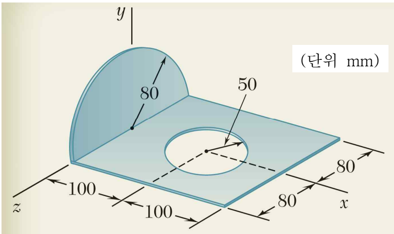
정 역 학　　2020년 학기말시험 (가/나/다반) [28점]　　2020. 12. 3.

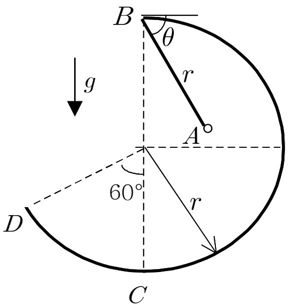
1.[4점(=1+3)] 도심과 중심에 관한 다음 문제에 답하여라.

(a) 임신한 침팬지는 두 다리로 서 있을 수 없다. 이에 비해 캥거루는 두 다리로 서 있을 수 있는 원리를 설명하여라.

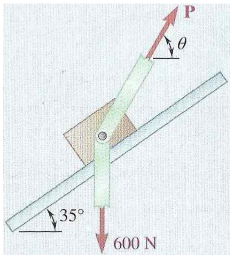
(b) A thin steel plate is cut and bent to form the machine part shown. Determine $\overline{X}$ of the centroid of this machine part.



2.[4점] The uniform and homogeneous wire $ABCD$ is bent into a circular arc and a straight section as shown and is attached to a hinge at A. Determine by <u>analysis</u> the <u>value of θ</u> for which the wire is in equilibrium for the indicated position.



3.[6점(=2+3+1)] 블록과 레일 간의 마찰계수가 μ_s = 0.30 이고 μ_k = 0.25 이다. 최소 크기의 힘 P로써 블록이 레일의 오른쪽 위 방향으로 미끄러지려 할 때, 마찰각을 사용하고 S.M.A.R.T. 과정에 따라, 힘 P의 <u>크기와 방향</u>을 구하여라.

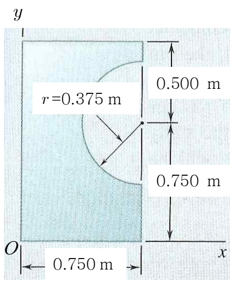


(a) 전략(strategy)과 모델링(modeling)

(b) 해석(analysis)

(c) 과정의 타당성 검토(reflect)와 결과의 의미 검토(think)

4.[6점] 어떤 보(beam)의 단면이 그림과 같다.

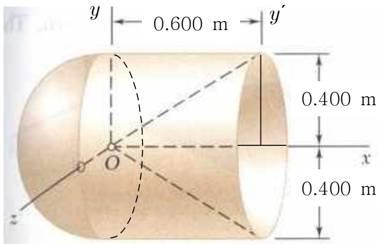


(a) 도심(centroid)의 $\overline{X}$ 좌표를 구하여라.

(b) 이 단면의 x축에 관한 <u>면적 관성모멘트</u> I_x와 <u>회전반경</u>(radius of gyration) k_x를 구하여라.

(c) 이 단면의 y축에 관한 <u>면적 관성모멘트</u> I_y와 O점에 관한 <u>극관성모멘트</u>(polar moment of inertia) J_O를 구하여라.

5.[8점] A homogeneous body of revolution shown was obtained by joining a hemisphere and a cylinder and carving out a cone. 재질의 밀도는 2800 kg/m^{3}이다.



(a) x축과 y축에 관한 <u>질량 관성모멘트</u> I_x와 I_y를 각각 구하여라.

(b) y'축에 관한 <u>질량 관성모멘트</u> $I_{y'}$를 구하여라.

