정 역 학　　연습과제 풀이　　제2장　　(2022년)

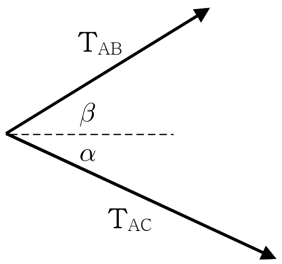
[2.1절]

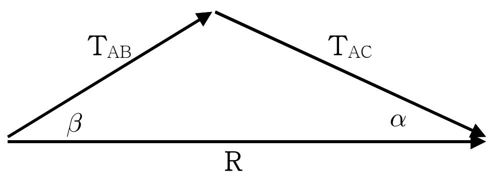
2.9　S; T_{AB} = 3 kN, β = 30°, R = 4.8 kN →

known : T_{AB}, β, R,　　unknown : T_{AC}, α

⇒ 삼각법(trigonometry) sine 공식, cosine 공식 사용하여 힘의 합성

M; 자유물체도 (F.B.D.),　　힘 삼각형 (force triangle)





A; 장력 T_{AC} (cosine 공식)

$$T_{AC}^2 = T_{AB}^2 + R^2 - 2\ T_{AB}\ R\ \cos\beta$$

$$= (3\text{ kN})^2 + (4.8\text{ kN})^2 - 2\ (3\text{ kN})\ (4.8\text{ kN})\ \cos 30° = 7.098\text{ kN}^2$$

$$T_{AC} = \sqrt{7.098\text{ kN}^2} = 2.664\text{ kN} \qquad \Rightarrow \qquad T_{AC} = 2.66\text{ kN}$$

각도 α (sine 공식)

$$\frac{T_{AB}}{\sin\alpha} = \frac{T_{AC}}{\sin\beta} \quad \Rightarrow \quad \sin\alpha = \frac{T_{AB}}{T_{AC}}\sin\beta = \frac{3\text{ kN}}{2.664\text{ kN}}\sin 30° = 0.5630$$

$$\Rightarrow \quad \alpha = \sin^{-1}(0.5630) = 34.26° \qquad \Rightarrow \qquad \alpha = 34.3°$$

R; (과정의 타당성 서술)

(가령, 장력 T_{AC} 계산에 sine 공식 대신 cosine 공식을 사용한 이유, 또는 각도 α를 장력 T_{AC} 보다 먼저 구할 수 없는 이유)

T; (결과의 의미 서술)

(가령, $\beta<\alpha$ 와 $T_{AC}<T_{AB}$ 의 연관성)

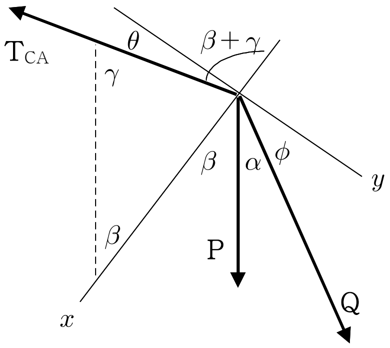
[2.2절]

2.41 S; P = 50 N, Q = 75 N, α = 25°, β = 35°, γ = 65°

known : R direction = BC (R_y = 0), unknown : T_{AC}, R

⇒ 직각성분 합에 의한 힘의 합성

M; 자유물체도 (F.B.D.)



θ = β + γ - 90° = 35° + 65° - 90° = 10°

ϕ = 90° - ($\alpha+\beta$) = 90° - (25°+35°) = 30°

A; $R_x = \Sigma F_x = T_{AC}\sin\theta + P\cos\beta + Q\sin\phi$

$R_y = \Sigma F_y = -T_{AC}\cos\theta + P\sin\beta + Q\cos\phi$

(a) $R_y = 0 = -T_{AC}\cos\theta + P\sin\beta + Q\cos\phi$

$$\Rightarrow \quad T_{AC} = \frac{1}{\cos\theta}(P\sin\beta + Q\cos\phi) = \frac{1}{\cos 10^\circ}[(50\text{ N})\sin 35° + (75\text{ N})\cos 30°]$$

$$= \frac{1}{\cos 10^\circ}(93.63\text{ N}) = 95.08\text{ N} \qquad \Rightarrow \qquad T_{AC} = 95.1\text{ N}$$

