정 역 학 　　2019년 학기말시험 (가/나/다반) [25점] 　　2019. 12. 5.

1.[4점] 정역학에 관한 다음 문제에 답하여라.

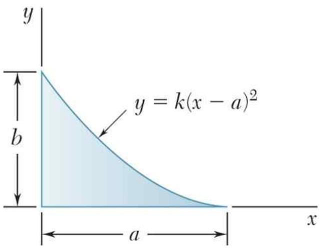
(a) 다음 글의 (　)에 적절한 영어 단어를 기입하여라.

① The center of gravity of a homogeneous body is also known as the (　　　　　) of the volume of the body.

② The product of the mass and the square of the distance provides a measure of the inertia of the system; i.e., a measure of the (　　　　　) the system offers when we try to set it in motion.

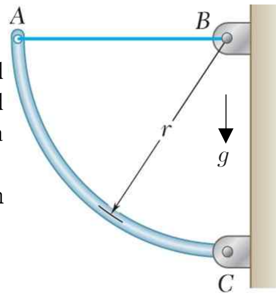
(b) 사진과 같이 소년이 수직 기둥에 정지상태를 유지하고 있다. 소년의 신체에 작용하는 힘들을 나타내는 자유물체도를 작성하여라. (치수 불필요, 빈 칸에 작성)

(c,d) 적분하여, x축에 관한 면적 관성모멘트를 구하여라.



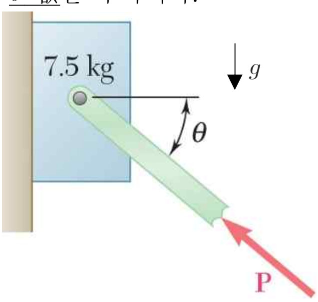
2.[3점] A uniform circular rod with a weight of 200 N and radius of 50 cm is attached to a pin at C and to the cable AB.

(a) Draw the free-body diagram of the rod. (치수 필요)



(b) Determine the tension in the cable AB by analysis.

3.[6점] 그림에 보인 7.5 kg의 블록과 수직면의 마찰계수가 μ_s = 0.40이고 μ_k = 0.30이다. 교재의 S.M.A.R.T 과정에 따라서, 블록이 평형상태를 유지하기 위한 힘 P의 최소값과 그 경우의 θ 값을 구하여라.

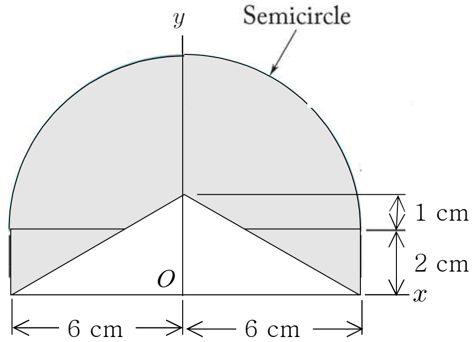


(a,b) 전략(strategy)과 모델링(modeling

(c,d,e) 해석(analysis)

(f) 과정의 타당성 검토(reflect)와 결과의 의미 검토(think)

4.[6점] 어떤 보(beam)의 단면이 그림과 같다. (음영 부분)

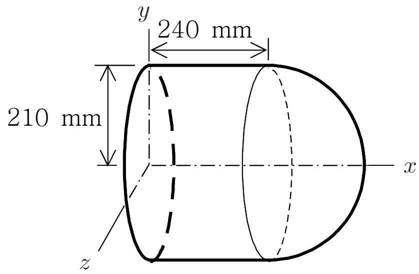


(a) 단면(area) 도심의 $\overline{Y}$좌표를 구하여라.

(b) 이 단면의 y축에 관한 면적 관성모멘트 I_y와 회전반경(radius of gyration) k_y를 구하여라.

(c) 이 단면의 O점에 관한 극관성모멘트(polar moment of inertia) J_O를 구하여라.

