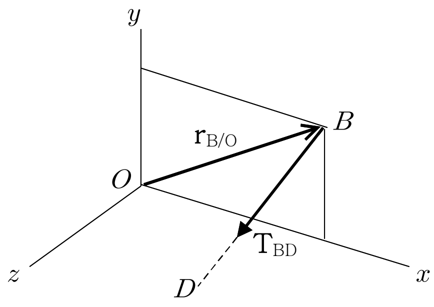
[3.1절]

3.26　T_{BD} = 900 N

S; known　T_{BD},　unknown 점 O에 관한 모멘트 $\mathbf{M}_O$

⇒ 위치벡터와 힘벡터의 벡터곱

M; 자유물체도(F.B.D.)



A; ① 위치벡터

$\mathbf{r}_{B/O} = l_x\ \mathbf{i} + l_y\ \mathbf{j} + l_z\ \mathbf{k}$

$= (2.5\ \text{m})\ \mathbf{i} + (2\ \text{m})\ \mathbf{j}$

② 힘벡터

d_x = -1 m,　d_y = -2 m,　d_z = 2 m

$d = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{(-1)^2 + (-2)^2 + 2^2}$ m = 3.00 m

$\boldsymbol{\lambda}_{BD} = \dfrac{1}{3.00}[(-1)\ \mathbf{i} + (-2)\ \mathbf{j} + 2\ \mathbf{k}] = -0.3333\ \mathbf{i} - 0.6667\ \mathbf{j} + 0.6667\ \mathbf{k}$

$\mathbf{T}_{BD} = T_{BD}\ \boldsymbol{\lambda}_{BD} = (900\ \text{N})\ \dfrac{1}{3.00}[(-1)\ \mathbf{i} + (-2)\ \mathbf{j} + 2\ \mathbf{k}]$

$= -300\ \mathbf{i} - 600\ \mathbf{j} + 600\ \mathbf{k}$ (N)

③ 벡터곱

$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BD}$

$= [2.5\ \mathbf{i} + 2\ \mathbf{j}\ (\text{m})] \times [300\ \mathbf{i} - 600\ \mathbf{j} + 600\ \mathbf{k}\ (\text{N})]$

$= [(2)(600) - 0]\ \mathbf{i} + [0 - (2.5)(600)]\ \mathbf{j}$

$+ [(2.5)(-600) - (2)(-300)]\ \mathbf{k}$ (N・m)

$= 1200\ \mathbf{i} - 1500\ \mathbf{j} - 900\ \mathbf{k}$ (N・m)

R(과정의 타당성) : 서술

(가령 T_x<0, T_y<0, T_z>0, 힘의 각 직각성분의 방향)

T(결과의 의미) : 서술

(가령, M_x>0, M_y<0, M_z<0, 각 좌표축에 관한 모멘트의 방향)

[3.2절]

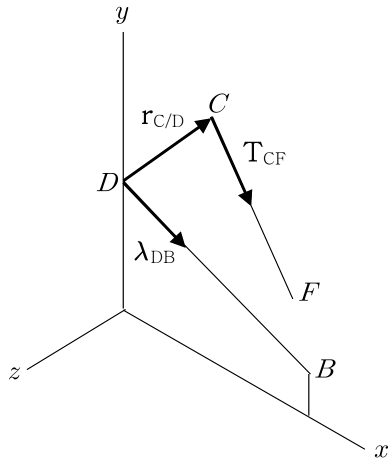
3.60 T_{CF} = 33 N

S; known T_{CF}, unknown 축 DB에 관한 모멘트 M_{DB}

⇒ 힘벡터, 위치벡터, 단위벡터의 혼합3중곱

$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r} \times \mathbf{T}_{CF})$]

M; 자유물체도(F.B.D.)



