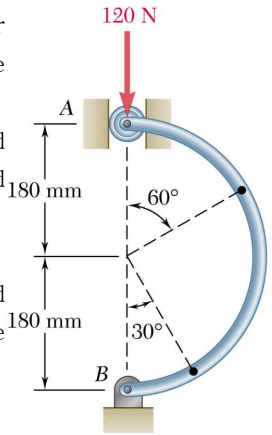
정 역 학 2014년 학기말시험 (가반) [25점] 2014. 12. 12.

1.[2점] 척추동물인 원숭이 중 챔팬지는 인간과 가장 유사하다고 하며, 때때로 인간의 2족 보행과 비슷한 동작을 하기도 한다. 임신한 인간은 배가 불룩해도 여전히 두 발로 서있을 수 있지만, 임신한 침팬지는 그러기가 더 어렵다. 그 이유를 2~3문장으로 서술하여라.

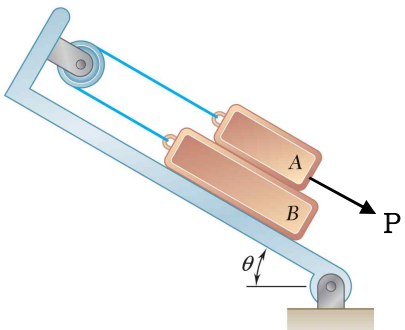
2.[5점] A uniform and homogeneous circular rod of weight 700 N and radius 180 mm is supported by a roller at A and attached to a pin at B in a vertical plane. The weight of the roller at A is 120 N as shown in the figure.

(a) Determine the magnitude and direction of the reaction exerted on the rod at A.

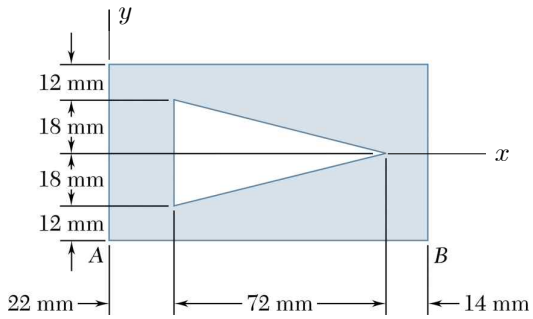
(b) Determine the horizontal and vertical components of the reaction at B.



3.[6점] 무게 30 N인 블록 A와 40 N인 블럭 B가 그림과 같이 경사면에 놓여 있다. 도르래 이외의 모든 접촉면 간의 정지마찰계수는 0.20이고, 경사각 θ는 30°이다. 힘 P가 그림과 같이 블럭 A에 가해져서 두 블럭이 움직이려 할 때, 줄에 작용하는 장력(tension)의 크기와 힘 P의 크기를 구하여라.



4.[6점] 단면이 그림과 같은 beam이 있다.



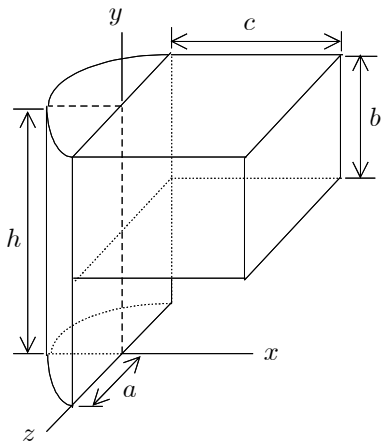
(a) 도심(centroid)의 $\bar{X}$좌표를 구하여라.

(b) 이 단면의 y축에 관한 면적 관성모멘트 I_y를 구하여라.

(삼각형 $I = \frac{1}{12}bh^3$, $\bar{I} = \frac{1}{36}bh^3$)

(c) 이 단면의 y축에 관한 회전반경(radius of gyration) k_y를 구하여라.

