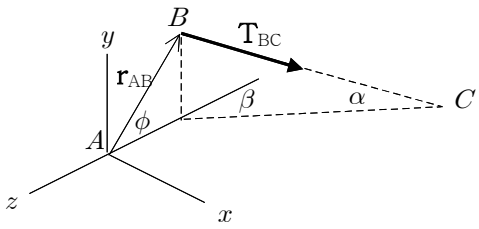
[3.1절]

3.25　l_{AB} = 2 m,　T_{BC} = 30 N,　α = 8°,　β = 30°,　ϕ = 45°

S; given l_{AB}, T_{BC}, α, β, ϕ,　required $\mathbf{M}_A$

M; 자유물체도(F.B.D.)



⇒ 점에 관한 모멘트, 위치벡터와 힘벡터의 벡터곱

A; ① 위치벡터

$\mathbf{r}_{AB} = l_{AB} \sin\phi\ \mathbf{j} - l_{AB} \cos\phi\ \mathbf{k}$

$= (2\text{ m}) \sin45°\ \mathbf{j} - (2\text{ m}) \cos45°\ \mathbf{k}$

$= 1.4142\ \mathbf{j} - 1.4142\ \mathbf{k}$ (m)

② 힘벡터

$T_y = -T_{BC} \sin\alpha = -(30\text{ N}) \sin8° = -4.175\text{ N}$

$T_h = T_{BC} \cos\alpha$

$T_x = T_h \sin\beta = T_{BC} \cos\alpha \sin\beta = (30\text{ N}) \cos8° \sin30° = 14.854\text{ N}$

$T_z = -T_h \cos\beta = -T_{BC} \cos\alpha \cos\beta = -(30\text{ N}) \cos8° \cos30° = -25.73\text{ N}$

$\mathbf{T}_{BC} = 14.854\ \mathbf{i} - 4.175\ \mathbf{j} - 25.73\ \mathbf{k}$ (N)

③ 벡터곱

$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{T}_{BC}$

$= [1.4142\ \mathbf{j} - 1.4142\ \mathbf{k}\ (\text{m})] \times [14.854\ \mathbf{i} - 4.175\ \mathbf{j} - 25.73\ \mathbf{k}\ (\text{N})]$

$= [(1.4142)(-25.73) - (-1.4142)(-4.175)]\ \mathbf{i} + [(-1.4142)(14.854) - 0]\ \mathbf{j}$

$+ [0 - (1.4142)(14.854)]\ \mathbf{k}$ (N · m)

$= -42.3\ \mathbf{i} - 21.0\ \mathbf{j} - 21.0\ \mathbf{k}$ (N · m)

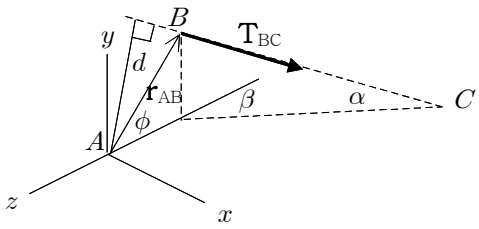
R(과정의 타당성) : (가령 T_x>0, T_y<0, T_z<0, 각 힘의 직각성분의 방향)

T(결과의 의미) ; (가령, M_x<0, M_y<0, M_z<0, 각 좌표축에 관한 모멘트의 방향)

3.30　T_{BC} = 30 N,　$\mathbf{M}_A = -42.3\ \mathbf{i} - 21.0\ \mathbf{j} - 21.0\ \mathbf{k}$ (N · m)

S; given T_{BC}, $\mathbf{M}_A$, required d

M; 자유물체도(F.B.D.)