A; ① 위치벡터

$\mathbf{r}_{C/D} = 0\mathbf{i} + (0.9 - 0.7)\mathbf{j} + (-0.4)\mathbf{k}$ (m)

$= 0.2\ \mathbf{j} - 0.4\ \mathbf{k}$ (m)

② 힘벡터

$\mathbf{T}_{CF} = T_{CF}\ \boldsymbol{\lambda}_{CF}$

$$= (33\text{ N})\frac{(0.6\text{ m})\mathbf{i} + (-0.9\text{ m})\mathbf{j} + (-0.6\text{ m} + 0.4\text{ m})\mathbf{k}}{\sqrt{(0.6\text{ m})^2 + (-0.9\text{ m})^2 + (-0.2\text{ m})^2}}$$

$$= \frac{33\text{ N}}{1.1}(0.6\ \mathbf{i} - 0.9\ \mathbf{j} - 0.2\ \mathbf{k}) = 18\ \mathbf{i} - 27\ \mathbf{j} - 6\ \mathbf{k}\ (\text{N})$$

①② 점 D에 관한 모멘트

$\mathbf{r}_{C/D} \times \mathbf{T}_{CF} = [0.2\ \mathbf{j} - 0.4\ \mathbf{k}\ (\text{m})] \times [18\ \mathbf{i} - 27\ \mathbf{j} - 6\ \mathbf{k}\ (\text{N})]$

$= [(0.2)(-6) - (-0.4)(-27)]\ \mathbf{i} + (-0.4)(18)\ \mathbf{j} + (-0.2)\times(18)\ \mathbf{k}$ (N · m)

$= -12\ \mathbf{i} - 7.2\ \mathbf{j} - 3.6\ \mathbf{k}$ (N · m)

③ 단위벡터

$$\boldsymbol{\lambda}_{DB} = \frac{(1.2\text{ m})\mathbf{i} - (0.35\text{ m})\mathbf{j}}{\sqrt{(1.2\text{ m})^2 + (-0.35\text{ m})^2}} = \frac{1}{1.25}(1.2\ \mathbf{i} - 0.35\ \mathbf{j}) = 0.96\ \mathbf{i} - 0.28\ \mathbf{j}$$

④ 축 DB에 관한 모멘트

$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot \mathbf{M}_B = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{C/D} \times \mathbf{T}_{CF})$

$= (0.96\ \mathbf{i} - 0.28\ \mathbf{j}) \cdot [-12\ \mathbf{i} - 7.2\ \mathbf{j} - 3.6\ \mathbf{k}$ (N·m)]

$= (0.96)(-12) + (-0.28)(-7.2) + 0 = -9.504$ (N·m) ⇒ $M_{DB} = -9.50$ N·m

R(과정의 타당성) : 서술

(가령, 모멘트 계산에 사용될 수 있는 위치벡터 $\mathbf{r}_{C/D}$, $\mathbf{r}_{F/D}$, $\mathbf{r}_{C/B}$, $\mathbf{r}_{E/B}$ 중 선택한 벡터의 타당성)

T(결과의 의미) ; 서술

(가령, M_{DB}<0인 결과는 모멘트 회전축 방향이 선 BD 방향)

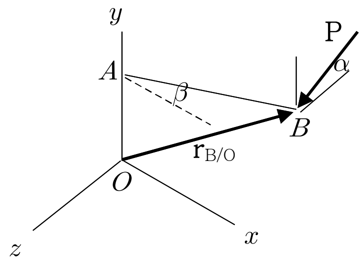
[3.3절]

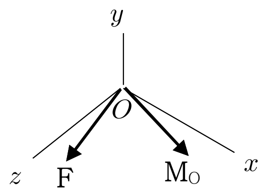
3.98 P = 110 N, α = 15°, β = 35°, l_{AB} = 0.220 m, l_{OA} = 0.150 m

S; known **P**, unknown **F**, $\mathbf{M}_O$

⇒ 3차원 등가 힘-우력 계

M; 자유물체도(F.B.D.)





A; ΣF ; **F** = **P** = P (−sinα **j** + cosα **k**)

= (110 N) (−sin15° **j** + cos15° **k**)

= (−28.47 N) **j** + (106.25 N) **k**

⇒ **F** = (−28.5 N) **j** + (106.3 N) **k**

$\mathbf{r}_{B/O} = l_{OA}\,\mathbf{j} + l_{AB}\,(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{k})$

= (0.150 m) **j** + (0.220 m) (cos35° **i** − sin35° **k**)

= 0.1802 **i** + 0.1500 **j** − 0.1262 **k** (m)

ΣM_O ; $\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{P}$

= [0.1802 **i** + 0.1500 **j** − 0.1262 **k** (m)] × [−28.5 **j** + 106.3 **k** (N)]

= [(0.1500)(106.3)−(−0.1262)(−28.5)] **i** + [0 − (0.1802)(106.3)] **j**

+ [(0.1802)(−28.5) − 0] **k** (N·m)

