<5.10~5.11절>

5.93 ① $V = L\ a\ b\ ,\quad \bar{x} = \frac{1}{2}L$

② $V = \frac{1}{3}\left(\frac{b}{2}\right) a\ h = \frac{1}{6}\ a\ b\ h\ ,\quad \bar{x} = L + \frac{1}{4}\ h$

$\Sigma V = L\ a\ b + \frac{1}{6}\ a\ b\ h = a\ b\ (L + \frac{1}{6}h)$

$\Sigma(\bar{x}\,V) = (\frac{1}{2}L)(L\ a\ b) + (L + \frac{1}{4}\ h)(\frac{1}{6}\ a\ b\ h) = \frac{1}{6}a\ b\ (3L^2 + h\ L + \frac{1}{4}h^2)$

(a) $h = \frac{L}{2}$

$\Sigma V = a\ b\ (L + \frac{1}{6}\frac{L}{2}) = \frac{13}{12}\ a\ b\ L$

$\Sigma(\bar{x}\,V) = \frac{1}{6}a\ b\ [3L^2 + \frac{L}{2}\ L + \frac{1}{4}(\frac{L}{2})^2] = \frac{57}{96}\ a\ b\ L^2$

$\bar{X} = \frac{\Sigma(\bar{x}\,V)}{\Sigma V} = \frac{\frac{57}{96}abL^2}{\frac{13}{12}\,a\,b\,L} = \frac{57}{104}\ L = 0.548\ L$

(b) $\bar{X} = L$

$\bar{X}\ \Sigma V = \Sigma(\bar{x}\,V)$

$L\ a\ b\ (L + \frac{1}{6}h) = \frac{1}{6}a\ b\ (3L^2 + h\ L + \frac{1}{4}h^2)$

$1 + \frac{1}{6}\left(\frac{h}{L}\right) = \frac{1}{2} + \frac{1}{6}\left(\frac{h}{L}\right) + \frac{1}{24}\left(\frac{h}{L}\right)^2$

$\left(\frac{h}{L}\right)^2 = 12 \qquad \Rightarrow \qquad \frac{h}{L} = 2\sqrt{3} = 3.46$

5.96 ① $V = (120 + 40\ \text{mm})(54\ \text{mm})(18\ \text{mm}) = 155{,}520\ \text{mm}^3$

$\bar{x} = \frac{1}{2}(120 + 40\ \text{mm}) = 80\ \text{mm}$

② $V = \frac{1}{2}(120\ \text{mm})(60 - 18\ \text{mm})(54\ \text{mm}) = 136{,}080\ \text{mm}^3$

$\bar{x} = \frac{1}{3}(120\ \text{mm}) = 40\ \text{mm}$

③ $V = \frac{1}{2}\pi\ (27\ \text{mm})^2\ (18\ \text{mm}) = 20{,}612\ \text{mm}^3$

$\bar{x} = (160\ \text{mm}) + \frac{4}{3\pi}(27\ \text{mm}) = 171.46\ \text{mm}$

④ $V = -\pi\ (16\ \text{mm})^2\ (18\ \text{mm}) = -14{,}476\ \text{mm}^3$

$\bar{x} = 160\ \text{mm}$

$\Sigma V = 155{,}520 + 136{,}080 + 20{,}612 - 14{,}476\ (\text{mm}^3)$

$= 297{,}736\ \text{mm}^3$

$\Sigma(\bar{x}\,V) = (80)(155{,}520) + (40)(136{,}080) + (171.46)(20{,}612) + (160)(-14{,}476)\ (\text{mm}^4)$

$= 19{,}102{,}774\ \text{mm}^4$

$\bar{X} = \frac{\Sigma(\bar{x}\,V)}{\Sigma V} = \frac{19{,}102{,}774\ \text{mm}^4}{297{,}736\ \text{mm}^3} = 64.2\ \text{mm}$

5.100 ① $A = \frac{1}{2}(90\text{ mm})(60\text{ mm}) = 2{,}700\text{ mm}^2$

$\bar{x} = \frac{1}{3}(90\text{ mm}) = 30\text{ mm}$

$\bar{y} = (120\text{ mm}) + \frac{1}{3}(60\text{ mm}) = 140\text{ mm}$

$\bar{z} = 0$

② $A = (90\text{ mm})](200\text{ mm}) = 18{,}000\text{ mm}^2$

$\bar{x} = 45\text{ mm}$

$\bar{y} = \frac{1}{2}(120\text{ mm}) = 60\text{ mm}$

$\bar{z} = \frac{1}{2}(160\text{ mm}) = 80\text{ mm}$

③ $A = -(45\text{ mm})(100\text{ mm}) = -4{,}500\text{ mm}^2$

$\bar{x} = \frac{1}{2}(45\text{ mm}) = 22.5\text{ mm}$

$\bar{y} = \frac{1}{2}(60\text{ mm}) = 30\text{ mm}$

$\bar{z} = (160\text{ mm}) - \frac{1}{2}(80\text{ mm}) = 120\text{ mm}$

④ $A = \frac{1}{2}[\pi(45\text{ mm})^2] = 3{,}181\text{ mm}^2$

$\bar{x} = 45\text{ mm}$

$\bar{y} = 0$

$\bar{z} = (160\text{ mm}) + \frac{4}{3\pi}(45\text{ mm}) = 179.1\text{ mm}$

$\Sigma A = 2{,}700 + 18{,}000 + (-4{,}500) + 3{,}181 = 19{,}381\text{ mm}^2$

$\Sigma(\bar{x}A) = (30)(2{,}700) + (45)(18{,}000) + (22.5)(-4{,}500) + (45)(3{,}181)$
$= 932{,}895\text{ mm}^3$

$\Sigma(\bar{y}A) = (140)(2{,}700) + (60)(18{,}000) + (30)(-4{,}500) + (0)(3{,}181)$
$= 1{,}323{,}000\text{ mm}^3$

$\Sigma(\bar{z}A) = (0)(2{,}700) + (80)(18{,}000) + (120)(-4{,}500) + (179.1)(3{,}181)$
$= 1{,}469{,}717\text{ mm}^3$

$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{932{,}895\text{ mm}^3}{19{,}381\text{ mm}^2} = 48.1\text{ mm}$

$\bar{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{1{,}323{,}000\text{ mm}^3}{19{,}381\text{ mm}^2} = 68.3\text{ mm}$

$\bar{Z} = \frac{\Sigma(\bar{z}A)}{\Sigma A} = \frac{1{,}469{,}717\text{ mm}^3}{19{,}381\text{ mm}^2} = 75.8\text{ mm}$