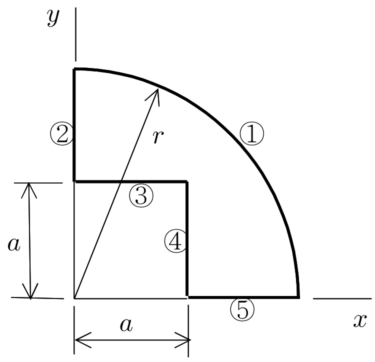
[5.1절]



5.26 r = 10 cm, a = 5 cm, 대칭 $\overline{X} = \overline{Y}$

① 1/4 원호

$$L = \frac{1}{4}(2\pi r) = \frac{1}{2}\pi\ (10\ \text{cm}) = 15.708\ \text{cm}$$

$$\overline{x} = \frac{2}{\pi}r = \frac{2}{\pi}\ (10\ \text{cm}) = 6.366\ \text{cm}$$

$$\overline{x}L = (\frac{2}{\pi}r)(\frac{\pi}{2}r) = r^2$$

② $L = r - a$ = (10 cm) - (5 cm) = 5 cm

$\overline{x} = 0$

③ $L = a$ = 5 cm

$$\overline{x} = \frac{1}{2}a = \frac{1}{2}\ (5\ \text{cm}) = 2.5\ \text{cm}$$

④ L = 5 cm

$\overline{x} = a$ = 5 cm

⑤ L = 5 cm

$$\overline{x} = \frac{1}{2}(r - a) + a = \frac{1}{2}(r + a) = \frac{1}{2}[(10\ \text{cm})+(5\ \text{cm})] = 7.5\ \text{cm}$$

ΣL = 15.708 + 5 + 5 + 5 + 5 cm = 35.708 cm

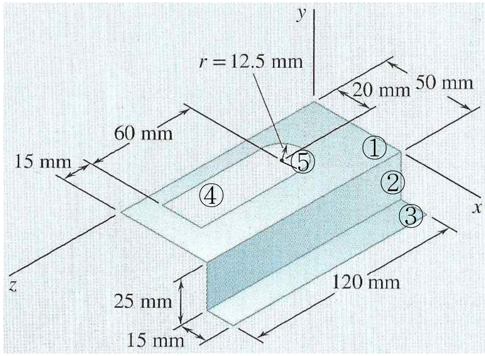
$\Sigma(\overline{x}L) = (10)^2 + (5)(0) + (5)(2.5) + (5)(5) + (5)(7.5)\ \text{cm}^2 = 175\ \text{cm}^2$

$$\overline{X} = \frac{\Sigma(\overline{x}L)}{\Sigma L} = \frac{175\ \text{cm}^2}{35.708\ \text{cm}} = 4.901\ \text{cm}$$ ⇒ 무게중심 = (4.90 cm, 4.90 cm)

[5.4절]

5.112 ① 직사각형 50×120, ② 직사각형 25×120, ③ 직사각형 15×120

④ 직사각형 구멍, ⑤ 반원 구멍



① A = (50 mm)(120 mm) = 6 000 mm^2

$\bar{x}$ = $\frac{1}{2}$(50 mm) = 25 mm

$\bar{y}$ = 0

$\bar{z}$ = $\frac{1}{2}$(120 mm) = 60 mm

② A = (25 mm)(120 mm) = 3 000 mm^2

$\bar{x}$ = 50 mm

$\bar{y}$ = $-\frac{1}{2}$(25 mm) = -12.5 mm

$\bar{z}$ = ①$\bar{z}$ = 60 mm

③ A = (15 mm)(120 mm) = 1 800 mm^2

$\bar{x}$ = 50 mm + $\frac{1}{2}$(15 mm) = 57.5 mm

$\bar{y}$ = -25 mm

$\bar{z}$ = ①$\bar{z}$ = 60 mm

④ A = -[2(12.5 mm)](60 mm) = -1 500 mm^2

$\bar{x}$ = 20 mm

$\bar{y}$ = 0

$\bar{z}$ = (120 mm) - (15 mm) - $\frac{1}{2}$(60 mm) = 75 mm

⑤ A = $-\frac{1}{2}\pi$(12.5 mm)^2 = -245.4 mm^2

$\bar{x}$ = 20 mm

$\bar{y}$ = 0

$\bar{z}$ = (120 mm) - (15 mm) - (60 mm) - $\frac{4}{3\pi}$(12.5 mm) = 39.70 mm

ΣA = 6,000 + 3,000 + 1,800 + (-1,500) + (-245.4) = 9,054.6 mm^2

$\Sigma(\bar{x}A)$ = (25)(6,000) + (50)(3,000) + (57.5)(1,800) + (20)(-1,500) + (20)(-245.4)
= 368,600 mm^3