⇒ $M_A = T_{BC}\, d$

A; $M_A = \sqrt{(-42.3)^2 + (-21.0)^2 + (-21.0)^2}$ N · m

$= 51.68$ N · m

$d = \dfrac{M_A}{T_{BC}} = \dfrac{51.68\text{ N} \cdot \text{m}}{30\text{ N}} = 1.7228\text{ m}$

⇒ $d = 1.723$ m

R; 모멘트 = 수직거리 × 힘 크기

T; 수직거리 $d < r_{AB}$ ($r_{AB} = \sqrt{0 + (1.4142)^2 + (-1.4142)^2}$ m = 2.00 m)

[3.2절]

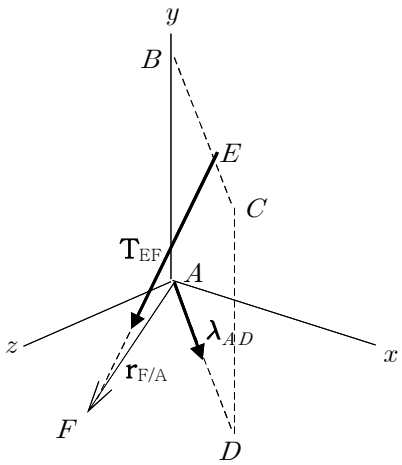
3.55 S; T_{EF} = 230 N,

M_{AD} = ?

⇒ 축 AD에 관한 모멘트

$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r} \times \mathbf{T}_{EF})$]

M; 자유물체도(F.B.D.)



A; ① 위치벡터

$\mathbf{r}_{F/A}$ = (525) **i** + (−350) **j** + (1425) **k** (mm)

= 0.525 **i** − 0.350 **j** + 1.425 **k** (m)

② 힘벡터를 구하기 위해 E의 좌표 필요

$BC = \sqrt{1200^2 + 900^2}$ mm = 1500 mm

$\frac{BE}{BC} = \frac{1125}{1500} = 0.750$

x = 0.750(1,200 mm) = 900 mm, y = 2,400 mm, z = 0.750(900 mm) = 675 mm

$\mathbf{r}_{F/E}$ = (525 − 900) **i** + (−350 − 2,400) **j** + (1,425 − 675) **k** (mm)

= (−375) **i** + (−2,750) **j** + (750) **k** (mm)

$d_{EF} = \sqrt{(-375)^2 + (-2{,}750)^2 + (750)^2}$ mm = 2,875 mm

$\boldsymbol{\lambda}_{EF} = \frac{1}{2{,}875}$ (−375) **i** + (−2,750) **j** + (750) **k**

$\mathbf{T}_{EF} = T_{EF}\ \boldsymbol{\lambda}_{EF}$

$= \frac{230\ \text{N}}{2{,}875}$ [(−375) **i** + (−2,750) **j** + (750) **k**]

= −30.0 **i** − 220 **j** + 60.0 **k** (N)

①② $\mathbf{M}_A = \mathbf{r}_{F/A} \times \mathbf{T}_{EF}$

= [0.525 **i** − 0.350 **j** + 1.425 **k** (m)] × [−30.0 **i** − 220 **j** + 60.0 **k** (N)]

= [(−0.350)(60.0)−(1.425)(−220)] **i** + [(1.425)(−30.0)−(0.525)(60.0)] **j**

+ [(0.525)(−220)−(−0.350)(−30.0)] **k** (N · m)

= 292.5 **i** − 74.25 **j** − 126.0 **k** (N · m)

③ $AD = \sqrt{(1{,}200\,\text{mm})^2 + (-300\,\text{mm})^2 + (900\,\text{mm})^2}$ = 1,529.7 mm

$\boldsymbol{\lambda}_{AD} = \frac{(1{,}200\,\text{mm})\mathbf{i} + (-300\,\text{mm})\mathbf{j} + (900\,\text{mm})\mathbf{k}}{1{,}529.7\,\text{mm}}$ = 0.7844 **i** − 0.1961 **j** + 0.5884 **k**

④ $M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \mathbf{M}_A = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{F/A} \times \mathbf{T}_{EF})$

= (0.7844 **i** − 0.1961 **j** + 0.5884 **k**) · [292.5 **i** − 74.25 **j** − 126.0 **k** (N · m)]

= (0.7844)(292.5) + (−0.1961)(−74.25) + (0.5884)(−126.0) = 98.50 (N · m)

