*.**\\ *.**\end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix}*.**\\ *.**\end{Bmatrix}$$

$$\lambda_2^{(2)} = *.** \;\frac{GJ}{IL^2} \qquad \rightarrow \qquad \omega_2^{(2)} = \sqrt{\lambda_2^{(2)}} = *.**\sqrt{\frac{GJ}{IL^2}}$$

$$\left\{a^{(2)}\right\}_2 = \begin{Bmatrix}1\\ 0.***\end{Bmatrix} \qquad \rightarrow \qquad \left\{a^{(2)}\right\}_2 = \frac{1}{\sqrt{IL}}\begin{Bmatrix}*.**\\ *.**\end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix}*.**\\ *.**\end{Bmatrix}$$

$$\Theta_1^{(2)}(x) = *.** \;\frac{x}{L} + *.** \left(\frac{x}{L}\right)^2$$

$$\Theta_2^{(2)}(x) = *.** \;\frac{x}{L} + *.** \left(\frac{x}{L}\right)$$

[5] (a) Galerkin's method

procedure :

advantage :

disadvantage :

(b) collocation method

procedure :

advantage : relatively easy to evaluate the coefficients k_{ij} and m_{ij}

disadvantage : The matrices $[k]$ and $[m]$ can be nonsymmetric, and the solution of the EVP is complicated because the orthogonality is not valid.

(c) assumed-modes method

procedure :

advantage : deriving the response of a system to external forces or initial excitation in terms of only *admissible* functions.

disadvantage :

[6] (x_1, y_1) = (0, 0), (x_2, y_2) = (8, 2), (x_3, y_3) = (4, 10)

p_1 = 100 Pa, p_2 = 150 Pa, p_3 = 200 Pa

$p(x, y) = \alpha_1 + \alpha_2\, x + \alpha_3\, y$

$p(0,\ 0) = \alpha_1 = 100\ \text{Pa}$

$p(8,\ 2) = (100\ \text{Pa}) + \alpha_2\ (8\ \text{m}) + \alpha_3\ (2\ \text{m}) = (150\ \text{Pa})$

$\Rightarrow\ (8\ \text{m})\ \alpha_2 + (2\ \text{m})\ \alpha_3 = (50\ \text{Pa}) \quad \cdots$ ①

$p(4,\ 10) = (100\ \text{Pa}) + \alpha_2\ (4\ \text{m}) + \alpha_3\ (10\ \text{m}) = (200\ \text{Pa})$

$\Rightarrow\ (4\ \text{m})\ \alpha_2 + (10\ \text{m})\ \alpha_3 = (100\ \text{Pa}) \quad \cdots$ ②

①, ② $\Rightarrow\ \alpha_2 = \dfrac{25}{3}\ \text{Pa/m}\ , \quad \alpha_3 = \dfrac{25}{6}\ \text{Pa/m}$

$$p(x, y) = (100\ \text{Pa}) + (\frac{25}{3}\ \text{Pa/m})\ x + (\frac{25}{6}\ \text{Pa/m})\ y$$

$A(x,y) = (6\ \text{m},\ 4\ \text{m})$

$$p(6\ \text{m},\ 4\ \text{m}) = (100\ \text{Pa}) + (\frac{25}{3}\ \text{Pa/m})\ (6\ \text{m}) + (\frac{25}{6}\ \text{Pa/m})\ (4\ \text{m})$$

$$= 166\frac{2}{3}\ \text{Pa} = 166.7\ \text{Pa}$$