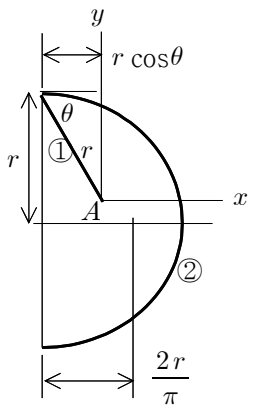
[5.1절]

5.28 S; 직선 + 반원호 M;



⇒ 선재의 무게와 무게중심, 강체의 평형

$\Sigma M_A = 0 \Rightarrow \bar{X} L = 0$

$\Rightarrow \Sigma(\bar{x} L) = 0$

A; 선재의 도심(= 무게중심)의 x 좌표

① $L = r$, $\bar{x} = -\frac{1}{2} r \cos\theta$

② $L = \pi r$, $\bar{x} = \frac{2r}{\pi} - r\cos\theta$

$\Sigma(\bar{x} L) = (-\frac{1}{2} r \cos\theta)\, r + (\frac{2r}{\pi} - r\cos\theta)\, \pi r = 0$

$\Rightarrow (\frac{1}{2} r + \pi r)\cos\theta = 2r$

$\Rightarrow \cos\theta = \frac{2}{\frac{1}{2}+\pi} = \frac{4}{1+2\pi} = 0.5492$

$\Rightarrow \theta = \cos^{-1}(0.5492) = 56.69° \Rightarrow \theta = 56.7°$

R(과정의 타당성 검토); (가령, 무게중심의 y좌표는 불필요)

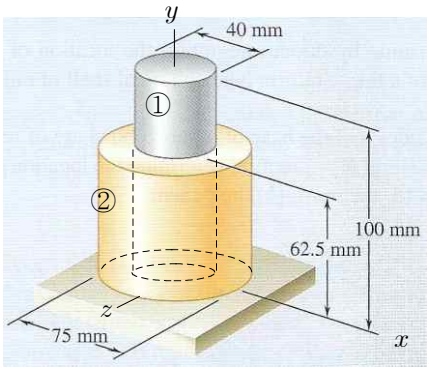
T(결과의 의미 검토); (가령, 힌지 A의 반력 중 $A_x = 0$)

[5.4절]

5.120 ρ_b = 8,470 kg/m^3 = 8.47×10^{-6} kg/mm^3, ρ_a = 2,800 kg/m^3 = 2.80×10^{-6} kg/mm^3

h_b = 62.5 mm, h_a = 100 mm,

d_b = 75 mm, d_a = 40 mm



S; 대칭구조 $\overline{X} = \overline{Z} = 0$ M;

$W = mg = \rho V g \Rightarrow m = \rho V$

$$\overline{Y} = \frac{\Sigma(\overline{y}W)}{\Sigma W} = \frac{\Sigma(\overline{y}m)}{\Sigma m}$$

A; ① 알루미늄 원기둥 + ② 황동 원통

① $V = \frac{\pi}{4} d_a^2 h_a = \frac{\pi}{4}(40\text{ mm})^2(100\text{ mm}) = 125{,}664\text{ mm}^3 = 125.664\times10^3\text{ mm}^3$

$m = \rho_a V = (2.80\times10^{-6}\text{ kg/mm}^3)(125.664\times10^3\text{ mm}^3) = 0.3519\text{ kg}$

$\overline{y} = \frac{1}{2} h_a = \frac{1}{2}(100\text{ mm}) = 50\text{ mm}$

② $V = \frac{\pi}{4}\left(d_b^2 - d_a^2\right) h_b = \frac{\pi}{4}[(75\text{ mm})^2 - (40\text{ mm})^2](62.5\text{ mm}) = 197.577\times10^3\text{ mm}^3$

$m = \rho_b V = (8.47\times10^{-6}\text{ kg/mm}^3)(197.577\times10^3\text{ mm}^3) = 1.6734\text{ kg}$

$\overline{y} = \frac{1}{2} h_b = \frac{1}{2}(62.5\text{ mm}) = 31.25\text{ mm}$

$\Sigma m = (0.3519 + 1.6734)\text{ kg} = 2.025\text{ kg}$

$\Sigma(\overline{y}m) = [(50)(0.3519) + (31.25)(1.6734)]\text{ kg·mm}$

$= 69.89\text{ kg·mm}$

$$\overline{Y} = \frac{\Sigma(\overline{y}m)}{\Sigma m} = \frac{69.89\text{ kg·mm}}{2.025\text{ kg}} = 34.51\text{ mm} \quad \Rightarrow \quad \text{중심} = (0,\ 34.5\text{ mm},\ 0)$$