⇒ M_{AD} = 169.9 N · m

R(과정의 타당성) : (가령 $\mathbf{M}_A$ 계산에 사용될 수 있는 위치벡터 $\mathbf{r}_{F/A}$, $\mathbf{r}_{F/D}$, $\mathbf{r}_{E/A}$, $\mathbf{r}_{E/D}$ 중 선택)

T(결과의 의미) ; (가령, M_{AD}>0, 선 AD에 관한 모멘트의 방향)

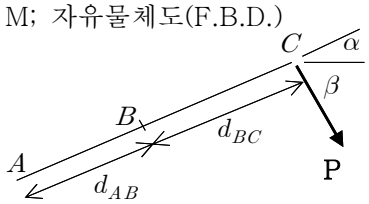
[3.3절]

3.82 P = 250 N, α = 30°, β = 60°, d_{AB} = 0.2 m, d_{BC} = 0.3 m

S; $\mathbf{P}$ = 250 N ↘60°

2차원 등가 힘-우력 계

M; 자유물체도(F.B.D.)



A; (a) $\Sigma\mathbf{F} = \mathbf{F}_B = \mathbf{P}$

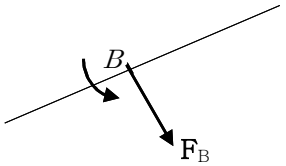
$\Rightarrow \quad \mathbf{F}_B = 250$ N ↘60°

$\circlearrowleft \Sigma M_B = M_B$

$= - d_{BC} P$

$= -(0.3\text{ m})(250\text{ N}) = -75.0\text{ N}\cdot\text{m}$

$\Rightarrow \quad \mathbf{M}_B = 75.0\text{ N}\cdot\text{m}\ \circlearrowleft$



(b) $\Sigma\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B = \mathbf{P}$

ΣF_x; $F_A \cos\phi + F_B \cos\phi = 0$

$\Rightarrow \quad (F_A + F_B)\cos\phi = 0$

$\Rightarrow \quad F_A + F_B = 0$, 또는 $\cos\phi = 0$

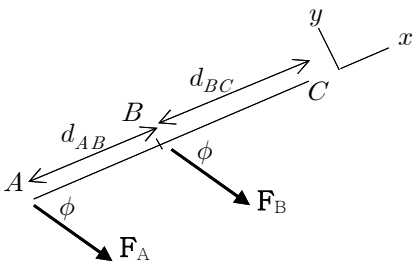
ΣF_y; $F_A \sin\phi + F_B \sin\phi = P$

$\Rightarrow \quad (F_A + F_B)\sin\phi = P$

$\Rightarrow \quad F_A + F_B \neq 0$, $\cos\phi = 0 \Rightarrow \phi = 90°$

$\Rightarrow \quad F_A + F_B = P \quad \cdots ①$

ΣM_B; $d_{AB} F_A = -d_{BC} P \quad \cdots ②$



② $\Rightarrow \quad F_A = -\dfrac{d_{BC}}{d_{AB}} P = -\dfrac{300\text{ mm}}{200\text{ mm}}(250\text{ N}) = -375\text{ N}$

① $\Rightarrow \quad F_B = P - F_A = (250\text{ N}) - (-375\text{ N}) = 625\text{ N}$

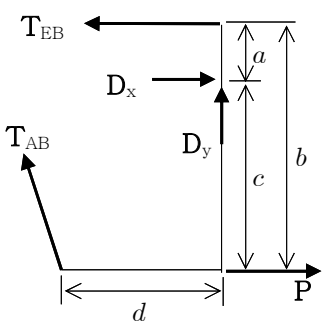
$\Rightarrow \quad \mathbf{F}_A = 375$ N ↖60.0°, $\mathbf{F}_B = 625$ N ↘60.0°

R(과정의 타당성) ; (가령, $\circlearrowleft \Sigma M_B$ 대신 $\circlearrowleft \Sigma M_C$ 을 비교하면?)

T(결과의 의미) ; (가령, $\circlearrowleft \Sigma M_A$ 는?)