$\Sigma(\bar{y}A)$ = (25)(6,000) + (-12.5)(3,000) + (-25)(1,800) + (0)(-1,500) + (0)(-245.4)
= -82,500 mm^3

$\Sigma(\bar{z}A)$ = (60)(6,000) + (60)(3,000) + (60)(1,800) + (75)(-1,500) + (39.70)(-245.4)
= 525,757 mm^3

$\overline{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{368,600\,\mathrm{mm}^3}{9,054.6\,\mathrm{mm}^2}$ = 40.7 mm

$\overline{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{-82,500\,\mathrm{mm}^3}{9,054.6\,\mathrm{mm}^2}$ = -9.11 mm

$\overline{Z} = \frac{\Sigma(\bar{z}A)}{\Sigma A} = \frac{525,757\,\mathrm{mm}^3}{9,054.6\,\mathrm{mm}^2}$ = 58.1 mm

중심 = (40.7 mm, -9.11 mm, 58.1 mm)

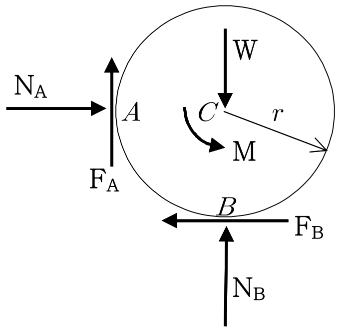
[8.1절]

8.21 S; known $W,\ r,\ \mu_A,\ \mu_B$

unknown largest M

⇒ 2차원 평형방정식, 최대 정지마찰력

M;



A; $F_A = \mu_A N_A,\ F_B = \mu_B N_B$

$\rightarrow \Sigma F_x = 0\ ;\quad N_A - F_B = 0$

$\Rightarrow\quad N_A = F_B = \mu_B N_B$

$F_A = \mu_A N_A = \mu_A \mu_B N_B$

$\uparrow \Sigma F_y = 0\ ;\quad N_B + F_A - W = 0$

$\Rightarrow\quad N_B + \mu_A \mu_B N_B - W = 0 \quad\Rightarrow\quad (1+\mu_A \mu_B) N_B - W = 0$

$\Rightarrow\quad N_B = \dfrac{1}{1+\mu_A \mu_B} W,\quad F_B = \dfrac{\mu_B}{1+\mu_A \mu_B} W,\quad F_A = \dfrac{\mu_A \mu_B}{1+\mu_A \mu_B} W$

$\circlearrowright \Sigma M_C = 0\ ;\quad M - r(F_A + F_B) = 0$

$\Rightarrow\quad M = r(F_A + F_B) = \dfrac{(1+\mu_A)\mu_B}{1+\mu_A \mu_B} rW$

(a) $\mu_A = 0,\ \mu_B = 0.30$

$\dfrac{(1+0)(0.30)}{1+(0)(0.30)} = 0.300 \qquad\Rightarrow\qquad M = 0.300\ rW$

(b) $\mu_A = 0.25,\ \mu_B = 0.30$

$\dfrac{(1+0.25)(0.30)}{1+(0.25)(0.30)} = \dfrac{0.375}{1.075} = 0.349 \qquad\Rightarrow\qquad M = 0.349\ rW$

R; (과정의 타당성) (가령, 모멘트 식 보다 힘 평형식을 먼저 사용한 이유를 서술)

T; (결과의 의미) (가령, (a)의 M 보다 (b)의 M이 더 큰 이유를 서술)

8.10 W = 600 N, α = 35°, μ_s = 0.25, μ_k = 0.20, θ = 60°

S; known W, α, μ_s, μ_k, unknown P, θ $\Rightarrow$ 마찰각 ϕ_s, ϕ_k, 힘 삼각형, 삼각법

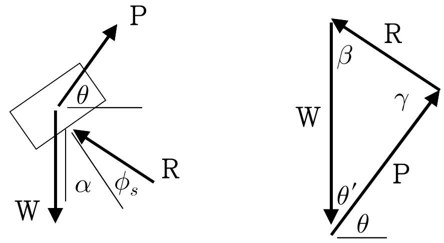
A; 마찰각 $\phi_s = \tan^{-1}(0.25) = 14.04°$

