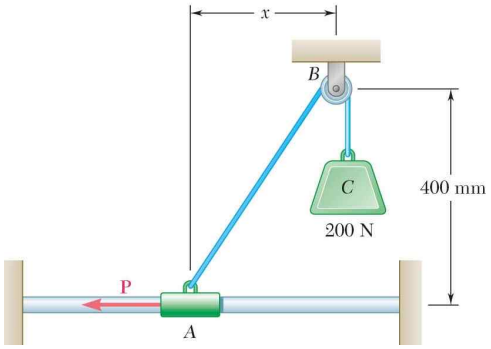
정 역 학 2011년 시험1 (가반) [20점] 2011. 9. 29.

1.[4점] 다음 물음에 답하라.

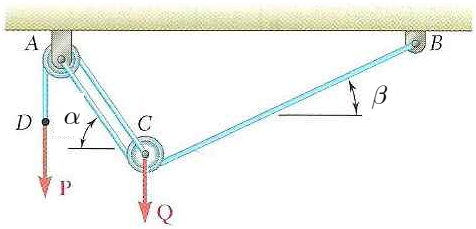
(a) 어느 영화에서 활을 잘 쏘는 주인공이 화살을 비틀어 쏘는 장면이 있다. 주인공을 포함해 세 사람이 일렬로 서 있는 상황에서 화살이 곡선으로 날아가며 앞 사람을 피해서 뒷사람에게 맞았다. 이 장면은 역학적으로 타당하지 않다고 하는데, 그 이유를 Newton의 제1법칙과 제2법칙을 인용하여 설명하라.

(b) '기계공학(mechanical engineering)'을 풍력발전기에 적용할 가능성들을 제시하라.

2.[2점] 아래 그림의 문제에서, 점 A의 자유물체도(free-body diagram)와 점 B의 자유물체도를 아래 여백에 각각 그려라.



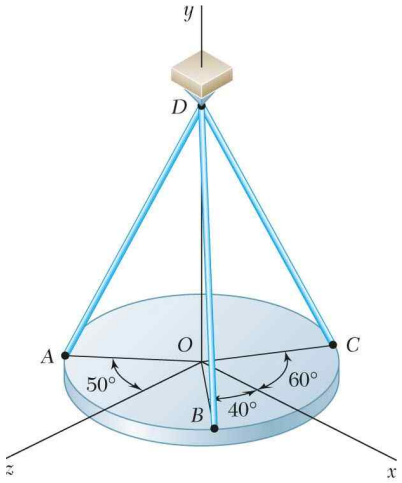
3.[6점] A load **Q** is applied to the pulley C, which can roll on the cable ACB. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load **P**. We know that α = 50°, β = 30°.



(a) Knowing that P = 375 N, determine the magnitude of load **Q** by the rectangular components.

(b) Knowing that P = 600 N, determine the tension in cable ACB by the force triangle.

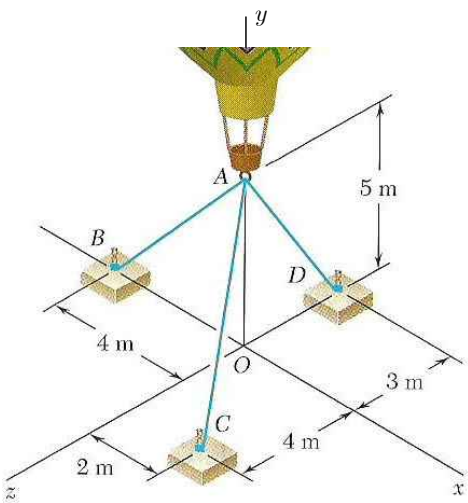
4.[4점] 수평 원판이 세 개의 줄에 매달려 있다. 세 줄이 수직 축과 이루는 각도는 각각 35°이다.



(a) 줄 AD의 장력이 55 N일 때, 줄 AD에 의해 점 A에 가해지는 힘의 직각성분 $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{F}_z$를 구하여라.

(b) 줄 CD에 의해 점 D에 가해지는 힘의 방향을 나타내기 위한 단위벡터 $\boldsymbol{\lambda}_{DC}$를 구하여라.

5.[4점] 그림과 같이 세 개의 줄로 열기구를 매어 놓았다. 줄 AB의 장력이 270 N이다.