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1\. (a) 침팬지의 무게중심을 지나는 수직선은 뒷발의 앞 끝 보다 앞을 지나 고꾸라지는 모멘트가 생김, 캥거루의 무게중심을 지나는 수직선은 뒷 다리의 무릎과 발끝 사이를 지나므로 회전시키려는 경향이 없음

(b) ① 사각 판

A = (200 mm)(160 mm) = 32,000 mm^2 , $\bar{x}$ = 100 mm

② 원판 구멍

$A = -\pi$ (50 mm)^2 = −7,854 mm^2 , $\bar{x}$ = 100 mm

③ 반원 판

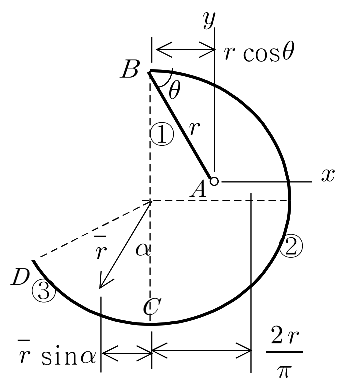
$A = \dfrac{1}{2}\pi$ (80 mm)^2 = 10,053 mm^2 , $\bar{x}$ = 0,

ΣA = 32,000 + (−7,854) + 10,053 (mm^2) = 34,199 mm^2

$\Sigma(\bar{x}A)$ = (100)(32,000) + (100)(−7,854) + (0)(10,053) = 2,414,600 mm^3

$$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{2{,}414{,}600\,\text{mm}^3}{34{,}199\,\text{mm}^2} = 70.60\ \text{mm} \qquad \Rightarrow \qquad \bar{X} = 70.6\ \text{mm}$$

2\. $\alpha = \dfrac{\pi}{6}$, $\sin\alpha = \dfrac{1}{2}$



$\Sigma M_A = 0 \ \Rightarrow\ \bar{X}L = 0 \ \Rightarrow\ \Sigma(\bar{x}L) = 0$

① AB; $L = r$, $\bar{x} = -\dfrac{1}{2}r\cos\theta$

② BC; $L = \pi r$, $\bar{x} = \dfrac{2}{\pi}r - r\cos\theta$

③ CD; $L = \dfrac{\pi}{3}r$,

$$\bar{r} = \frac{\sin\alpha}{\alpha}r = \frac{\sin 30^\circ}{\pi/6}r = \frac{3}{\pi}r$$

$$\bar{x} = -\bar{r}\sin\alpha - r\cos\theta$$

$$= -\left(\frac{3}{\pi}r\right)\left(\frac{1}{2}\right) - r\cos\theta = -\frac{3}{2\pi}r - r\cos\theta$$

$$\Sigma(\bar{x}L) = \left(-\frac{1}{2}r\cos\theta\right)r + \left(\frac{2r}{\pi} - r\cos\theta\right)\pi r - \left(\frac{3}{2\pi}r + r\cos\theta\right)\frac{\pi}{3}r = 0$$

$$\Rightarrow \left(\frac{1}{2}r + \pi r + \frac{\pi}{3}r\right)\cos\theta = 2r - \frac{1}{2}r$$

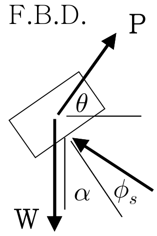
$$\Rightarrow \cos\theta = \frac{2-\frac{1}{2}}{\frac{1}{2}+\pi+\frac{\pi}{3}} = \frac{\frac{3}{2}}{\frac{1}{2}+\frac{4}{3}\pi} = \frac{3}{1+\frac{8}{3}\pi} = 0.3199$$

$$\Rightarrow \theta = \cos^{-1}(0.3199) = 71.34^\circ \qquad \Rightarrow \qquad \theta = 71.3^\circ$$

3. μ_s = 0.30, α = 35°, W = 600 N

(a) S; 미끄러져 올라가려 할 때, 마찰각, 삼각법 사용

M; F.B.D.



(b) A; $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.30$ = 16.70°

$\beta = \alpha + \phi_s$ = 35° + 16.70° = 51.70°

P의 크기 최소 ⇒ P⊥R ⇒ γ = 90°

θ' = 180° − γ − β = 180° − 90° − 51.70° = 38.30°

⇒ θ = 90° − θ' = 90° − 38.30° = 51.70°

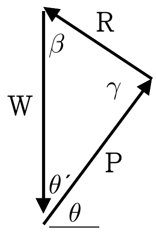
⇒ θ = 51.7°

$P = W\cos\theta'$ = (600 N) cos38.30° = 470.9 N

⇒ P = 471 N ⇒ P = 471 N ↗51.7°

($R = W\cos\beta$ = (600 N) cos51.70° = 371.9 N)

