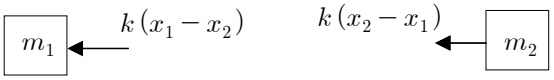
[4.1절]

4.13 자유물체도



운동방정식

$$m_1 \ddot{x}_1 = -k(x_1 - x_2) \quad \Rightarrow \quad m_1 \ddot{x}_1 + k x_1 - k x_2 = 0$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1) \quad \Rightarrow \quad m_2 \ddot{x}_2 - k x_1 + k x_2 = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

고유진동수

$$\det(-\omega_n^2 M + K) = \det \begin{bmatrix} -\omega_n^2 m_1 + k & -k \\ -k & -\omega_n^2 m_2 + k \end{bmatrix}$$

$$= (-\omega_n^2 m_1 + k)(-\omega_n^2 m_2 + k) - (-k)(-k)$$

$$= m_1 m_2 \omega_n^4 - (m_1 + m_2) k \omega_n^2 + k^2 - k^2$$

$$= m_1 m_2 \omega_n^4 - (m_1 + m_2) k \omega_n^2$$

$$= [m_1 m_2 \omega_n^2 - (m_1 + m_2) k] \omega_n^2 = 0$$

$$\Rightarrow \quad \omega_1 = 0, \quad \omega_2^2 = \frac{(m_1 + m_2)k}{m_1 m_2}$$

$m_1 = m_2 = 2{,}100$ kg, $\quad k = 270{,}000$ N/m

$$\Rightarrow \quad \omega_2^2 = \frac{(2{,}100 + 2{,}100\,\text{kg})(270{,}000\,\text{N/m})}{(2{,}100\,\text{kg})(2{,}100\,\text{kg})} = 257.1\ (\text{rad/s})^2$$

$$\Rightarrow \quad \omega_2 = 16.04\ \text{rad/s}$$

모드 형상

$$\begin{bmatrix} -\omega_n^2 m_1 + k & -k \\ -k & -\omega_n^2 m_2 + k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \quad (-\omega_n^2 m_1 + k) u_1 - k u_2 = 0$$

$$\Rightarrow \quad u_2 = \frac{-\omega_n^2 m_1 + k}{k} u_1$$

$\omega_n = \omega_1 = 0$ 일 때,

$$u_2 = u_1 \quad \Rightarrow \quad \mathbf{u}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$\omega_n = \omega_2 = 16.04$ rad/s 일 때,

$$u_2 = \frac{-(257.1\,\text{rad}^2/\text{s}^2)(2{,}100\,\text{kg}) + (270{,}000\,\text{N/m})}{270{,}000\,\text{N/m}} u_1 = (-1)u_1 \quad \Rightarrow \quad \mathbf{u}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$