5.[6점] 그림과 같이 구조물이 반원기둥과 직육면체로 구성되어 있다. 재질은 균질(homogeneous)이고 밀도는 7500 kg/m^{3}이다. 치수는 h = 0.30 m, a = 0.10 m, b = 0.15 m, c = 0.25 m 이다.



(a) 무게중심의 $\bar{X}$좌표를 구하여라.

(b) y축에 관한 질량 관성모멘트 I_y를 구하여라.

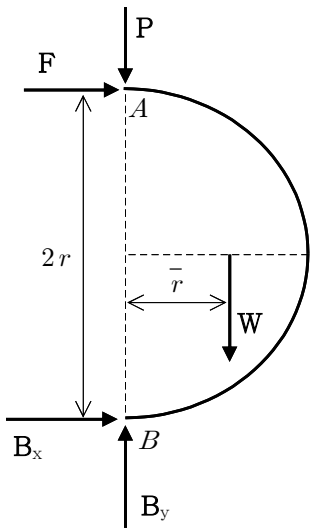
(c) x축에 관한 질량 관성모멘트 I_x를 구하여라.

정 역 학　　2014년 학기말시험 (가반) 해 답

1. 서술 [인간 – 요추 휨, 척추가 뒤로 젖힘, 무게 중심
침팬지 – 요추 구조 차이 없음, 몸무게, 무게 중심]

2. W = 700 N, P = 120 N, r = 180 mm

$\bar{r} = \frac{2r}{\pi} = \frac{2}{\pi}(180\text{ mm}) = 114.59\text{ mm}$



(a) $\Sigma M_B = 0$

$W\bar{r} + F(2r) = 0$

$\Rightarrow \quad F = -W\frac{\bar{r}}{2r} = -(700\text{ N})\frac{114.59}{2(180)} = -222.8\text{ N}$

$\Rightarrow \quad \mathbf{F} = 223\text{ N} \leftarrow$

(b) $\Sigma F_x = 0$

$B_x + F = 0$

$\Rightarrow \quad B_x = -F = 222.8\text{ N}$

$\Sigma F_y = 0$

$B_y - W - P = 0$

$\Rightarrow \quad B_y = W + P = (700\text{ N}) + (120\text{ N}) = 820\text{ N}$

$\mathbf{B_x} = 223\text{ N} \rightarrow$, $\mathbf{B_y} = 820\text{ N} \uparrow$

3. W_A = 30 N, W_B = 40 N, μ_s = 0.20, $\theta = 30°$

블럭 A

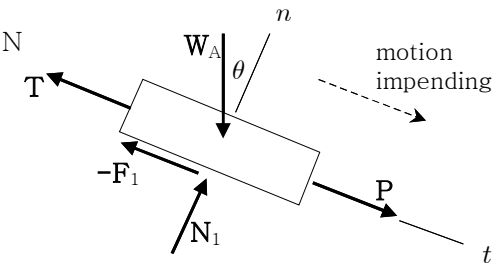
$\nearrow \Sigma F_n = 0$; $N_1 - W_A\cos\theta = 0$