R; (과정의 타당성 검토) (가령, 원통의 부피 및 질량 계산 과정)

T; (결과의 의미 검토) (가령, 중심의 위치)

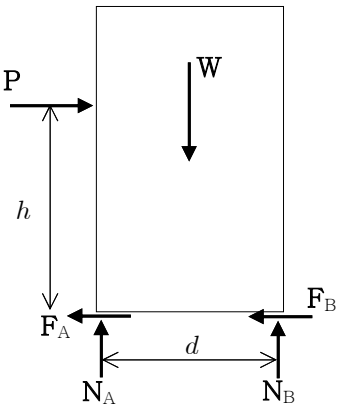
[8.1절]

8.18　W = 480 N,　μ_s = 0.30,　d = 0.6 m

S; known W, μ_s, d,　unknown **P**, h_{max}

⇒　힘의 평형방정식,　최대 정지마찰력

M;



A;

(a) 미끄러지려 할 때, 최대 정지마찰력

$F_A = \mu_s N_A$,　$F_B = \mu_s N_B$

$\uparrow \Sigma F_y = 0$

$N_A + N_B - W = 0$

$\Rightarrow \quad N_A + N_B = W$

$\rightarrow \Sigma F_x = 0$

$P - F_A - F_B = 0$

$\Rightarrow \quad P = F_A + F_B = \mu_s N_A + \mu_s N_B$

$= \mu_s (N_A + N_B) = \mu_s W$

$= (0.30)(480\ \text{N}) = 144\ \text{N}$

P = 144.0 N →

(b) 넘어지려 할 때,　$N_A = 0$,　$F_A = 0$

$\curvearrowleft \Sigma M_B = 0$

$$h P - \frac{d}{2} W = 0$$

$$\Rightarrow \quad h = \frac{d}{2} \frac{W}{P} = \frac{0.6\ \text{m}}{2} \frac{480\ \text{N}}{144\ \text{N}} = 1\ \text{m}$$

h_{max} = 1.000 m

R; (과정의 타당성 검토) (가령, 미끄러지려는 순간, 힘 평형방정식 사용
넘어지려는 순간, 모멘트 평형방정식 사용)

T; (결과의 의미 검토) (가령, 캐스터 개수에 무관)

8.9 W = 600 N, α = 35°, μ_s = 0.25, μ_k = 0.20, θ = 60°

S; known W, α, μ_s, μ_k, θ, unknown P ⇒ 마찰각 ϕ_s, ϕ_k, 힘 삼각형, 삼각법

A; 마찰각 $\phi_s = \tan^{-1}(0.25) = 14.04°$

$\phi_k = \tan^{-1}(0.20) = 11.31°$

M;

F.B.D. force triangle

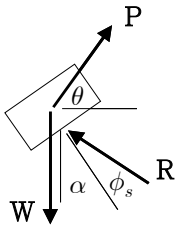
(a) 미끄러져 올라가려 할 때

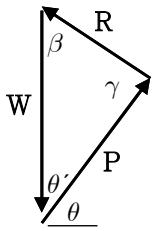
$\beta = \alpha + \phi_s = 35° + 14.04° = 49.04°$

$\theta' = 90° - \theta = 90° - 60° = 30°$

$\gamma = 180° - \beta - \theta'$

$= 180° - 49.04° - 30° = 100.96°$





$$\frac{P}{\sin\beta} = \frac{W}{\sin\gamma} \Rightarrow P = W\frac{\sin\beta}{\sin\gamma} = (600\text{ N})\frac{\sin 49.04^\circ}{\sin 100.96^\circ} = 461.5\text{ N} \Rightarrow P = 462\text{ N}$$

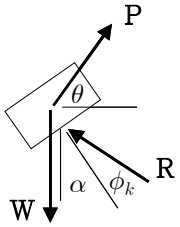
(b) 미끄러져 올라가고 있을 때

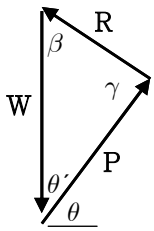
$\beta = \alpha + \phi_k = 35° + 11.31° = 46.31°$