(a) 줄 AC에 의해 점 A에 가해지는 힘이 x축, y축과 이루는 각도 θ_x, θ_y를 각각 구하여라.

(b) 줄 AC의 장력을 구하여라.

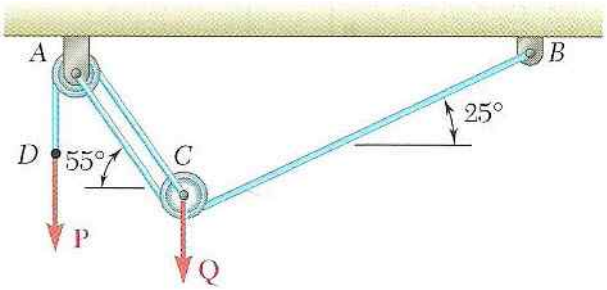
정 역 학 2011년 시험1 (나반) [20점] 2011. 9. 28.

1.[4점] 다음 물음에 답하라.

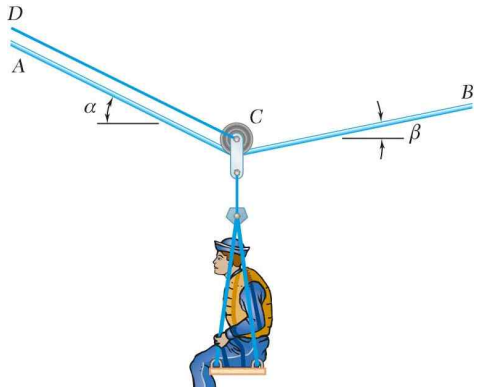
(a) 마술사가 <공중부양> 묘기를 보인다. '정역학(statics)'의 정의와 '평형(equilibrium)'의 정의에 따라 이 상황을 설명하라.

(b) 서양의 힘 단위에 다음과 같은 관계가 있다. 1 lb = 0.4536 kgf, 중력가속도 g = 9.81 m/s^2 = 32.2 ft/s^2. 무게 300 N은 몇 lb인가?

2.[2점] 아래 그림의 문제에서, 점 A의 자유물체도(free-body diagram)와 점 C의 자유물체도를 아래 여백에 각각 그려라.



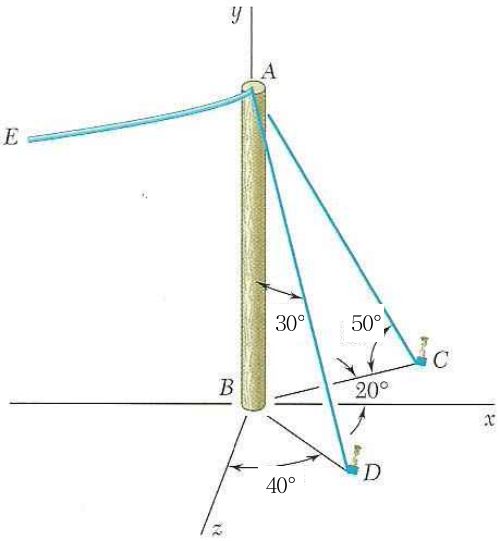
3.[6점] A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable ACB and is pulled at a constant speed by cable CD. We know that α = 25° and β = 15°.



(a) Knowing that the tension in cable CD is 80 N, determine the combined weight of the boatswain's chair and the sailor by the rectangular components.

(b) Knowing that the tension in cable CD is 100 N, determine the combined weight of the boatswain's chair and the sailor by the force triangle.

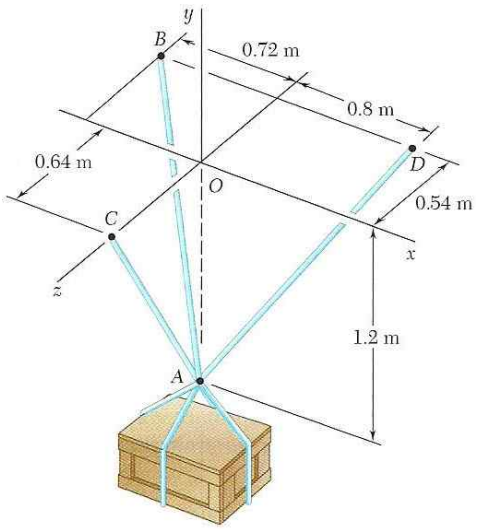
4.[4점] 동축 케이블 AE의 끝이 기둥 AB에 고정되어 있고, 그 기둥은 버팀줄 AC와 AD에 의해 보강되어 있다.