5.[6점] 그림의 3차원 입체 구조물은 알루미늄 원기둥과 철(steel) 반구(hemisphere)로 구성되어 있다. 알루미늄의 밀도는 2,700 kg/m^{3}이고, 철의 밀도는 7,860 kg/m^{3}이다.



(a) 무게중심의 $\overline{X}$ 좌표를 구하여라.

(b) x축에 관한 질량 관성모멘트 I_x를 구하여라.

(c) y축에 관한 질량 관성모멘트 I_y를 구하여라.

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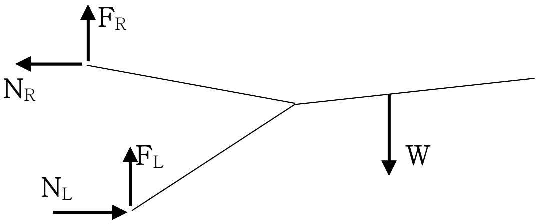
1. (a) ① centroid, ② resistance

(b) free-body diagram



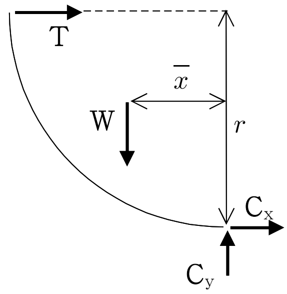
(c,d) $y = k\,(x-a)^2$, 　 $(0,\ b)$ $\Rightarrow$ $b = k\,a^2$ $\Rightarrow$ $k = \dfrac{b}{a^2}$ $\Rightarrow$ $y = \dfrac{b}{a^2}(x-a)^2$

$$dI_x = \frac{1}{3}y^3\,dx = \frac{1}{3}\frac{b^3}{a^6}(x-a)^6\,dx$$

$$I_x = \int dI_x = \int_0^a \frac{1}{3}\frac{b^3}{a^6}(x-a)^6\,dx = \frac{1}{3}\frac{b^3}{a^6}\left[\frac{1}{7}(x-a)^7\right]_0^a = \frac{1}{3}\frac{b^3}{a^6}\left\{\frac{1}{7}\left[0-(-a)^7\right]\right\} = \frac{1}{21}ab^3$$

$$\Rightarrow \quad I_y = 0.0476\ ab^3$$

2. (a) W = 200 N, r = 50 cm, $\bar{x} = \dfrac{2}{\pi}r = \dfrac{2}{\pi}$ (50 cm) = 31.8 cm



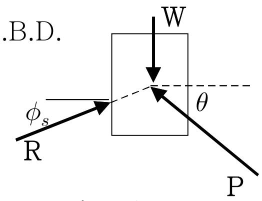
(b) $\Sigma M_C = 0$; $\bar{x}\ W - r\ T = 0$

$$\Rightarrow \quad T = \frac{\frac{2}{\pi}r}{r}W = \frac{2}{\pi}(200\ \text{N}) = 127.32\ \text{N} \quad \Rightarrow \quad T = 127.3\ \text{N}$$

3. m = 7.5 kg, μ_s = 0.40, μ_k = 0.30

(a,b) S; 미끄러져 내려가려 할 때, 마찰각 사용

M; F.B.D.



(c,d,e) A; W = (7.5 kg)(9.81 m/s^2) = 73.58 N

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40 = 21.80°$

$\alpha + \phi_s = 90°$

P의 크기 최소 $\Rightarrow$ P⊥R

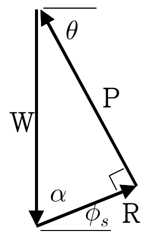
$\Rightarrow$ $\theta = \alpha = 90° - \phi_s = 90° - 21.80° = 68.20°$

$\Rightarrow$ 　 θ = 68.2°

$P = W\ \sin\alpha$ = (73.58 N) sin68.20° = 68.32 N

$\Rightarrow$ 　 P = 68.3 N

force triangle



($R = W\ \cos\alpha$ = (73.58 N) cos68.20° = 27.32 N)