$\theta' = 90° - \theta = 90° - 60° = 30°$

$\gamma = 180° - \beta - \theta'$

$= 180° - 46.31° - 30° = 103.69°$





$$\frac{P}{\sin\beta} = \frac{W}{\sin\gamma} \Rightarrow P = W\frac{\sin\beta}{\sin\gamma} = (600\text{ N})\frac{\sin 46.31^\circ}{\sin 103.69^\circ} = 446.5\text{ N} \Rightarrow P = 447\text{ N}$$

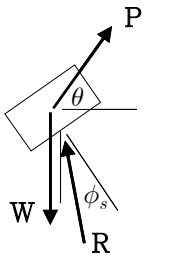
(c) 미끄러져 내려가려 할 때

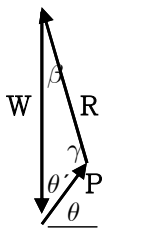
$\beta = \alpha - \phi_s = 35° - 14.04° = 20.96°$

$\theta' = 90° - \theta = 90° - 60° = 30°$

$\gamma = 180° - \beta - \theta'$

$= 180° - 20.96° - 30° = 129.04°$





$$\frac{P}{\sin\beta} = \frac{W}{\sin\gamma} \Rightarrow P = W\frac{\sin\beta}{\sin\gamma} = (600\text{ N})\frac{\sin 20.96^\circ}{\sin 129.04^\circ} = 276.3\text{ N} \Rightarrow P = 276\text{ N}$$

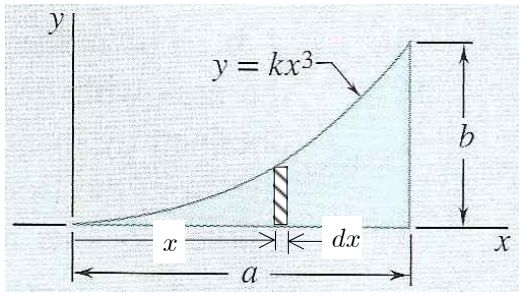
R;(과정의 타당성 검토) (가령, 수직반력과 마찰력의 합력 R을 몰라도 풀이할 수 있는 이유)

T;(결과의 의미 검토) (가령, 462 N > 447 N ⇒ 운동마찰력이 정지마찰력 최대값 보다 작음)

[9.1절]

9.4&8　$y = k x^3$,　　$(a,\ b)$　$\Rightarrow$　$b = k a^3$

$\Rightarrow$　$k = \dfrac{b}{a^3}$,　　$y = \dfrac{b}{a^3} x^3$



$dA = y\,dx = \dfrac{b}{a^3} x^3\ dx$

(문제 외 $A = \int dA = \int_0^a \dfrac{b}{a^3} x^3\,dx$

$= \dfrac{b}{a^3}\left[\dfrac{1}{4}x^4\right]_0^a = \dfrac{b}{a^3}\left[\dfrac{1}{4}(a^4 - 0)\right] = \dfrac{1}{4}ab$)

9.4　$dI_y = x^2\,dA = x^2\ y\ dx = x^2\ \dfrac{b}{a^3} x^3\ dx = \dfrac{b}{a^3} x^5\ dx$

$I_y = \int dI_y = \int_0^a \dfrac{b}{a^3} x^5\,dx = \dfrac{b}{a^3}\left[\dfrac{1}{6}x^6\right]_0^a = \dfrac{b}{a^3}\left[\dfrac{1}{6}(a^6 - 0)\right] = \dfrac{1}{6}a^3 b$

$\Rightarrow$　$I_y = 0.1667\ a^3 b$

(문제 외 $k_y^2 = \dfrac{I_y}{A} = \dfrac{\frac{1}{6}a^3 b}{\frac{1}{4}ab} = \dfrac{2}{3}a^2$　$\Rightarrow$　$k_y = \sqrt{\dfrac{2}{3}a^2} = 0.816\ a$)

9.8　$dI_x = \dfrac{1}{3}y^3\,dx$

$I_x = \int dI_x = \int_0^a \dfrac{1}{3}y^3\,dx = \dfrac{1}{3}\int_0^a \dfrac{b^3}{a^9}x^9\,dx = \dfrac{1}{3}\dfrac{b^3}{a^9}\left[\dfrac{1}{10}x^{10}\right]_0^a = \dfrac{1}{30}ab^3$

