[3.2절]

3.28 사각하중 응답 (유한 시간 계단함수 하중에 대한 응답)

F_0 = 30 N, $t_1 = \dfrac{\pi}{\omega_n}$, k = 1,000 N/m, ζ = 0.1, ω_n = 3.16 rad/s

$\zeta\omega_n$ = (0.1) (3.16 rad/s) = 0.316 rad/s, $\omega_d = \sqrt{1-0.1^2}$ (3.16 rad/s) = 3.144 rad/s

$$t_1 = \frac{\pi\,\text{rad}}{3.16\,\text{rad/s}} = 0.994 \text{ s}$$

$0 \le t \le 0.994$ s 일 때,

$$x(t) = \frac{F_0}{k}\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \phi)\right]$$

$$\phi = \tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}} = \tan^{-1}\frac{0.1}{\sqrt{1-0.1^2}} = 0.1002 \text{ rad}$$

$$\frac{F_0}{k} = \frac{30\text{ N}}{1{,}000\text{ N/m}} = 0.03\text{ m}, \quad \frac{1}{\sqrt{1-0.1^2}} = 1.005$$

$\Rightarrow$ $x(t)$ = 0.0300 [1 - 1.005 $e^{-0.316t}$ cos(3.14 t - 0.1002)] (m)

$t > 0.994$ s 일 때,

[방법1] 예제 3.2.2 교재의 풀이 방법

$$x(t) = \frac{F_0}{k}\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \phi)\right] - \frac{F_0}{k}\left\{1 - \frac{e^{-\zeta\omega_n (t-t_1)}}{\sqrt{1-\zeta^2}}\cos[\omega_d (t-t_1) - \phi]\right\}$$

$$= \frac{F_0}{k\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\left\{e^{\zeta\omega_n t_1}\cos[\omega_d(t-t_1) - \phi] - \cos(\omega_d t - \phi)\right\}$$

$$\frac{F_0}{k\sqrt{1-\zeta^2}} = \frac{(30\text{ N})(1.005)}{1000\text{ N/m}} = 0.03015\text{ m}$$

$\zeta\omega_n t_1$ = (0.316 rad/s) (0.994 s) = 0.314 rad

$e^{\zeta\omega_n t_1} = e^{0.314}$ = 1.369

$\omega_d(t - t_1) - \phi$ = (3.144 rad/s) [t - (0.994 s)] - (0.1002 rad)

= (3.144 rad/s) t - (3.225 rad)

$\Rightarrow$ $x(t)$ = 0.0302 $e^{-0.316t}$ [1.369 cos(3.14 t - 3.23)

- cos(3.14 t - 0.1002)] m

[방법2] 예제 3.2.2 노트의 풀이 방법

$F(t-\tau) = F_0$ $0 < t-\tau < t_1$ $\Rightarrow$ $-t < -\tau < t_1 - t$ $\Rightarrow$ $t > \tau > t - t_1$

$$x(t) = \frac{1}{m\omega_d}\int_{t-t_1}^{t} F_0\, e^{-\zeta\omega_n \tau}\sin\omega_d \tau\, d\tau = \cdots$$

또는 $x(t) = \dfrac{1}{m\omega_d}\displaystyle\int_0^{t_1} F_0\, e^{-\zeta\omega_n (t-\tau)}\sin\omega_d (t-\tau)\, d\tau = \cdots$