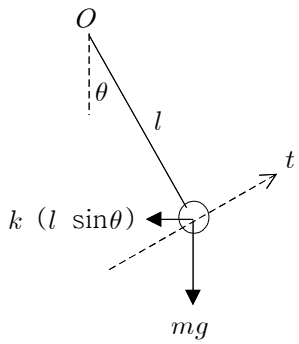
[1.2절]

1.21 [뉴튼 법칙 사용]

<방법 1: 오일러 법칙>



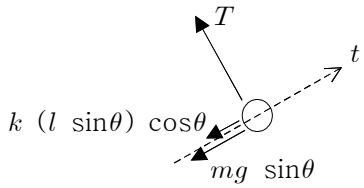
$\Sigma M_O = J\ddot{\theta}$ (질량관성모멘트 $J = m l^2$)

$\Rightarrow \; -mg\,(l\sin\theta) - k\,(l\sin\theta)(l\cos\theta) = (m l^2)\,\ddot{\theta}$

$\Rightarrow \; m l^2\,\ddot{\theta} + (mg + kl\cos\theta)\, l\sin\theta = 0\,, \qquad l \neq 0,$

$\Rightarrow \; ml\,\ddot{\theta} + (mg + kl\cos\theta)\sin\theta = 0$

<방법 2>



$\Sigma F_t = m a_t$ (가속도 $a_t = l\ddot{\theta}$)

$\Rightarrow \; -mg\sin\theta - k\,(l\sin\theta)\cos\theta = m\,(l\ddot{\theta})$

$\Rightarrow \; ml\,\ddot{\theta} + (mg + kl\cos\theta)\sin\theta = 0$

$\theta \simeq 0$ 이면 $\sin\theta \simeq \theta, \;\; \cos\theta \simeq 1$

$\Rightarrow \; ml\ddot{\theta} + (mg + kl)\,\theta = 0$

$\Rightarrow \; \ddot{\theta} + (\frac{g}{l} + \frac{k}{m})\,\theta = 0$ 고유진동수 $\omega_n = \sqrt{\frac{g}{l} + \frac{k}{m}}$

[에너지방법]

운동에너지 $T = \frac{1}{2} m v^2 = \frac{1}{2} m\,(l\dot{\theta})^2 = \frac{1}{2} m l^2\,\dot{\theta}^2$

위치에너지 $U = mgl\,(1 - \cos\theta) + \frac{1}{2} k\,(l\,\theta)^2$

$\frac{d}{dt}(T + U) = \frac{d}{dt}[\frac{1}{2} m l^2\,\dot{\theta}^2 + mgl\,(1 - \cos\theta) + \frac{1}{2} k l^2\,\theta^2] = 0$

$\Rightarrow \; m l^2\,\dot{\theta}\,\ddot{\theta} + mgl\sin\theta\;\dot{\theta} + k l^2\,\theta\,\dot{\theta} = 0,$

$\theta \simeq 0$ 이면 $\sin\theta \simeq \theta$

$[m l\,(l\ddot{\theta} + g\,\theta) + k l^2\,\theta]\;\dot{\theta} = 0 \quad \Rightarrow \quad ml\,\ddot{\theta} + (mg + kl)\,\theta = 0$

$\Rightarrow \; \ddot{\theta} + (\frac{g}{l} + \frac{k}{m})\,\theta = 0$

고유진동수 $\omega_n = \sqrt{\frac{g}{l} + \frac{k}{m}}$