(b) $R = R_x = T_{AC}\sin\theta + P\cos\beta + Q\sin\phi$

$= (95.08\text{ N})\sin 10° + (50\text{ N})\cos 35° + (75\text{ N})\sin 30° = 94.97\text{ N}$

$\Rightarrow \qquad R_x = 95.0\text{ N}$

R; (과정의 타당성 서술)

(가령, 직각성분 방법을 다각형 방법과 비교, 또는

직각성분 좌표 축을 수평 수직 방향으로 설정하는 경우와 비교)

T; (결과의 의미 서술)

(가령, R_x>0의 의미, 또는 R_y=0의 의미)

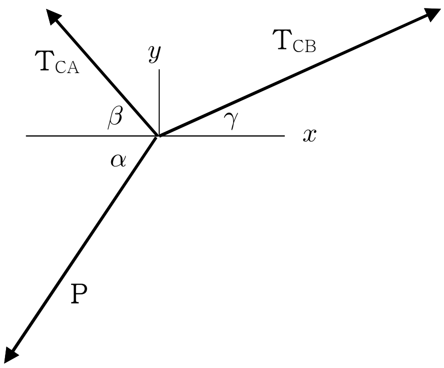
[2.3절]

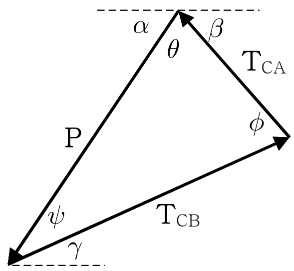
2.46 P = 500 N, α = 60°, β = 45°, γ = 25°

S; known P, α, β, γ, unknown T_{AC}, T_{BC},

질점의 평형 문제 ⇒ 직각성분 방법 또는 힘 삼각형 방법

M; 자유물체도 (F.B.D.) 힘 삼각형(force triangle)





θ = 180° - (α+β)
= 180° - (60°+45°) = 75°

ψ = α - γ
= 60° - 25° = 35°

ϕ = 180° - (θ+ψ)
= 180° - (75°+35°) = 70°

A; <방법1 : 직각 성분>

$\Sigma F_x = 0$; $-P\cos\alpha - T_{AC}\cos\beta + T_{BC}\cos\gamma = 0$ ··· ①

$\Sigma F_y = 0$; $-P\sin\alpha + T_{AC}\sin\beta + T_{BC}\sin\gamma = 0$ ··· ②

(a) ①×sinγ - ②×cosγ

$-P(\cos\alpha\sin\gamma - \sin\alpha\cos\gamma) - T_{AC}(\cos\beta\sin\gamma + \sin\beta\cos\gamma) = 0$

$$\Rightarrow\quad T_{AC} = P\frac{\sin(\alpha-\gamma)}{\sin(\beta+\gamma)} = (500\text{ N})\frac{\sin(60^\circ-25^\circ)}{\sin(45^\circ+25^\circ)} = 305.2\text{ N}$$

⇒ T_{AC} = 305 N

$$\text{(b) ① }\Rightarrow\quad T_{BC} = \frac{1}{\cos\gamma}(P\cos\alpha + T_{AC}\cos\beta)$$

$$= \frac{1}{\cos 25^\circ}[(500\text{ N})\cos 60^\circ + (305.2\text{ N})\cos 45^\circ] = 514.0\text{ N}$$

⇒ T_{BC} = 514 N

<방법2 : 힘 삼각형>

$$\frac{T_{AC}}{\sin\psi} = \frac{T_{BC}}{\sin\theta} = \frac{P}{\sin\phi}$$

(a) $T_{AC} = P\dfrac{\sin\psi}{\sin\phi} = (500\text{ N})\dfrac{\sin 35^\circ}{\sin 70^\circ} = 305.2\text{ N}$ ⇒ T_{AC} = 305 N