$\phi_k = \tan^{-1}(0.20) = 11.31°$

smallest P $\Rightarrow$ $\gamma = 90°$

M;

F.B.D. force triangle



(a) 미끄러져 올라가려 할 때

$\beta = \alpha + \phi_s = 35° + 14.04° = 49.04°$

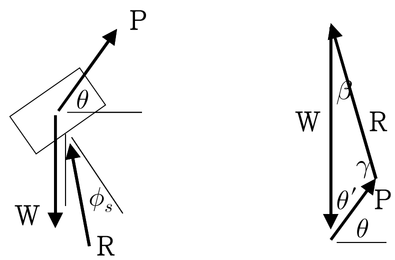
$\theta' = 180° - \gamma - \beta$

$= 180° - 90° - 49.04° = 40.96°$

$\theta = 90° - \theta' = \beta = 49.04°$

$P = W \sin\beta = (600\text{ N}) \sin 49.04° = 453\text{ N}$ $\Rightarrow$ P = 453 N ⦣49.0°

(b) 미끄러져 내려가려 할 때



$\beta = \alpha - \phi_s = 35° - 14.04° = 20.96°$

$\theta' = 180° - \gamma - \beta$

$= 180° - 90° - 20.96° = 69.04°$

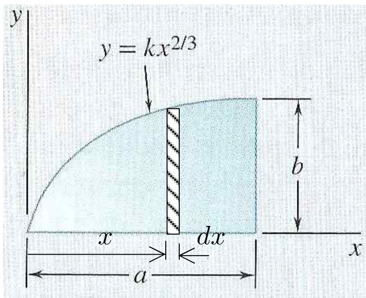
$\theta = 90° - \theta' = \beta = 20.96°$

$P = W \sin\beta = (600\text{ N}) \sin 20.96° = 215\text{ N}$ $\Rightarrow$ P = 215 N ⦣21.0°

R;(과정의 타당성) (가령, 올라가려 할 때와 내려가려 할 때 마찰각 방향)

T;(결과의 의미) (가령, 올라가려 할 때 P가 내려가려 할 때 P 보다 큼)

[9.1절]



9.15&17 $y = k\,x^{2/3}, \quad (a,\ b) \ \Rightarrow \ b = k\,a^{2/3}$

$$\Rightarrow \quad k = \frac{b}{a^{2/3}}, \qquad y = \frac{b}{a^{2/3}}\,x^{2/3}$$

$$dA = y\,dx = \frac{b}{a^{2/3}}\,x^{2/3}\ dx$$

$$A = \int dA = \int_0^a \frac{b}{a^{2/3}}\,x^{2/3}\,dx$$

$$= \frac{b}{a^{2/3}}\left[\frac{3}{5}\,x^{5/3}\right]_0^a = \frac{b}{a^{2/3}}\left[\frac{3}{5}\,(a^{5/3}-0)\right] = \frac{3}{5}\,a\,b$$

9.15 $d\,I_x = \frac{1}{3}\,y^3\,dx$

$$I_x = \int dI_x = \int_0^a \frac{1}{3}\,y^3\,dx = \frac{1}{3}\int_0^a \frac{b^3}{a^2}\,x^2\,dx = \frac{1}{3}\,\frac{b^3}{a^2}\left[\frac{1}{3}x^3\right]_0^a = \frac{1}{9}\,a\,b^3$$

$$\Rightarrow \quad I_x = 0.1111\ a\,b^3$$

$$k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{9}a\,b^3}{\frac{3}{5}a\,b} = \frac{5}{27}\,b^2 \quad \Rightarrow \quad k_x = \sqrt{\frac{5}{27}}\,b \quad \Rightarrow \quad k_x = 0.430\ b$$

9.17 $dI_y = x^2\,dA = x^2\ \frac{b}{a^{2/3}}\,x^{2/3}\ dx = \frac{b}{a^{2/3}}\,x^{8/3}\ dx$

