[1.3절]

1.54 ω_n = 2 rad/s, ζ = 0.1, v_0 = 0이고, 초기변위 x_0 = 10, 100 mm에 대해 $x(t)$ plot.

$x(t) = A\ e^{-\zeta\omega_n t}\ \sin(\omega_d t + \phi), \quad \dot{x}(t) = A\ e^{-\zeta\omega_n t}\ [-\zeta\omega_n \sin(\omega_d t + \phi) + \omega_d \cos(\omega_d t + \phi)]$

$x(0) = A\ \sin\phi = x_0 \ > 0 \ \cdots \ ①$

$\dot{x}(0) = A\ (-\zeta\omega_n\ \sin\phi + \omega_d\ \cos\phi) = v_0 = 0$

$$\Rightarrow \quad A\ \cos\phi = \frac{\zeta\omega_n x_0}{\omega_d} = \frac{\zeta}{\sqrt{1-\zeta^2}}\ x_0 = \frac{0.1}{\sqrt{1-0.1^2}}\ x_0 = 0.1005\ x_0 > 0 \ \cdots \ ②$$

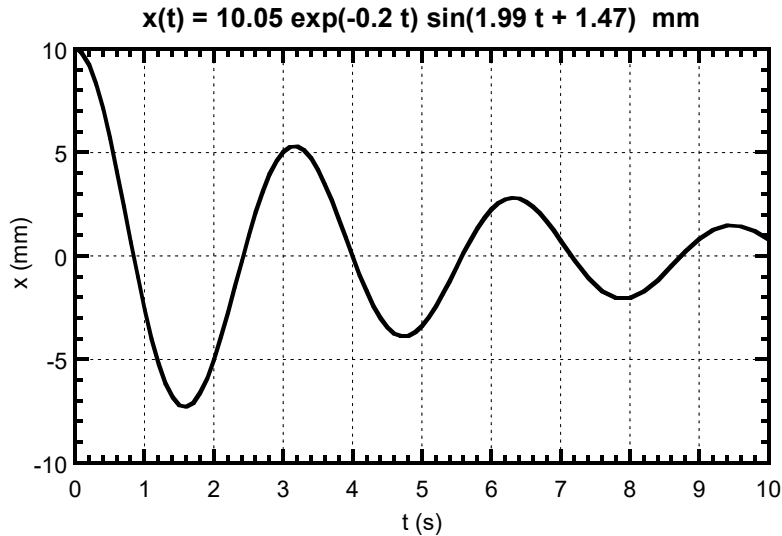$①^2 + ②^2 \quad \Rightarrow \quad A = \sqrt{1^2 + 0.1005^2}\ x_0 = 1.005\ x_0$

$① \div ② \quad \Rightarrow \quad \phi = \tan^{-1}\frac{1}{0.1005} = \tan^{-1}(9.95) = 84.2° = 1.471\ \text{rad}$

$\zeta\omega_n = (0.1)\,(2\ \text{rad/s}) = 0.2\ \text{rad/s}, \quad \omega_d = \sqrt{1-0.1^2}\,(2\ \text{rad/s}) = 1.990\ \text{rad/s}$

$x(t) = 1.005\ x_0\ e^{-0.2\,t}\ \sin(1.990\,t + 1.471)$

x_0 = 10 mm일 때, $x(t) = 1.005\ (10\ \text{mm})\ e^{-0.2\,t}\ \sin(1.990\,t + 1.471)$

$\qquad\qquad\qquad\qquad = 10.05\ e^{-0.2\,t}\ \sin(1.990\,t + 1.471) \quad \text{mm}$

x(t) = 10.05 exp(-0.2 t) sin(1.99 t + 1.47) mm

x_0 = 100 mm일 때, $x(t) = 1.005\ (100\ \text{mm})\ e^{-0.2\,t}\ \sin(1.990\,t + 1.471)$

$\qquad\qquad\qquad\qquad = 100.5\ e^{-0.2\,t}\ \sin(1.990\,t + 1.471) \quad \text{mm}$

x(t) = 100.5 exp(-0.2 t) sin(1.99 t + 1.47) mm