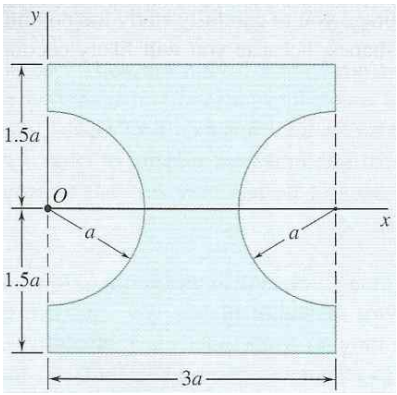
$\Rightarrow$　$I_x = 0.0333\ ab^3$

(문제 외 $k_x^2 = \dfrac{I_x}{A} = \dfrac{\frac{1}{30}ab^3}{\frac{1}{4}ab} = \dfrac{2}{15}b^2$　$\Rightarrow$　$k_x = \sqrt{\dfrac{2}{15}b^2} = 0.365\ b$)

[9.2절]

9.35 a = 20 mm

① 정사각형, ② 왼쪽 반원 구멍, ③ 오른쪽 반원 구멍



$I_{x1} = \frac{1}{12} b h^3 = \frac{1}{12}(3a)(3a)^3 = \frac{27}{4} a^4 = \frac{27}{4}$ (20 mm)^4

$= 1{,}080{,}000$ mm^4 $= 1{,}080 \times 10^3$ mm^4

$I_{x2} = I_{x3} = \frac{1}{8}\pi a^4 = \frac{1}{8}\pi(20 \text{ mm})^4$

$= 62{,}830$ mm^4 $= 62.83 \times 10^3$ mm^4

$I_x = I_{x1} - I_{x2} - I_{x3}$

$= (1{,}080 - 62.83 - 62.83) \times 10^3$ mm^4 $= 954.3 \times 10^3$ mm^4

$I_{y1} = \frac{1}{3} b^3 h = \frac{1}{3}(3a)^3(3a) = 27\ a^4 = 27$ (20 mm)^4

$= 4{,}320{,}000$ mm^4 $= 4{,}320 \times 10^3$ mm^4

$I_{y2} = \frac{1}{8}\pi a^4 = \frac{1}{8}\pi(20 \text{ mm})^4$

$= 62{,}830$ mm^4 $= 62.83 \times 10^3$ mm^4

$I_{y3} = \frac{1}{8}\pi a^4 - \left(\frac{1}{2}\pi a^2\right)\left(\frac{4}{3\pi}a\right)^2 + \left(\frac{1}{2}\pi a^2\right)\left(3a - \frac{4}{3\pi}a\right)^2$

$= \left[\frac{1}{8}\pi - \frac{8}{9\pi} + \frac{\pi}{2}\left(3 - \frac{4}{3\pi}\right)^2\right] a^4$

$= (10.530)$ (20 mm)^4 $= 1{,}684{,}800$ mm^4 $= 1{,}684 \times 10^3$ mm^4

$I_y = I_{y1} - I_{y2} - I_{y3} = (4{,}320 - 62.83 - 1{,}684) \times 10^3$ mm^4 $= 2{,}573 \times 10^3$ mm^4

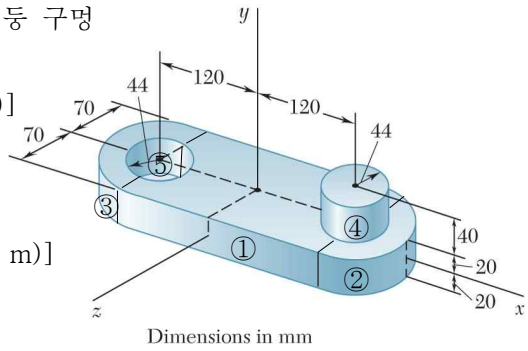
$\Rightarrow I_x = 954 \times 10^3$ mm^4 $= 0.954 \times 10^6$ mm^4

$I_y = 2{,}570 \times 10^3$ mm^4 $= 2.57 \times 10^6$ mm^4

[9.5절]

9.142 $\rho = 7{,}850 \text{ kg/m}^3$

① 사각기둥, ②,③ 반원기둥, ④ 원기둥, ⑤ 원기둥 구멍



Dimensions in mm

$m = \rho V$

$m_1 = (7{,}850 \text{ kg/m}^3)\,[(0.24 \text{ m})(0.14 \text{ m})(0.04 \text{ m})]$

$= (7.85\times10^3 \text{ kg/m}^3)\,(1.344\times10^{-3} \text{ m}^3)$

$= 10.550 \text{ kg}$

$m_2 = m_3 = (7{,}850 \text{ kg/m}^3)\,[\frac{\pi}{2}\,(0.07 \text{ m})^2\,(0.04 \text{ m})]$