$\Rightarrow \quad N_1 = W_A\cos\theta = (30\text{ N})\cos30° = 25.98\text{ N}$

$F_1 = \mu_s N_1 = (0.20)(25.98\text{ N}) = 5.196\text{ N}$

$\searrow \Sigma F_t = 0$; $P - T - F_1 + W_A\sin\theta = 0$

$\Rightarrow \quad P = T + F_1 - W_A\sin\theta \quad \cdots ①$



블록 B

$\nearrow \Sigma F_n = 0$; $N_2 - N_1 - W_B\cos\theta = 0$

$\Rightarrow \quad N_2 = N_1 + W_B\cos\theta$

$= (25.98\text{ N}) + (40\text{ N})\cos30° = 60.62\text{ N}$

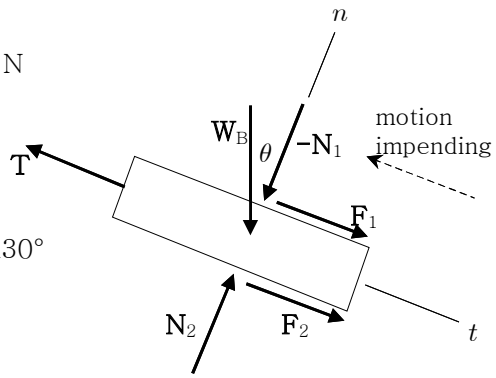
$F_2 = \mu_s N_2 = (0.20)(60.62\text{ N}) = 12.124\text{ N}$

$\searrow \Sigma F_t = 0$; $-T + F_1 + F_2 + W_B\sin\theta = 0$

$\Rightarrow \quad T = F_1 + F_2 + W_B\sin\theta$

$= (5.196\text{ N}) + (12.124\text{ N}) + (40\text{ N})\sin30°$

$= 37.32\text{ N}$



① $\Rightarrow \quad P = (37.32\text{ N}) + (5.196\text{ N}) - (30\text{ N})\sin30°$

$= 27.52\text{ N}$

$\Rightarrow \quad T = 37.3\text{ N}, \quad P = 27.5\text{ N}$

4. (a) ① 직사각형

$a_1 = 22 + 72 + 14 \text{ mm} = 108 \text{ mm}, \quad b_1 = 2\,(12 + 18) \text{ mm} = 60 \text{ mm}$

$A_1 = a_1 b_1 = (108 \text{ mm})(60 \text{ mm}) = 6{,}480 \text{ mm}^2$

$\bar{x}_1 = \frac{1}{2} a_1 = \frac{1}{2}(108 \text{ mm}) = 54 \text{ mm}$

② 삼각형

$A_2 = -\frac{1}{2} b_2 h_2 = -\frac{1}{2}(36 \text{ mm})(72 \text{ mm}) = -1{,}296 \text{ mm}^2$

$\bar{x}_2 = (22 \text{ mm}) + \frac{1}{3} h_2 = (22 \text{ mm}) + \frac{1}{3}(72 \text{ mm}) = 46 \text{ mm}$

$\Sigma A = (6{,}480 \text{ mm}^2) + (-1{,}296 \text{ mm}^2) = 5{,}184 \text{ mm}^2$

$\Sigma(\bar{x} A) = (54 \text{ mm})(6{,}480 \text{ mm}^2) + (46 \text{ mm})(-1{,}296 \text{ mm}^2) = 290{,}304 \text{ mm}^3$

$\bar{X} = \frac{\Sigma(\bar{x} A)}{\Sigma A} = \frac{290{,}304 \text{ mm}^3}{5{,}184 \text{ mm}^2} = 56.0 \text{ mm}$

(b) $I_{y1} = \frac{1}{3} b_1 a_1^3 = \frac{1}{3}(60 \text{ mm})\,(108 \text{ mm})^3 = 25.19 \times 10^6 \text{ mm}^4$

$I_{y2} = \frac{1}{36} b_2 h_2^3 + A_2 \bar{x}_2^2 = \frac{1}{36}\,(36 \text{ mm})\,(72 \text{ mm})^3 + (1{,}296 \text{ mm})\,(46 \text{ mm})^2$

$= 3.116 \times 10^6 \text{ mm}^4$

$I_y = I_{y1} - I_{y2} = (25.19 \times 10^6 \text{ mm}^4) - (3.116 \times 10^6 \text{ mm}^4) = 22.07 \times 10^6 \text{ mm}^4$

$\Rightarrow \quad I_y = 22.1 \times 10^6 \text{ mm}^4$

(c) $k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{22.07 \times 10^6 \text{ mm}^4}{5{,}184 \text{ mm}^2}} = 65.24 \text{ mm}$