= 12.35 **i** − 19.16 **j** − 5.14 **k** (N·m)

R(과정의 타당성) ; 서술

(가령, ΣM_B 또는 ΣM_A 대신 ΣM_O를 사용한 과정의 장점)

T(결과의 의미) ; 서술

(가령, M_x>0, M_y<0, M_z<0, 각 좌표축에 관한 모멘트의 방향)

정 역 학 연습문제 풀이 제4장 (2022년)

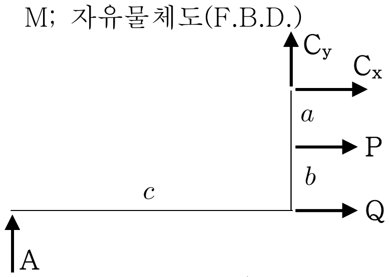
[4.1절]

4.28 S; known ; P = 300 N →, Q = 300 N →, a = 0.2 m, b = 0.2 m, c = 0.8 m, α

unknown ; reaction A, C

⇒ 모멘트 평형, 힘의 평형, 반력 유형 1&2

M; 자유물체도(F.B.D.)



A; (a) α = 0

$+\circlearrowleft\Sigma M_C = 0$;

$-c\ A + a\ P + (a+b)\ Q = 0$

$\Rightarrow\ A = \dfrac{aP + (a+b)Q}{c}$

$= \dfrac{(0.2\ \mathrm{m})(300\ \mathrm{N}) + (0.4\ \mathrm{m})300\ \mathrm{N}}{0.8\ \mathrm{m}} = 225\ \mathrm{N}$ ⇒ A = 225 N ↑

$\rightarrow\Sigma F_x = 0$;

$C_x + P + Q = 0 \quad \Rightarrow \quad C_x = -P - Q = -(300\ \mathrm{N}) - (300\ \mathrm{N}) = -600\ \mathrm{N}$

$\uparrow\Sigma F_y = 0$;

$C_y + A = 0 \quad \Rightarrow \quad C_y = -A = -225\ \mathrm{N}$

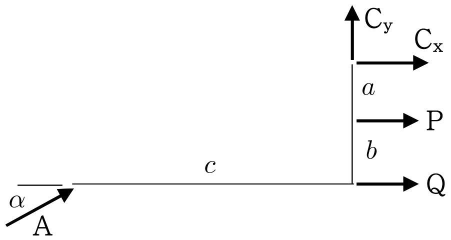
$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-600\ \mathrm{N})^2 + (-225\ \mathrm{N})^2} = 640.8\ \mathrm{N}$

$\theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{-225\ \mathrm{N}}{-600\ \mathrm{N}} = \tan^{-1}(0.375) = 20.56°$

⇒ C = 641 N ↙20.6°

(b) α = 30°

M; 자유물체도(F.B.D.)



$+\circlearrowleft\Sigma M_C = 0$;

$-c\ A\ \cos\alpha + (a+b)\ A\ \sin\alpha$

$+ a\ P + (a+b)\ Q = 0$

$\Rightarrow\ A = \dfrac{aP + (a+b)Q}{c\cos\alpha - (a+b)\sin\alpha}$

$= \dfrac{(0.2\ \mathrm{m})(300\ \mathrm{N}) + (0.4\ \mathrm{m})300\ \mathrm{N}}{(0.8\ \mathrm{m})\cos 30^\circ - (0.4\ \mathrm{m})\sin 30^\circ}$

$= 365.2\ \mathrm{N}$ ⇒ A = 365 N ∠60.0°

$\rightarrow\Sigma F_x = 0$;

$C_x + P + Q + A\ \sin\alpha = 0$

$\Rightarrow\ C_x = -P - Q - A\ \sin\alpha$

$= -(300\ \mathrm{N}) - (300\ \mathrm{N}) - (365.2\ \mathrm{N})\sin 30° = -782.6\ \mathrm{N}$

$\uparrow\Sigma F_y = 0$;

$C_y + A\ \cos\alpha = 0$

$\Rightarrow\ C_y = -A\ \cos\alpha = -(365.2\ \mathrm{N})\cos 30° = -316.3\ \mathrm{N}$

$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-782.6\ \mathrm{N})^2 + (-316.3\ \mathrm{N})^2} = 844.1\ \mathrm{N}$

$\theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{-316.3\ \mathrm{N}}{-782.6\ \mathrm{N}} = \tan^{-1}(0.404) = 22.0°$