(f) R(과정의 타당성 검토) ; 가령, 힘 삼각형 방법에 의해 최소 P 확인 cf. 직각성분 방법

T(결과의 의미 검토) ; 가령, P와 R의 수직성분과 W가 평형, P와 R의 수평성분이 평형을 이룸

(27.32 N) sin21.80° + (68.32 N) sin68.20° = 78.58 N

(27.32 N) cos21.80° - (68.32 N) cos68.20° = 0

4. (a)① 사각형, $A = (12\ \text{cm})(2\ \text{cm}) = 24\ \text{cm}^2$, $\bar{y} = \dfrac{1}{2}(2\ \text{cm}) = 1.0\ \text{cm}$

② 반원, $A = \dfrac{1}{2}\pi(6\ \text{cm})^2 = 56.55\ \text{cm}^2$, $\bar{y} = (2\ \text{cm}) + \dfrac{4}{3\pi}(6\ \text{cm}) = 4.546\ \text{cm}$

③ 삼각형, $A = -\dfrac{1}{2}(12\ \text{cm})(3\ \text{cm}) = -18.0\ \text{cm}^2$, $\bar{y} = \dfrac{1}{3}(3\ \text{cm}) = 1.0\ \text{cm}$

$\Sigma A = 24 + 56.55 + (-18.0)\ (\text{cm}^2) = 62.55\ \text{cm}^2$

$\Sigma(\bar{y}A) = (1.0)(24) + (4.546)(56.55) + (1.0)(-18.0)\ (\text{cm}^3) = 263.1\ \text{cm}^3$

$\overline{Y} = \dfrac{\Sigma(\bar{y}A)}{\Sigma A} = \dfrac{263.1\ \text{cm}^3}{62.55\ \text{cm}^2} = 4.206\ \text{cm} \quad \Rightarrow \quad \overline{Y} = 4.21\ \text{cm}$

(b) $I_{y1} = \dfrac{1}{12}(2\ \text{cm})(12\ \text{cm})^3 = 288.0\ \text{cm}^4$

$I_{y2} = \dfrac{1}{2}\dfrac{\pi}{4}(6\ \text{cm})^4 = 508.9\ \text{cm}^4$

$I_{y3} = 2[\dfrac{1}{12}(3\ \text{cm})(6\ \text{cm})^3] = 108.0\ \text{cm}^4$

$I_y = I_{y1} + I_{y2} - I_{y3} = 288.0 + 508.9 - 108.0\ (\text{cm}^4) = 688.9\ \text{cm}^4 \quad \Rightarrow \quad I_y = 689\ \text{cm}^4$

$k_y = \sqrt{\dfrac{I_y}{A}} = \sqrt{\dfrac{688.9\ \text{cm}^4}{62.55\ \text{cm}^2}} = 3.319\ \text{cm} \quad \Rightarrow \quad k_y = 3.32\ \text{cm}$

(c) $I_{x1} = \dfrac{1}{3}(12\ \text{cm})\ (2\ \text{cm})^3 = 32.0\ \text{cm}^4$

$I_{x2} = \dfrac{1}{2}\dfrac{\pi}{4}(6\ \text{cm})^4 - (56.55\ \text{cm}^2)\ [\dfrac{4}{3\pi}(6\ \text{cm})]^2 + (56.55\ \text{cm}^2)\ [2 + \dfrac{4}{3\pi}(6\ \text{cm})]^2$
$= 1{,}311.1\ \text{cm}^4$

$I_{x3} = \dfrac{1}{12}(12\ \text{cm})\ (3\ \text{cm})^3 = 27.0\ \text{cm}^4$

$I_x = I_{x1} + I_{x2} - I_{x3} = 32.0 + 1{,}311.1 - 27.0\ (\text{cm}^4) = 1316.1\ \text{cm}^4$

$\Rightarrow \quad I_x = 1{,}316\ \text{cm}^4$

$J_O = I_x + I_y = 1{,}316.1 + 688.9\ (\text{cm}^4) = 2{,}005\ \text{cm}^4 \quad \Rightarrow \quad J_O = 2{,}010\ \text{cm}^4$

