

[1.4절]

1.80    statics     $J = \frac{1}{2} m r^2$ ,    kinematics     $x = r \theta$ ,     $\dot{x} = r \dot{\theta}$

spring     $\Delta l = (r + a) \theta$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 = \frac{1}{2} m (r \dot{\theta})^2 + \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \dot{\theta}^2 = \frac{3}{4} m r^2 \dot{\theta}^2$$

$$U = 2 \left[ \frac{1}{2} k (\Delta l)^2 \right] = k [(r + a) \theta]^2 = k (r + a)^2 \theta^2$$

$$\frac{d}{dt}(T + U) = \frac{d}{dt} \left[ \frac{3}{4} m r^2 \dot{\theta}^2 + k (r + a)^2 \theta^2 \right]$$

$$= \left[ \frac{3}{2} m r^2 \ddot{\theta} + 2 k (r + a)^2 \theta \right] \dot{\theta} = 0, \quad \dot{\theta} \neq 0$$

equation of motion     $\frac{3}{2} m r^2 \ddot{\theta} + 2 k (r + a)^2 \theta = 0$ ,

natural frequency     $\omega_n = \sqrt{\frac{4 k (a + r)^2}{3 m r^2}} = 2 \frac{a + r}{r} \sqrt{\frac{k}{3 m}}$