⇒ C = 844 N ↙22.0°

R; (예: $+\circlearrowleft\Sigma M_A$에 비해 $+\circlearrowleft\Sigma M_C$ 을 사용하는 장점

힘 평형 식 ($\rightarrow\Sigma F_x = 0$, $\uparrow\Sigma F_y = 0$)보다 모멘트 식($\Sigma M_C = 0$)을 먼저 사용하는 장점)

T; (예: a와 b의 A 비교, C 비교)

4.50 P = 900 N, Q = 4000 N, T = 1950 N, a = 2.1 m, b = 3 m, c = 7.2 m

$\alpha = \tan^{-1}\dfrac{7.2}{3} = \tan^{-1}(2.4) = 67.38°$

$\overline{BC} = \sqrt{3^2+7.2^2} = 7.8\ (\mathrm{m})$

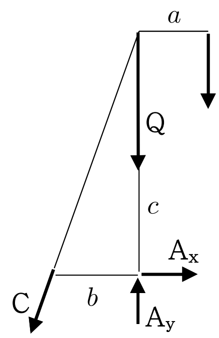
(a) S; known $P,\ Q,\ a,\ b,\ c$

unknown reaction C, A

유형 1&2 (줄, 힌지)

method 힘의 평형, 모멘트 평형

M; 자유물체도(F.B.D.)



A; $+\circlearrowleft\Sigma M_B = 0;\quad c\ A_x - a\ P = 0$

$\Rightarrow\quad A_x = \dfrac{a}{c}\ P = \dfrac{2.1}{7.2}\ (900\ \mathrm{N}) = 262.5\ \mathrm{N}$

$\Sigma F_x = 0;\quad -T\cos\alpha + A_x = 0$

$\Rightarrow\quad T = \dfrac{7.8}{3}\ A_x = 2.6\ (262.5\ \mathrm{N}) = 682.5\ \mathrm{N}$ $\Rightarrow$ C = 683 N ↙67.4°

$\Sigma F_y = 0;\ A_y - T\sin\alpha - (P+Q) = 0$

$\Rightarrow\quad A_y = \dfrac{7.2}{7.8}\ T + P + Q = \dfrac{12}{13}(682.5\ \mathrm{N}) + (4900\ \mathrm{N}) = 5530\ \mathrm{N}$

$A = \sqrt{A_x^2+A_y^2} = \sqrt{(262.5)^2+(5530)^2} = 5536\ \mathrm{N}$

$\theta = \tan^{-1}\dfrac{A_y}{A_x} = \tan^{-1}\dfrac{5530}{262.5} = \tan^{-1}(21.07) = 87.28°$ $\Rightarrow$ A = 5540 N ∠87.3°

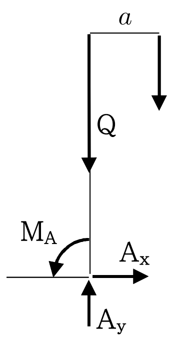
(b) S; known $P,\ Q,\ a$

unknown reaction A, M_A

유형 3 (고정)

method 힘의 평형, 모멘트 평형

M; 자유물체도(F.B.D.)



A; $+\circlearrowleft\Sigma M_A = 0;\quad M_A - a\ P = 0$

$\Rightarrow\quad M_A = a\ P = (2.1\ \mathrm{m})\ (900\ \mathrm{N}) = 1890.0\ \mathrm{N\cdot m}$

$\Rightarrow$ M_A = 1890 N・m ↺

$\Sigma F_x = 0;\quad A_x = 0$

$\Sigma F_y = 0;\ A_y - (P+Q) = 0$

$\Rightarrow\quad A_y = P + Q = 4900\ \mathrm{N}$ $\Rightarrow$ A = 4900 N ↑

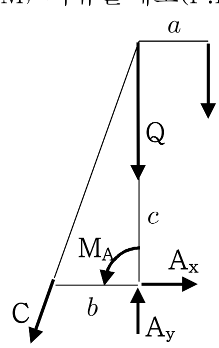
(c) S; known $P,\ Q,\ T,\ a,\ b,\ c$

unknown reaction C, A, M_A

유형 1&3 (줄, 고정)

method 힘의 평형, 모멘트 평형

M; 자유물체도(F.B.D.)