$= (7.85\times10^3 \text{ kg/m}^3)\,(0.3079\times10^{-3} \text{ m}^3)$

$= 2.417 \text{ kg}$

$m_4 = m_5 = (7{,}850 \text{ kg/m}^3)\,[\pi\,(0.044 \text{ m})^2\,(0.04 \text{ m})]$

$= (7.85\times10^3 \text{ kg/m}^3)\,(0.2433\times10^{-3} \text{ m}^3) = 1.9098 \text{ kg}$

(a) $I_{x1} = \frac{1}{12} m_1 (b^2 + c^2)$

$= \frac{1}{12}(10.550 \text{ kg})[(0.14 \text{ m})^2 + (0.04 \text{ m})^2] = 18.638\times10^{-3} \text{ kg} \cdot \text{m}^2$

$I_{x2} = I_{x3} = \frac{1}{12} m_2 (3r_2^2 + L_2^2)$

$= \frac{1}{12}(2.417 \text{ kg})[3\,(0.07 \text{ m})^2 + (0.04 \text{ m})^2] = 3.283\times10^{-3} \text{ kg} \cdot \text{m}^2$

$I_{x4} = \frac{1}{12} m_4 (3r_4^2 + L_4^2) + m_4 d_4^2$

$= (1.9098 \text{ kg})\,\{\frac{1}{12}[3\,(0.044 \text{ m})^2 + (0.04 \text{ m})^2] + (0.04)^2\} = 4.234\times10^{-3} \text{ kg} \cdot \text{m}^2$

$I_{x5} = \frac{1}{12} m_5 (3r_5^2 + L_5^2)$

$= \frac{1}{12}(1.9098 \text{ kg})[3\,(0.044 \text{ m})^2 + (0.04 \text{ m})^2] = 1.179\times10^{-3} \text{ kg} \cdot \text{m}^2$

$I_x = I_{x1} + I_{x2} + I_{x3} + I_{x4} - I_{x5}$

$= [(18.638) + 2\,(3.283) + (4.234) - (1.179)]\times10^{-3} \text{ kg} \cdot \text{m}^2$

$= 28.259\times10^{-3} \text{ kg} \cdot \text{m}^2 \quad \Rightarrow \quad I_x = 28.3\times10^{-3} \text{ kg} \cdot \text{m}^2$

(b) $I_{y1} = \frac{1}{12} m_1 (a^2 + b^2)$

$= \frac{1}{12}(10.550 \text{ kg})[(0.24 \text{ m})^2 + (0.14 \text{ m})^2] = 67.87\times10^{-3} \text{ kg} \cdot \text{m}^2$

$I_{y2} = I_{y3} = \frac{1}{2} m_2 r_2^2 - m_2 d_{2a}^2 + m_2 d_{2b}^2$

$= (2.417 \text{ kg})\,\{\frac{1}{2}(0.07 \text{ m})^2 - [\frac{4}{3\pi}(0.07 \text{ m})]^2 + [(0.12 \text{ m}) + \frac{4}{3\pi}(0.07 \text{ m})]^2\}$

$= 57.96\times10^{-3} \text{ kg} \cdot \text{m}^2$

$I_{y4} = I_{y5} = \frac{1}{2} m_4 r_4^2 + m_4 d_4^2$

$= (1.9098 \text{ kg})\,[\frac{1}{2}(0.044 \text{ m})^2 + (0.12 \text{ m})^2] = 29.34\times10^{-3} \text{ kg} \cdot \text{m}^2$

$$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4} - I_{y5}$$

$$= [(67.87) + 2\,(57.96) + 0]\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= 183.79\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \qquad \Rightarrow \qquad I_y = 183.8\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

(c) $$m = m_1 + m_2 + m_3 + m_4 - m_5$$

$$= (10.550\ \text{kg}) + 2\,(2.417\ \text{kg}) + 0 = 15.384\ \text{kg}$$

$$k_x^2 = \frac{I_x}{m} = \frac{28.259\times 10^{-3}\ \text{kg}\cdot\text{m}^2}{15.384\ \text{kg}} = 1.8369\times 10^{-3}\ \text{m}^2$$

$$\Rightarrow \qquad k_x = 0.0429\ \text{m} = 42.9\ \text{mm}$$

$$k_y^2 = \frac{I_y}{m} = \frac{183.79\times 10^{-3}\ \text{kg}\cdot\text{m}^2}{15.384\ \text{kg}} = 11.947\times 10^{-3}\ \text{m}^2$$

$$\Rightarrow \qquad k_y = 0.1093\ \text{m} = 109.3\ \text{mm}$$