force triangle



(c) R(과정의 타당성 검토) ; 가령, 힘 삼각형 방법에 의해 최소 P 확인 cf. 직각성분 방법

T(결과의 의미 검토) ; 가령, P와 R의 수직성분과 W가 평형, P와 R의 수평성분이 평형을 이룸

(470.9 N) cos38.30° + (371.9 N) cos51.70° = 600 N

(470.9 N) sin38.30° − (371.9 N) sin51.70° = 0

4. (a)① 사각형, A = (0.750 m)(1.250 m) = 0.9375 m^2,

$\bar{x} = \frac{1}{2}$(0.750 m) = 0.375 m

② 반원, $A = -\frac{1}{2}\pi$(0.375 m)^2 = −0.2209 m^2,

$\bar{x}$ = (0.750 m) − $\frac{4}{3\pi}$(0.375 m) = 0.5908 m

ΣA = 0.9375 + (−0.2209) m^2 = 0.7166 m^2

$\Sigma(\bar{x}A)$ = (0.375)(0.9375) + (0.5908)(−0.2209) m^3 = 0.22105 m^3

$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{0.22105\ \text{m}^3}{0.7166\ \text{m}^2}$ = 0.3084 m ⇒ $\bar{X}$ = 0.308 m

(b) $I_{x1} = \frac{1}{3}$(0.750 m)(1.250 m)^3 = 0.4883 m^4

$I_{x2} = \frac{1}{2}\frac{\pi}{4}$(0.375 m)^4 + (0.2209 m^2) (0.750 m)^2 = 0.13202 m^4

$I_x = I_{x1} - I_{x2}$ = 0.4883−0.13202 m^4 = 0.3563 m^4 ⇒ I_x = 0.356 m^4

$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{0.3563\ \text{m}^4}{0.7166\ \text{m}^2}}$ = 0.7051 m ⇒ k_x = 0.705 m

(c) $I_{y1} = \frac{1}{3}$(1.250 m)(0.750 m)^3 = 0.17578 m^4

$I_{y2} = \frac{1}{2}\frac{\pi}{4}$(0.375 m)^4 − (0.2209 m^2) [$\frac{4}{3\pi}$(0.375 m)]^2

+ (0.2209 m^2) [0.750 − $\frac{4}{3\pi}$(0.375) m]^2 = 0.07929 m^4

$I_y = I_{y1} - I_{y3}$ = 0.17578 − 0.07929 m^4 = 0.09649 m^4 ⇒ I_y = 0.0965 m^4

$J_O = I_x + I_y$ = 0.3563 + 0.09649 m^4 = 0.4528 m^4 ⇒ J_O = 0.453 m^4

5. (a) $a = 0.400\ \text{m},\quad b = 0.600\ \text{m},\quad \rho = 2{,}800\ \text{kg/m}^3$

① 반구, $m_1 = \rho V_1 = \rho\left(\frac{1}{2}\frac{4}{3}\pi a^3\right) = \frac{2}{3}\pi a^3 \rho = \frac{2}{3}\pi(0.400\ \text{m})^3(2{,}800\ \text{kg/m}^3)$

$= 375.3\ \text{kg}$

② 원기둥, $m_2 = \rho V_2 = \rho(\pi a^2 b) = \pi a^2 b \rho = \pi(0.400\ \text{m})^2(0.600\ \text{m})(2{,}800\ \text{kg/m}^3)$

$= 844.5\ \text{kg}$

③ 원뿔, $m_3 = \rho V_3 = \rho\left(\frac{1}{3}\pi a^2 b\right) = \frac{1}{3}\pi a^2 b \rho = \frac{1}{3}\pi(0.400\ \text{m})^2(0.600\ \text{m})(2{,}800\ \text{kg/m}^3)$

$= 281.5\ \text{kg}$

$I_{x1} = \frac{2}{5} m_1 a^2 = \frac{2}{5}(375.3\ \text{kg})(0.400\ \text{m})^2 = 24.02\ \text{kg}\cdot\text{m}^2$

$I_{x2} = \frac{1}{2} m_2 a^2 = \frac{1}{2}(844.5\ \text{kg})(0.400\ \text{m})^2 = 67.56\ \text{kg}\cdot\text{m}^2$