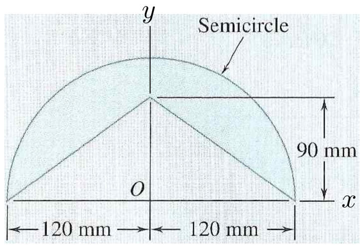
$$I_y = \int dI_y = \int_0^a \frac{b}{a^{2/3}}\,x^{8/3}\,dx = \frac{b}{a^{2/3}}\left[\frac{3}{11}x^{11/3}\right]_0^a = \frac{b}{a^{2/3}}\left[\frac{3}{11}(a^{11/3}-0)\right] = \frac{3}{11}\,a^3\,b$$

$$\Rightarrow \quad I_y = 0.273\ a^3\,b$$

$$k_y^2 = \frac{I_y}{A} = \frac{\frac{3}{11}\,a^3\,b}{\frac{3}{5}\,a\,b} = \frac{5}{11}\,a^2 \quad \Rightarrow \quad k_y = \sqrt{\frac{5}{11}}\,a \quad \Rightarrow \quad k_y = 0.674\ a$$

[9.2절]

9.45 ① 반원, ② 삼각 구멍



(a) $(J_O)_1 = \dfrac{1}{4}\pi(120\text{ mm})^4 = 162.86\times10^6\text{ mm}^4$

$(\bar{I}_x)_2 = \dfrac{1}{12}(240\text{ mm})(90\text{ mm})^3 = 14.58\times10^6\text{ mm}^2$

$(\bar{I}_y)_2 = 2\ \dfrac{1}{12}(90\text{ mm})(120\text{ mm})^3 = 25.92\times10^6\text{ mm}^2$

$(J_O)_2 = (\bar{I}_x)_2 + (\bar{I}_y)_2 = (25.92 + 14.58)\times10^6\text{ mm}^2 = 40.50\times10^6\text{ mm}^2$

$J_O = (J_O)_1 - (J_O)_2 = (162.86-40.50)\times10^6\text{ mm}^4 = 122.36\times10^6\text{ mm}^4$

$\Rightarrow\quad J_O = 122.4\times10^6\text{ mm}^4$

(b) ① $A = \dfrac{\pi}{2}(120\text{ mm})^2 = 22.62\times10^3\text{ mm}^2$

$\bar{y} = \dfrac{4}{3\pi}(120\text{ mm}) = 50.93\text{ mm}$

② $A = -\dfrac{1}{2}(240\text{ mm})(90\text{ mm}) = -10.80\times10^3\text{ mm}^2$

$\bar{y} = \dfrac{1}{3}(90\text{ mm}) = 30\text{ mm}$

$\Sigma A = (22.62 - 10.80)\times10^3\text{ mm}^2 = 11.82\times10^3\text{ mm}^2$

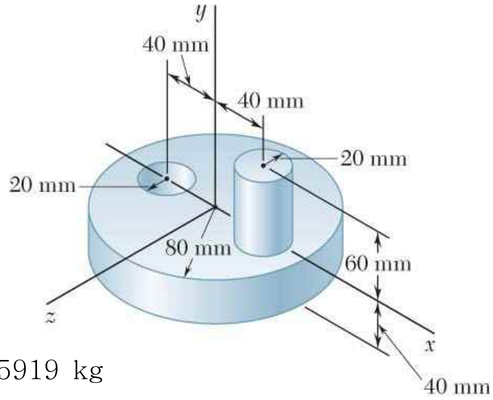
$\Sigma(\bar{y}A) = [(50.93)(22.62) + (30)(-10.80)]\times10^3\text{ mm}^3 = 828.0\times10^3\text{ mm}^3$

$\overline{Y} = \dfrac{\Sigma(\bar{y}A)}{\Sigma A} = \dfrac{828.0\times10^3\text{ mm}^3}{11.82\times10^3\text{ mm}^2} = 70.05\text{ mm},$ symmetry $\Rightarrow\ \overline{X} = 0$

$\bar{J}_C = J_O - A\,\overline{Y}^2 = (122.36\times10^6\text{ mm}^4) - (11.82\times10^3\text{ mm}^2)(70.05\text{ mm})^2 = 64.36\times10^6\text{ mm}^4$

$\Rightarrow\quad \bar{J}_C = 64.4\times10^6\text{ mm}^4$

[9.5절]