(b) $T_{BC} = P\dfrac{\sin\theta}{\sin\phi} = (500\text{ N})\dfrac{\sin 75^\circ}{\sin 70^\circ} = 514.0\text{ N}$ ⇒ T_{BC} = 514 N

R; (과정의 타당성 서술)

(가령, 두 가지 방법 비교)

T; (결과의 의미 서술)

(가령, $T_{AC} < T_{BC}$ 인 이유)

[2.4절]

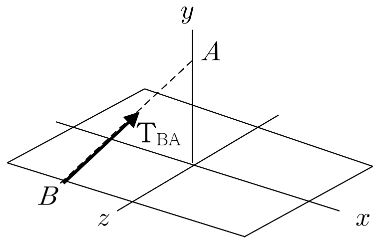
2.89 T_{BA} = 408 N, A(0, 480, 0), B(-320, 0, 360) mm

S; known T_{BA}, A, B

unknown $(T_{BA})_x$, $(T_{BA})_y$, $(T_{BA})_z$

⇒ 공간에서 힘의 직각성분 (두 점과 힘 크기)

M; 자유물체도 (F.B.D.)



A; $d_x = x_A - x_B$ = [0 - (-320)] mm = 320 mm

$d_y = y_A - y_B$ = (480 - 0) mm = 480 mm

$d_z = z_A - z_B$ = (0 - 360) mm = -360 mm

$d_{BA} = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{(320\,\text{mm})^2 + (480\,\text{mm})^2 + (-360\,\text{mm})^2}$ = 680 mm

$\boldsymbol{\lambda}_{BA} = \frac{1}{680}(320\ \mathbf{i} + 480\ \mathbf{j} - 360\ \mathbf{k}) = \frac{1}{17}(8\ \mathbf{i} + 12\ \mathbf{j} - 9\ \mathbf{k})$

$= -0.4706\ \mathbf{i} + 0.7059\ \mathbf{j} + 0.5294\ \mathbf{k}$

$\mathbf{T}_{BA} = T_{BA}\ \boldsymbol{\lambda}_{BA} = (408\ \text{N})\ \frac{1}{17}(8\ \mathbf{i} + 12\ \mathbf{j} - 9\ \mathbf{k}) = (192\ \mathbf{i} + 288\ \mathbf{j} - 216\ \mathbf{k})$ N

⇒ $(T_{BA})_x$ = 192 N, $(T_{BA})_y$ = 288 N, $(T_{BA})_z$ = -216 N

R; (과정의 타당성 서술)

(가령, 공간에서 단위벡터의 방향, 또는 $\boldsymbol{\lambda}_{BA}$의 크기 = 1)

T; (결과의 의미 서술)

(가령, 각 성분의 부호의 의미)

[2.5절]

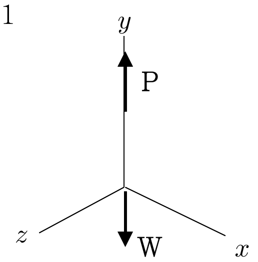
2.120 W = 300 N, α = 30°, θ_A = 50°, θ_B = 40°, θ_C = 60°

S; known W, α, θ_A, θ_B, θ_C

unknown T_{DA}, T_{DB}, T_{DC}

⇒ 공간에서 힘의 직각성분 (각도 이용)