[4.1절]

4.33 S; known ; **P** = 90 N →, a = 3 cm, b = 12 cm, d = 7 cm
unknown ; T, **D**　　M; 자유물체도(F.B.D.)



⇒ 모멘트 평형, 힘의 평형, 반력 유형 1&2

A; $c = b - a = (12\text{ cm}) - (3\text{ cm}) = 9\text{ cm}$

$T_{EB} = T_{AB} = T,\quad T_{ABx} = \frac{5}{13}T,\quad T_{ABy} = \frac{12}{13}T$

(a) $+\circlearrowleft\Sigma M_D = 0$;

$$a\ T_{EB} + c\ P - c\ T_{ABx} - d\ T_{ABy} = 0$$

$$\Rightarrow\ a\ T + c\ P - c\ \frac{5}{13}T - d\ \frac{12}{13}T = 0$$

$$\Rightarrow\ T = \frac{c}{\frac{5}{13}c + \frac{12}{13}d - a}P = \frac{(9\text{ cm})}{\frac{5}{13}(9\text{ cm}) + \frac{12}{13}(7\text{ cm}) - (3\text{ cm})}\ (90\text{ N})$$

$= (1.3)\ (90\text{ N}) = 117.0\text{ N}$　　⇒　　$T = 117.0$ N

(b) $\rightarrow\Sigma F_x = 0$;

$$D_x + P - T_{EB} - T_{ABx} = 0$$

$$\Rightarrow\ D_x = T + \frac{5}{13}T - P = \frac{18}{13}(117.0\text{ N}) - (90\text{ N}) = 72.0\text{ N}$$

$\uparrow\Sigma F_y = 0$;

$$D_y + T_{ABy} = 0$$

$$\Rightarrow\ D_y = -\frac{12}{13}T = -\frac{12}{13}(117.0\text{ N}) = -108.0\text{ N}$$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(72.0\text{ N})^2 + (-108.0\text{ N})^2} = 129.80\text{ N}$$

$$\theta = \tan^{-1}\frac{D_y}{D_x} = \tan^{-1}\frac{-108.0\text{ N}}{72.0\text{ N}} = \tan^{-1}(-1.500) = -56.3°$$

⇒　　**D** = 129.8 N ↘56.3°

R; (예: $+\circlearrowleft\Sigma M_A = 0$ 을 사용하면?
힘의 평형 방정식 ($\rightarrow\Sigma F_x = 0$, $\uparrow\Sigma F_y = 0$)을 먼저 사용하면?)

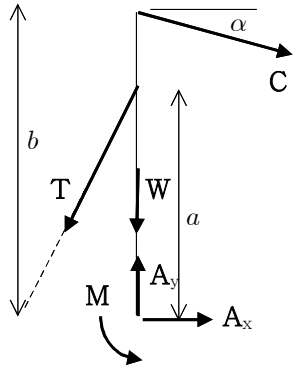
T; (예: 힌지 D의 역할)

4.45 m = 175 kg, C = 600 N, α = 15°, a = 3.6 m, b = 4.5 m, $M \le$ 500 N · m

S; 반력 유형 1&3(고정지지, 줄) M; 자유물체도(F.B.D.)

힘의 평형, 모멘트 평형,

$+\circlearrowleft \Sigma M_A = 0$



A; AD = 1.5 m, AB = 3.6 m

$BD = \sqrt{1.5^2 + 3.6^2}$ m = 3.90 m

$\frac{AD}{BD} = \frac{1.5 \text{ m}}{3.90 \text{ m}} = 0.3846$

M = $M\circlearrowleft$ 일 때, T_{max}

$+\circlearrowleft \Sigma M_A = 0$; $-M + a\ T_x - b\ C_x = 0$

$\Rightarrow \quad -M + a\ (\frac{AD}{BD}\ T) - b\ C\ \cos\alpha = 0$

$\Rightarrow \quad T = \frac{b\ C\cos\alpha + M}{a\ \frac{AD}{BD}} = \frac{(4.5 \text{ m})\,(600 \text{ N})\cos 15^\circ + (500 \text{ N} \cdot \text{m})}{(3.6 \text{ m})\,(0.3846)} = 2{,}244{,}7 \text{ N}$