A; C = T = 1950 N $\swarrow$67.4° $\Rightarrow$ C = 1950 N $\swarrow$67.4°

$\Sigma F_x = 0;\quad -T\cos\alpha + A_x = 0$

$\Rightarrow\quad A_x = T\cos\alpha = (1950\ \text{N})\ \dfrac{3}{7.8} = 750\ \text{N}$

$\Sigma F_y = 0;\ A_y - T\sin\alpha - (P+Q) = 0$

$\Rightarrow\quad A_y = \dfrac{7.2}{7.8}\ T + P + Q = \dfrac{12}{13}(1950\ \text{N}) + (4900\ \text{N}) = 6700\ \text{N}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(750)^2 + (6700)^2} = 6742\ \text{N}$

$\theta = \tan^{-1}\dfrac{A_y}{A_x} = \tan^{-1}\dfrac{6700}{750} = \tan^{-1}(8.933) = 83.61°$ $\Rightarrow$ A = 6740 N $\angle$83.6°

$+\circlearrowleft \Sigma M_B = 0;\quad M_A + c\ A_x - a\ P = 0$

$\Rightarrow\quad M_A = -c\ A_x + a\ P = -(7.2\ \text{m})\ (750\ \text{N}) + (2.1\ \text{m})\ (900\ \text{N})$

$= -3510\ \text{N}\cdot\text{m}$ $\Rightarrow$ M_A = 3510 N · m $\circlearrowleft$

R; (예: (a)와 (b)에서는 모멘트 식을 먼저 사용하지만, (c)에서는 힘 평형 식을 먼저 사용)

T; (예: 모멘트 반력 방향이 (b)에서는 반시계방향인데, (c)에서는 시계방향)

[4.2절]

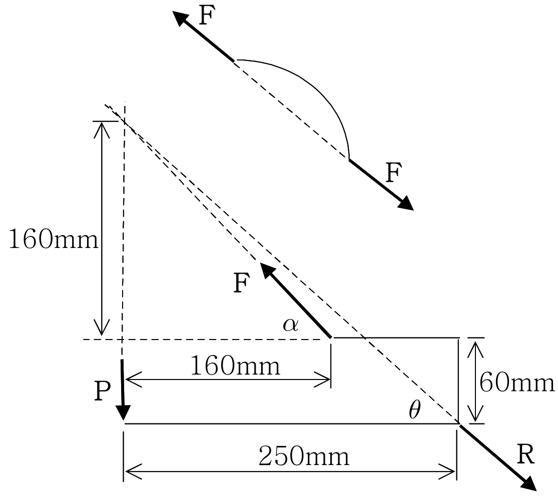
4.67 P = 80 N, α = 45°

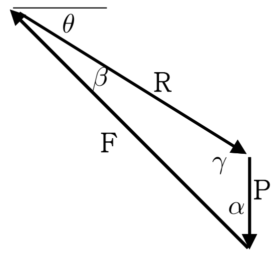
S; known P, α, unknown R, F

⇒ 두 힘의 평형, 세 힘의 평형, 반력 유형2

세 힘의 작용선이 한 점에서 만남

M; 자유물체도(F.B.D.)





A; $\tan\theta = \dfrac{160+60\ \text{mm}}{250\ \text{mm}} = 0.880$

⇒ $\theta = \tan^{-1}(0.880) = 41.348°$

$\gamma = 90° + \theta = 90° + 41.35° = 131.35°$

$\beta = 180° - \gamma - \alpha$

$= 180° - 131.35° - 45° = 3.65°$

$$\frac{R}{\sin\alpha} = \frac{P}{\sin\beta}$$

⇒ $R = P\dfrac{\sin\alpha}{\sin\beta} = (80\ \text{N})\dfrac{\sin 45°}{\sin 3.65°}$

$= 888.58$ N

⇒ R = 889 N ↘41.3°

$$\frac{F}{\sin\gamma} = \frac{P}{\sin\beta}$$

⇒ $F = P\dfrac{\sin\gamma}{\sin\beta} = (80\ \text{N})\dfrac{\sin 131.35°}{\sin 3.65°}$

$= 943.35$ N

⇒ F = 943 N ↖45.0°

R; (예: 미지수 개수 4(두 힘의 크기와 방향 각도)가 식 개수 3보다 많은데,

힘 삼각형 방법으로 미지수 개수를 하나 줄일 수 있음)

T; (예: B의 반력의 방향, D의 반력의 방향 검토)