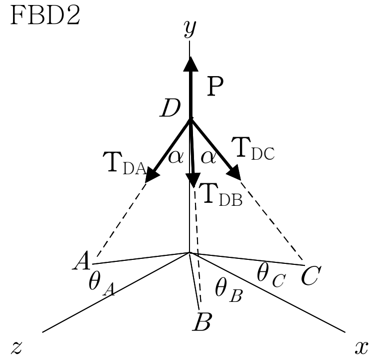
M; FBD1



A; FBD1에서 $\Sigma F_y = 0$, $P - W = 0$

⇒ $P = W$ = 300 N

FBD2



FBD2에서

$(T_{DA})_y = -T_{DA}\ \cos\alpha$, $(T_{DA})_h = T_{DA}\ \sin\alpha$

$(T_{DA})_x = -(T_{DA})_h\ \sin\theta_A = -T_{DA}\ \sin\alpha\ \sin\theta_A$

$(T_{DA})_z = (T_{DA})_h\ \cos\theta_A = T_{DA}\ \sin\alpha\ \cos\theta_A$

$(T_{DB})_y = -T_{DB}\ \cos\alpha$, $(T_{DB})_h = T_{DB}\ \sin\alpha$

$(T_{DB})_x = (T_{DB})_h\ \cos\theta_B = T_{DB}\ \sin\alpha\ \cos\theta_B$

$(T_{DB})_z = (T_{DB})_h\ \sin\theta_B = T_{DB}\ \sin\alpha\ \sin\theta_B$

$(T_{DC})_y = -T_{DC}\ \cos\alpha$, $(T_{DC})_h = T_{DC}\ \sin\alpha$

$(T_{DC})_x = (T_{DC})_h\ \cos\theta_C = T_{DC}\ \sin\alpha\ \cos\theta_C$

$(T_{DC})_z = -(T_{DC})_h\ \sin\theta_C = -T_{DC}\ \sin\alpha\ \sin\theta_C$

$\Sigma \mathrm{F} = 0$ ⇒ $\mathrm{P} + \mathrm{T_{DA}} + \mathrm{T_{DB}} + \mathrm{T_{DC}} = 0$

$\Sigma F_x = 0$, $(T_{DA})_x + (T_{DB})_x + (T_{DC})_x = 0$

$-T_{DA}\ \sin\alpha\ \sin\theta_A + T_{DB}\ \sin\alpha\ \cos\theta_B + T_{DC}\ \sin\alpha\ \cos\theta_C = 0$

$-T_{DA}\ \sin\theta_A + T_{DB}\ \cos\theta_B + T_{DC}\ \cos\theta_C = 0$

$-T_{DA}\ \sin 50° + T_{DB}\ \cos 40° + T_{DC}\ \cos 60° = 0$

⇒ $-0.7660\ T_{DA} + 0.7660\ T_{DB} + 0.5\,T_{DC} = 0$ ⋯ ①

$\Sigma F_y = 0$, $P + (T_{DA})_y + (T_{DB})_y + (T_{DC})_y = 0$

$T_{DA}\ \cos\alpha + T_{DB}\ \cos\alpha + T_{DC}\ \cos\alpha = P$

$$T_{DA} + T_{DB} + T_{DC} = \frac{P}{\cos\alpha} = \frac{300\ \mathrm{N}}{\cos 30^\circ}$$

⇒ $T_{DA} + T_{DB} + T_{DC}$ = 346.4 N ⋯ ②

$\Sigma F_z = 0$, $(T_{DA})_z + (T_{DB})_z + (T_{DC})_z = 0$

$T_{DA}\ \sin\alpha\ \cos\theta_A + T_{DB}\ \sin\alpha\ \sin\theta_B - T_{DC}\ \sin\alpha\ \sin\theta_C = 0$

$T_{DA}\ \cos\theta_A + T_{DB}\ \sin\theta_B - T_{DC}\ \sin\theta_C = 0$

$T_{DA}\ \cos 50° + T_{DB}\ \sin 40° - T_{DC}\ \sin 60° = 0$

⇒ $0.6428\ T_{DA} + 0.6428\ T_{DB} - 0.8660\ T_{DC} = 0$ ⋯ ③

연립방정식 ①, ②, ③의 해 ⇒ T_{DA} = 147.6N, T_{DB} = 51.3 N, T_{DC} = 147.6 N

R; (과정의 타당성 서술) (가령, 방향여현 표현)

T; (결과의 의미 서술) (가령, $T_{DA} + T_{DB} + T_{DC} > W$ 인 이유)