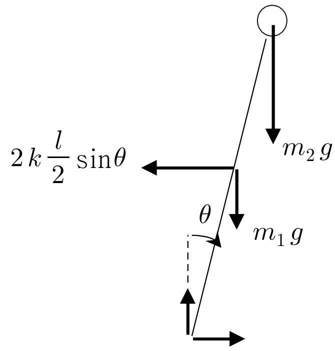
[1.8절]

1.113 <방법 1> Euler's 2nd law

F.B.D.



$$\Sigma M_O = J\ddot{\theta}$$

$$m_1 g\left(\frac{l}{2}\sin\theta\right) + m_2 g\,(l\sin\theta) - 2k\frac{l}{2}\sin\theta\left(\frac{l}{2}\cos\theta\right)$$

$$= \left(\frac{1}{3}m_1 l^2 + m_2 l^2\right)\ddot{\theta}$$

$$\Rightarrow \left(\frac{1}{2}m_1 + m_2\right) g\,l\sin\theta - \frac{1}{2}k\,l^2\sin\theta\cos\theta = \left(\frac{1}{3}m_1 + m_2\right) l^2\ddot{\theta}$$

$\theta \approx 0$ 일 때, $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2} \approx 1$

$$\Rightarrow \left(\frac{1}{2}m_1 + m_2\right) g\,l\,\theta - \frac{1}{2}k\,l^2\,\theta = \left(\frac{1}{3}m_1 + m_2\right) l^2\ddot{\theta}$$

$$\Rightarrow \left(\frac{1}{3}m_1 + m_2\right) l\,\ddot{\theta} + \left[\frac{1}{2}k\,l - \left(\frac{1}{2}m_1 + m_2\right) g\right]\theta = 0$$

<방법 2> energy method

$$T = \frac{1}{2}J_1\dot{\theta}^2 + \frac{1}{2}J_2\dot{\theta}^2 = \frac{1}{2}\left(\frac{1}{3}m_1 l^2\right)\dot{\theta}^2 + \frac{1}{2}\left(m_2 l^2\right)\dot{\theta}^2 = \frac{1}{2}\left(\frac{1}{3}m_1 + m_2\right) l^2\dot{\theta}^2$$

$$U = 2\left[\frac{1}{2}k\left(\frac{l}{2}\theta\right)^2\right] - m_1 g\frac{l}{2}(1-\cos\theta) - m_2 g\,l\,(1-\cos\theta)$$

$$= \frac{1}{4}k\,l^2\theta^2 - \left(\frac{1}{2}m_1 + m_2\right) g\,l\,(1-\cos\theta)$$

$$\frac{d}{dt}(T+U) = \frac{d}{dt}\left[\frac{1}{2}\left(\frac{1}{3}m_1 + m_2\right) l^2\dot{\theta}^2 + \frac{1}{4}k\,l^2\theta^2 - \left(\frac{1}{2}m_1 + m_2\right) g\,l\,(1-\cos\theta)\right] = 0$$

$$\Rightarrow \left(\frac{1}{3}m_1 + m_2\right) l^2\dot{\theta}\ddot{\theta} + \frac{1}{2}k\,l^2\theta\dot{\theta} - \left(\frac{1}{2}m_1 + m_2\right) g\,l\sin\theta\,\dot{\theta} = 0$$

$$\Rightarrow \left(\frac{1}{3}m_1 + m_2\right) l^2\ddot{\theta} + \frac{1}{2}k\,l^2\theta - \left(\frac{1}{2}m_1 + m_2\right) g\,l\sin\theta = 0$$

$\theta \approx 0$ 이면, $\sin\theta \approx \theta$

$$\Rightarrow \left(\frac{1}{3}m_1 + m_2\right) l^2\ddot{\theta} + \frac{1}{2}k\,l^2\theta - \left(\frac{1}{2}m_1 + m_2\right) g\,l\,\theta = 0$$

$$\Rightarrow \left(\frac{1}{3}m_1 + m_2\right) l\,\ddot{\theta} + \left[\frac{1}{2}k\,l - \left(\frac{1}{2}m_1 + m_2\right) g\right]\theta = 0$$

안정성 검토

$\frac{1}{2}k\,l - \left(\frac{1}{2}m_1 + m_2\right) g > 0$ 이면,

$$\ddot{\theta} + \omega_n^2\theta = 0 \qquad \omega_n = \sqrt{\frac{\frac{1}{2}k\,l - \left(\frac{1}{2}m_1 + m_2\right) g}{\left(\frac{1}{3}m_1 + m_2\right) l}}$$

$\theta(t) = a_1 e^{j\omega_n t} + a_2 e^{-j\omega_n t} = A_1 \sin\omega_n t + A_2 \cos\omega_n t$ 진동 ⇒ 안정(stable)

$\frac{1}{2}k\,l - \left(\frac{1}{2}m_1 + m_2\right) g = 0$ 이면,

$\ddot{\theta} = 0 \Rightarrow \theta(t) = a_1 + a_2 t$ 비진동 증폭 ⇒ 불안정(unstable)

$\frac{1}{2}k\,l - \left(\frac{1}{2}m_1 + m_2\right)g < 0$ 이면,

$$\ddot{\theta} - \lambda^2\,\theta = 0 \qquad \lambda = \sqrt{\frac{\left(\frac{1}{2}m_1 + m_2\right)g - \frac{1}{2}\,k\,l}{\left(\frac{1}{3}m_1 + m_2\right)l}}$$

$$\theta(t) = a_1\,e^{\lambda t} + a_2\,e^{-\lambda t}$$

비진동 증폭 ⇒ 불안정(unstable)