M = $M\circlearrowright$ 일 때, T_{min}

$+\circlearrowleft \Sigma M_A = 0$; $M + a\ T_x - b\ C_x = 0$

$\Rightarrow \quad M + a\ (\frac{AD}{BD}\ T) - b\ C\ \cos\alpha = 0$

$\Rightarrow \quad T = \frac{b\ C\cos\alpha - M}{a\ \frac{AD}{BD}} = \frac{(4.5 \text{ m})\,(600 \text{ N})\cos 15^\circ - (500 \text{ N} \cdot \text{m})}{(3.6 \text{ m})\,(0.3846)} = 1{,}522.5 \text{ N}$

$\Rightarrow \quad T_{max}$ = 2,240 N, T_{min} = 1,522 N

R; (예: ΣF_x 또는 ΣF_y 의 용도는?)

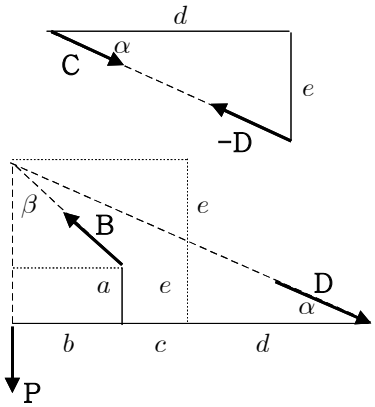
T; (예: T가 $T_{min} < T < T_{max}$ 이면?)

[4.2절]

4.65 P = 250 N, a = 30 mm, b = 60 mm, c = 40 mm, d = 100 mm, e = 60 mm

S; 두 힘의 평형, 세 힘의 평형, 반력 유형2
세 힘의 작용선이 한 점에서 만남, 힘 삼각형, 삼각법

M; 자유물체도(F.B.D.)



A; $\tan\alpha = \dfrac{e}{d} = \dfrac{60\text{ mm}}{100\text{ m}} = 0.6$

$\Rightarrow \quad \alpha = \tan^{-1}(0.6) = 31.0°$

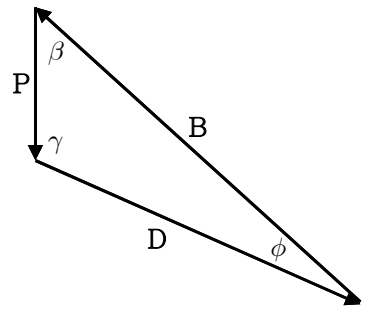
$\tan\beta = \dfrac{b}{2e-a} = \dfrac{60\text{ mm}}{90\text{ m}} = 0.667$

$\Rightarrow \quad \beta = \tan^{-1}(0.667) = 33.7°$

$\theta = 90° - 33.7° = 56.3°$

$\gamma = 90° + 31.0° = 121.0°$

$\phi = 180° - \beta - \gamma = 180° - 33.7° - 121.0° = 25.3°$



$\dfrac{B}{\sin\gamma} = \dfrac{P}{\sin\phi}$

$\Rightarrow \quad B = P\dfrac{\sin\gamma}{\sin\phi} = (250\text{ N})\ \dfrac{\sin 121.0^\circ}{\sin 25.3^\circ} = 501.4\text{ N}$

$\Rightarrow \quad \mathbf{B} = 501\text{ N} \nwarrow 56.3°$

$\dfrac{D}{\sin\beta} = \dfrac{P}{\sin\phi}$

$\Rightarrow \quad D = P\dfrac{\sin\beta}{\sin\phi} = (250\text{ N})\ \dfrac{\sin 33.7^\circ}{\sin 25.3^\circ} = 324.6\text{ N}$

$\Rightarrow \quad \mathbf{D} = 325\text{ N} \searrow 31.0°$

R; (예: 직각성분 방법으로 풀이 한다면 . . .)

T; (예: B의 반력의 방향, C의 반력의 방향 . . .)