$I_{x3} = \frac{3}{10} m_3 a^2 = \frac{3}{10}(281.5\ \text{kg})(0.400\ \text{m})^2 = 13.512\ \text{kg}\cdot\text{m}^2$

$I_x = I_{x1} + I_{x2} - I_{x3} = 24.02 + 67.56 - 13.512\ \text{kg}\cdot\text{m}^2 = 78.07\ \text{kg}\cdot\text{m}^2$

$\Rightarrow \quad I_x = 78.1\ \text{kg}\cdot\text{m}^2$

$I_{y1} = \frac{2}{5} m_1 a^2 = 24.02\ \text{kg}\cdot\text{m}^2$

$I_{y2} = \frac{1}{4} m_2 a^2 + \frac{1}{3} m_2 b^2 = \frac{1}{4}(844.5\ \text{kg})(0.400\ \text{m})^2 + \frac{1}{3}(844.5\ \text{kg})(0.600\ \text{m})^2$

$= 33.78 + 101.34\ \text{kg}\cdot\text{m}^2 = 135.12\ \text{kg}\cdot\text{m}^2$

(또는, $= \frac{1}{12} m_2(3a^2 + 4b^2) = \frac{1}{12}(844.5\ \text{kg})[3(0.400)^2 + 4(0.600)^2\ \text{m}^2] = 135.12\ \text{kg}\cdot\text{m}^2$)

$I_{y3} = \frac{3}{5} m_3\left(\frac{1}{4}a^2 + b^2\right) = \frac{3}{5}(281.5\ \text{kg})\left[\frac{1}{4}(0.400\ \text{m})^2 + (0.600\ \text{m})^2\right] = 67.56\ \text{kg}\cdot\text{m}^2$

$I_y = I_{y1} + I_{y2} - I_{y3} = 24.02 + 135.12 - 67.56\ \text{kg}\cdot\text{m}^2 = 91.58\ \text{kg}\cdot\text{m}^2$

$\Rightarrow \quad I_y = 91.6\ \text{kg}\cdot\text{m}^2$

(b) $I_{y'1} = I_{y1} - m_1 d_1^2 + m_1 d_2^2 \qquad d_1 = \frac{3}{8}a = \frac{3}{8}(0.400\ \text{m}) = 0.150\ \text{m}$

$d_2 = d_1 + b = 0.150 + 0.600\ \text{m} = 0.750\ \text{m}$

$= (24.02\ \text{kg}\cdot\text{m}^2) - (375.3\ \text{kg})(0.150\ \text{m})^2 + (375.3\ \text{kg})(0.750\ \text{m})^2$

$= 24.02 - 8.444 + 211.1\ \text{kg}\cdot\text{m}^2 = 226.7\ \text{kg}\cdot\text{m}^2$

$I_{y'2} = I_{y2} = 135.12\ \text{kg}\cdot\text{m}^2$

$I_{y'3} = \bar{I}_{y''3} + m_3 d_3^2 = \frac{3}{20} m_3\left(a^2 + \frac{1}{4}b^2\right) + m_3\left(\frac{1}{4}b\right)^2 = \frac{3}{20} m_3 a^2 + \frac{1}{10} m_3 b^2$

$= \frac{3}{20}(281.5\ \text{kg})(0.400\ \text{m})^2 + \frac{1}{10}(281.5\ \text{kg})(0.600\ \text{m})^2$

$= 6.756 + 10.134\ \text{kg}\cdot\text{m}^2 = 16.890\ \text{kg}\cdot\text{m}^2$

(또는, $= \frac{1}{20} m_3(3a^2 + 2b^2) = \frac{1}{20}(281.5\ \text{kg})[3(0.400)^2 + 2(0.600)^2\ \text{m}^2] = 16.890\ \text{kg}\cdot\text{m}^2$)

$I_{y'} = I_{y'1} + I_{y'2} - I_{y'3} = 226.7 + 135.12 - 16.890\ \text{kg}\cdot\text{m}^2 = 344.93\ \text{kg}\cdot\text{m}^2$

$\Rightarrow \quad I_{y'} = 345\ \text{kg}\cdot\text{m}^2$