9.141 $\rho = 7{,}850\ \text{kg/m}^3$

$m = \rho V$ ① 원판, ② 원기둥, ③ 원판 구멍

$m_1 = \rho V_1$

$= (7{,}850\ \text{kg/m}^3)\ [\pi\ (0.08\ \text{m})^2\ (0.04\ \text{m})]$

$= 6.313\ \text{kg}$

$m_2 = \rho V_2$

$= (7{,}850\ \text{kg/m}^3)\ [\pi\ (0.02\ \text{m})^2\ (0.06\ \text{m})] = 0.5919\ \text{kg}$

$m_3 = \rho V_3 = (7{,}850\ \text{kg/m}^3)\ [\pi\ (0.02\ \text{m})^2\ (0.04\ \text{m})] = 0.3946\ \text{kg}$

(a) $I_{x1} = \dfrac{1}{12} m_1 (3 r_1^2 + 4 L_1^2)$

$= \dfrac{1}{12}(6.313\ \text{kg})[3\ (0.08\ \text{m})^2 + 4\ (0.04\ \text{m})^2] = 13.467\times10^{-3}\ \text{kg·m}^2$

$I_{x2} = \dfrac{1}{12} m_2 (3 r_2^2 + 4 L_2^2)$

$= \dfrac{1}{12}(0.5919\ \text{kg})[3\ (0.02\ \text{m})^2 + 4\ (0.06\ \text{m})^2] = 0.7695\times10^{-3}\ \text{kg·m}^2$

$I_{x3} = \dfrac{1}{12} m_3 (3 r_3^2 + 4 L_3^2)$

$= \dfrac{1}{12}(0.3946\ \text{kg})[3\ (0.02\ \text{m})^2 + 4\ (0.04\ \text{m})^2] = 0.2499\times10^{-3}\ \text{kg·m}^2$

$I_x = I_{x1} + I_{x2} - I_{x3} = [(13.467) + (0.7695) - (0.2499)]\times10^{-3}\ \text{kg·m}^2$

$= 13.9886\times10^{-3}\ \text{kg·m}^2 \quad\Rightarrow\quad I_x = 13.99\times10^{-3}\ \text{kg·m}^2$

(b) $I_{y1} = \dfrac{1}{2} m_1 r_1^2$

$= \dfrac{1}{2}(6.313\ \text{kg})\ (0.08\ \text{m})^2 = 20.20\times10^{-3}\ \text{kg·m}^2$

$I_{y2} = \dfrac{1}{2} m_2 r_2^2 + m_2 d_2^2$

$= \dfrac{1}{2}(0.5919\ \text{kg})\ (0.02\ \text{m})^2 + (0.5919\ \text{kg})\ (0.04\ \text{m})^2 = 1.0654\times10^{-3}\ \text{kg·m}^2$

$I_{y3} = \dfrac{1}{2} m_3 r_3^2 + m_3 d_3^2$

$= \dfrac{1}{2}(0.3946\ \text{kg})\ (0.02\ \text{m})^2 + (0.3946\ \text{kg})\ (0.04\ \text{m})^2 = 0.7103\times10^{-3}\ \text{kg·m}^2$

$I_y = I_{y1} + I_{y2} - I_{y3} = [(20.20) + (1.0654) - (0.7103)]\times10^{-3}\ \text{kg·m}^2$

$= 20.5551\times10^{-3}\ \text{kg·m}^2 \quad\Rightarrow\quad I_y = 20.6\times10^{-3}\ \text{kg·m}^2$

(c) $I_{z1} = I_{x1} = 13.467\times10^{-3}\ \text{kg·m}^2$

$I_{z2} = \dfrac{1}{12} m_2 (3 r_2^2 + 4 L_2^2) + m_2 d_2^2$

$= (0.7695\times10^{-3}\ \text{kg·m}^2) + (0.5919\ \text{kg})\ (0.04\ \text{m})^2 = 1.7165\times10^{-3}\ \text{kg·m}^2$

$I_{z3} = \dfrac{1}{12} m_3 (3 r_3^2 + 4 L_3^2) + m_3 d_3^2$

$= (0.2499\times10^{-3}\ \text{kg·m}^2) + (0.3946\ \text{kg})\ (0.04\ \text{m})^2 = 0.88126\times10^{-3}\ \text{kg·m}^2$

$I_z = I_{z1} + I_{z2} - I_{z3} = [(13.467) + (1.7165) - (0.88126)]\times10^{-3}\ \text{kg·m}^2$

$= 14.3022\times10^{-3}\ \text{kg·m}^2 \quad\Rightarrow\quad I_z = 14.30\times10^{-3}\ \text{kg·m}^2$