$\Rightarrow \quad k_y = 65.2 \text{ mm}$

5. ① 반원기둥, ② 직육면체

(a) ① $V_1 = \frac{1}{2}\pi a^2 h = \frac{1}{2}\pi(0.10\ \text{m})^2\ (0.30\ \text{m}) = 4.712\times10^{-3}\ \text{m}^3$

$\bar{x} = -\frac{4}{3\pi}a = -\frac{4}{3\pi}(0.10\ \text{m}) = -42.44\times10^{-3}\ \text{m}$

② $V_2 = (2a)\,b\,c = (2\times0.10\ \text{m})(0.15\ \text{m})(0.25\ \text{m}) = 7.50\times10^{-3}\ \text{m}^3$

$\bar{x} = \frac{1}{2}c = \frac{1}{2}(0.25\ \text{m}) = 0.125\ \text{m} = 125.0\times10^{-3}\ \text{m}$

$\Sigma V = (4.712 + 7.50)\times10^{-3}\ \text{m}^3 = 12.212\times10^{-3}\ \text{m}^3$

$\Sigma(\bar{x}V) = [(-42.44)(4.712) + (125.0)(7.50)]\times10^{-6}\ \text{m}^4 = 737.5\times10^{-6}\ \text{m}^4$

$\bar{X} = \frac{\Sigma(\bar{x}V)}{\Sigma V} = \frac{737.5\times10^{-6}\ \text{m}^4}{12.212\times10^{-3}\ \text{m}^3} = 0.0604\ \text{m} = 60.4\ \text{mm}$

(b) $m_1 = \rho V_1 = (7{,}500\ \text{kg/m}^3)\ (4.712\times10^{-3}\ \text{m}^3) = 35.34\ \text{kg}$

$m_2 = \rho V_2 = (7{,}500\ \text{kg/m}^3)\ (7.50\times10^{-3}\ \text{m}^3) = 56.25\ \text{kg}$

$I_{y1} = \frac{1}{2}m_1 a^2 = \frac{1}{2}(35.34\ \text{kg})\ (0.10\ \text{m})^2 = 0.1767\ \text{kg·m}^2$

$I_{y2} = [\frac{1}{12}m_2(2a)^2 + \frac{1}{3}m_2 c^2] = \frac{1}{3}m_2\ (a^2 + c^2) = \frac{1}{3}(56.25\ \text{kg})\ [(0.10\ \text{m})^2 + (0.25\ \text{m})^2]$

$= 1.3594\ \text{kg·m}^2$

$I_y = I_{y1} + I_{y2} = (0.1767 + 1.3594)\ \text{kg·m}^2 = 1.5361\ \text{kg·m}^2$

$\Rightarrow \quad I_y = 1.536\ \text{kg·m}^2$

(c) $I_{x1} = \frac{1}{4}m_1 a^2 + \frac{1}{3}m_1 h^2 = m_1(\frac{1}{4}a^2 + \frac{1}{3}h^2) = (35.34\ \text{kg})\ [\frac{1}{4}(0.10\ \text{m})^2 + \frac{1}{3}(0.30\ \text{m})^2]$

$= 1.1486\ \text{kg·m}^2$

$I_{x2} = \frac{1}{12}m_2[(2a)^2 + b^2] + m_2 d_y^2 = m_2\ [\frac{1}{3}a^2 + \frac{1}{12}b^2 + (h - \frac{1}{2}b)^2]$

$= (56.25\ \text{kg})\ [\frac{1}{3}(0.10\ \text{m})^2 + \frac{1}{12}(0.15\ \text{m})^2 + (0.30\text{-}0.075\ \text{m})^2]$

$= 3.1406\ \text{kg·m}^2$

$I_x = I_{x1} + I_{x2} = (1.1486 + 3.1406)\ \text{kg·m}^2 = 4.289\ \text{kg·m}^2$

$\Rightarrow \quad I_x = 4.29\ \text{kg·m}^2$