5. ① 원기둥, 밀도 ρ_a = 2,700 kg/m³, 반지름 a = 0.210 m, 높이 h = 0.240 m

② 반구, 밀도 ρ_s = 7,860 kg/m³, 반지름 a = 0.210 m

(a) ① $m_1 = \rho_a V_1 = \rho_a (\pi a^2 h) = (2{,}700 \text{ kg/m}^3)\ [\pi\ (0.210 \text{ m})^2\ (0.240 \text{ m})] = 89.78 \text{ kg}$

$$\bar{x} = \frac{1}{2}h = \frac{1}{2}(0.240 \text{ m}) = 0.120 \text{ m}$$

② $m_2 = \rho_s V_2 = \rho_s \left(\frac{1}{2}\frac{4}{3}\pi a^3\right) = (7{,}860 \text{ kg/m}^3)\ \left[\frac{2}{3}\pi (0.210 \text{ m})^3\right] = 152.45 \text{ kg}$

$$\bar{x} = h + \frac{3}{8}a = (0.240 \text{ m}) + \frac{3}{8}(0.210 \text{ m}) = 0.3188 \text{ m}$$

$\Sigma m = 89.78 + 152.45 \text{ kg} = 242.2 \text{ kg}$

$\Sigma(\bar{x} m) = (0.120)(89.78) + (0.3188)(152.45) \text{ kg}\cdot\text{m} = 59.37 \text{ kg}\cdot\text{m}$

$$\bar{X} = \frac{\Sigma(\bar{x} m)}{\Sigma m} = \frac{59.37 \text{ kg}\cdot\text{m}}{242.2 \text{ kg}} = 0.2451 \text{ m} \qquad \Rightarrow \qquad \bar{X} = 0.245 \text{ m} = 245 \text{ mm}$$

(b) $I_{x1} = \frac{1}{2}m_1 a^2 = \frac{1}{2}(89.78 \text{ kg})\ (0.210 \text{ m})^2 = 1.9796 \text{ kg}\cdot\text{m}^2$

$I_{x2} = \frac{2}{5}m_2 a^2 = \frac{2}{5}(152.45 \text{ kg})\ (0.210 \text{ m})^2 = 2.689 \text{ kg}\cdot\text{m}^2$

$I_x = I_{x1} + I_{x2} = 1.9796 + 2.689 \text{ kg}\cdot\text{m}^2 = 4.669 \text{ kg}\cdot\text{m}^2 \qquad \Rightarrow \qquad I_x = 4.67 \text{ kg}\cdot\text{m}^2$

(c) $I_{y1} = \frac{1}{4}m_1 a^2 + \frac{1}{3}m_1 h^2 = m_1\left[\frac{1}{4}a^2 + \frac{1}{3}h^2\right]$

$= (89.78 \text{ kg})\ \left[\frac{1}{4}(0.210 \text{ m})^2 + \frac{1}{3}(0.240 \text{ m})^2\right] = 2.714 \text{ kg}\cdot\text{m}^2$

$I_{y2} = \frac{2}{5}m_2 a^2 - m_2\left(\frac{3}{8}a\right)^2 + m_2\left(h + \frac{3}{8}a\right)^2 = m_2\left[\frac{2}{5}a^2 - \left(\frac{3}{8}a\right)^2 + \left(h + \frac{3}{8}a\right)^2\right]$

$= (152.45 \text{ kg})\ \left\{\frac{2}{5}(0.210 \text{ m})^2 - \left[\frac{3}{8}(0.210 \text{ m})\right]^2 + \left[(0.240 \text{ m}) + \frac{3}{8}(0.210 \text{ m})\right]^2\right\}$

$= 17.233 \text{ kg}\cdot\text{m}^2$

$I_y = I_{y1} + I_{y2} = 2.714 + 17.233 \text{ kg}\cdot\text{m}^2 = 19.947 \text{ kg}\cdot\text{m}^2 \qquad \Rightarrow \qquad I_y = 19.95 \text{ kg}\cdot\text{m}^2$