

&lt;9.1~9.5 &gt;

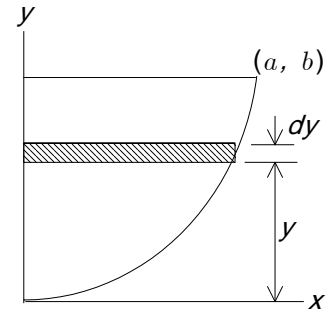
9.1  $y = kx^{5/2}$

$$x = a, y = b, \quad k = \frac{b}{a^{5/2}}$$

$$y = \frac{b}{a^{5/2}} x^{5/2} \quad x = \frac{a}{b^{2/5}} y^{2/5}$$

$$dI_y = \frac{1}{3} (dy) x^3 = \frac{1}{3} \left( \frac{a}{b^{2/5}} y^{2/5} \right)^3 dy = \frac{1}{3} \frac{a^3}{b^{6/5}} y^{6/5} dy$$

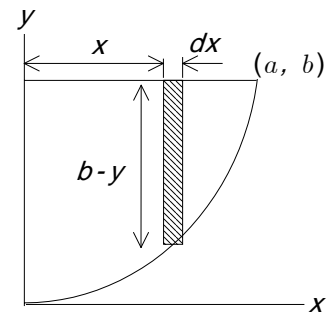
$$I_y = \frac{1}{3} \frac{a^3}{b^{6/5}} \int_0^b y^{6/5} dy = \frac{1}{3} \frac{5}{11} \frac{a^3}{b^{6/5}} y^{11/5} \Big|_0^b = \frac{5}{33} \frac{a^3}{b^{6/5}} b^{11/5} \\ = \frac{5}{33} a^3 b$$



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$$dI_y = x^2 (b - y) dx = x^2 \left( b - \frac{b}{a^{5/2}} x^{5/2} \right) dx$$

$$I_y = b \int_0^a \left( x^2 - \frac{1}{a^{5/2}} x^{9/2} \right) dx = b \left[ \frac{1}{3} x^3 - \frac{2}{11} \frac{1}{a^{5/2}} x^{11/2} \right]_0^a \\ = b \left( \frac{1}{3} a^3 - \frac{2}{11} \frac{1}{a^{5/2}} a^{11/2} \right) = \frac{5}{33} a^3 b$$



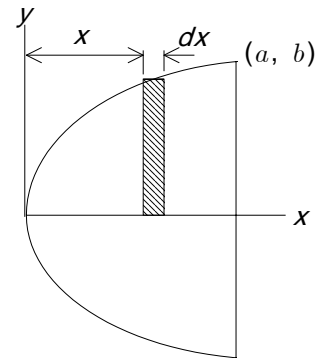
9.9  $y = kx^{1/2}$

$$x = a, y = b, \quad k = \frac{b}{a^{1/2}}$$

$$y = \frac{b}{a^{1/2}} x^{1/2} \quad x = \frac{a}{b^2} y^2$$

$$dI_x = \frac{1}{3} (dx) y^3 = \frac{1}{3} \left( \frac{b}{a^{1/2}} x^{1/2} \right)^3 dx = \frac{1}{3} \frac{b^3}{a^{3/2}} x^{3/2} dx$$

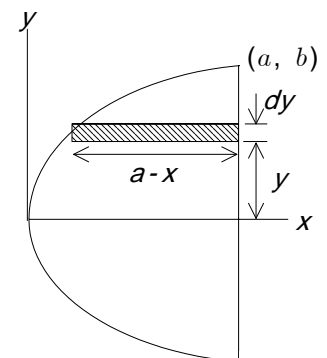
$$I_x = 2 \int_0^a dI_x = \frac{2}{3} \frac{b^3}{a^{3/2}} \int_0^a x^{3/2} dx = \frac{2}{3} \frac{2}{5} \frac{b^3}{a^{3/2}} x^{5/2} \Big|_0^a \\ = \frac{4}{15} \frac{b^3}{a^{3/2}} a^{5/2} = \frac{4}{15} ab^3$$



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$$dI_x = y^2 (a - x) dy = y^2 \left( a - \frac{a}{b^2} y^2 \right) dy$$

$$I_x = 2 \int_0^b y^2 \left( a - \frac{a}{b^2} y^2 \right) dy = 2a \left[ \frac{1}{3} y^3 - \frac{1}{5} \frac{1}{b^2} y^5 \right]_0^b \\ = 2a \left[ \frac{1}{3} b^3 - \frac{1}{5} \frac{1}{b^2} b^5 \right] = \frac{4}{15} ab^3$$



$$9.17 \quad y_1 = m_1 x^3, \quad y_2 = m_2 x$$

$$x = a, \quad y = b \quad , \quad m_1 = \frac{b}{a^3}, \quad m_2 = \frac{b}{a}$$

$$y_1 = \frac{b}{a^3} x^3, \quad y_2 = \frac{b}{a} x$$

$$dA = (y_2 - y_1) dx = \left( \frac{b}{a} x - \frac{b}{a^3} x^3 \right) dx$$

$$\begin{aligned} A &= 2 \int_0^a dA = 2 \int_0^a \left( \frac{b}{a} x - \frac{b}{a^3} x^3 \right) dx = 2 \left[ \frac{1}{2} \frac{b}{a} x^2 - \frac{1}{4} \frac{b}{a^3} x^4 \right]_0^a \\ &= 2 \left( \frac{1}{2} \frac{b}{a} a^2 - \frac{1}{4} \frac{b}{a^3} a^4 \right) = \frac{1}{2} ab \end{aligned}$$

$$dI_y = x^2 dA = x^2 \left( \frac{b}{a} x - \frac{b}{a^3} x^3 \right) dx = \frac{b}{a} \left( x^3 - \frac{1}{a^2} x^5 \right) dx$$

$$\begin{aligned} I_y &= 2 \int_0^a dI_y = 2 \frac{b}{a} \int_0^a \left( x^3 - \frac{1}{a^2} x^5 \right) dx = 2 \frac{b}{a} \left[ \frac{1}{4} x^4 - \frac{1}{6} \frac{1}{a^2} x^6 \right]_0^a \\ &= 2 \frac{b}{a} \left( \frac{1}{4} a^4 - \frac{1}{6} \frac{1}{a^2} a^6 \right) = \frac{1}{6} a^3 b \end{aligned}$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{1}{6} a^3 b}{\frac{1}{2} ab}} = \frac{a}{\sqrt{3}}$$