(a) 줄 AC의 장력이 290 N임을 알 때, 줄 AC에 의해 점 C에 가해지는 힘의 직각성분 $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{F}_z$를 구하여라.

(b) 줄 AD에 의해 점 A에 가해지는 힘의 방향을 나타내기 위한 단위벡터 $\boldsymbol{\lambda}_{AD}$를 구하여라.

5.[4점] 나무상자가 그림과 같이 세 개의 줄에 매달려 있다. 줄 AB의 장력이 3.00 kN이다.



(a) 줄 AB에 의해 점 A에 가해지는 힘이 y축, z축과 이루는 각도 θ_y, θ_z를 각각 구하여라.

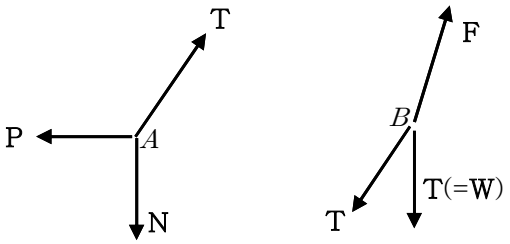
(b) 나무상자의 무게를 구하여라.

정 역 학　　2011년 시험1 (가반) 해 답

1. (a) Newton의 운동 제1법칙(관성의 법칙)에 따르면, 힘의 합력이 0인 경우 운동하던 질점은 직선상을 등속도 운동한다. 화살이 발사된 이후에 화살에 작용하는 힘이 중력 이외에는 없으므로 수평방향으로는 등속 직선운동을 해야 한다. 제2법칙(가속도 법칙)에 따르면, 가속도는 힘의 합에 비례한다. 합력이 0인 경우 가속도가 0이므로 가속도의 적분 값인 속도는 상수이어서 등속도 직선운동을 해야 한다.

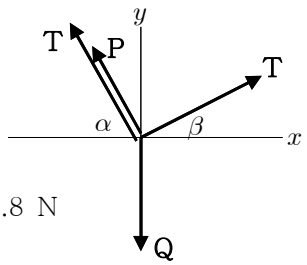
(b) 기계공학(mechanical engineering)이란 역학(mechanics)에 의한 설계(design) 및 생산(production)을 다루는 학문이다. (타워, 블레이드, 터빈 등에 역학을 적용할 가능성을 제시)

2. 자유물체도(free-body diagram)



3. $\alpha = 50°$, $\beta = 30°$

(a) $P = 375$ N



$\Sigma F_x = 0$: $-(P+T)\cos\alpha + T\cos\beta = 0$ … ①

$\Sigma F_y = 0$: $(P+T)\sin\alpha + T\sin\beta - Q = 0$ … ②

① ⇒ $-P\cos\alpha - T\cos\alpha + T\cos\beta = 0$

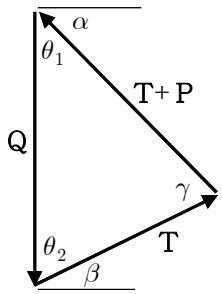
⇒ $T = P\dfrac{\cos\alpha}{\cos\beta - \cos\alpha} = (375\text{ N})\dfrac{\cos 50^\circ}{\cos 30^\circ - \cos 50^\circ} = 1079.8$ N

② ⇒ $Q = (P+T)\sin\alpha + T\sin\beta$

$= (375 + 1079.8\text{ N})\sin 50° + (1079.8\text{ N})\sin 30°$

$= 1654.3$ N ⇒ $Q = 1654$ N

(b) $P = 600$ N



$\theta_1 = 90° - \alpha = 90° - 50° = 40°$

$\theta_2 = 90° - \beta = 90° - 30° = 60°$

$\dfrac{T}{\sin\theta_1} = \dfrac{T+P}{\sin\theta_2}$

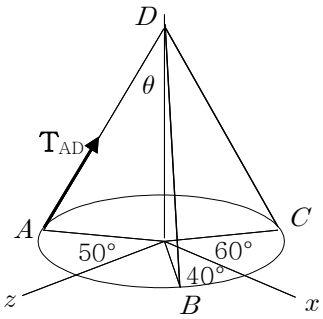
⇒ $T\sin\theta_2 = (T+P)\sin\theta_1$

⇒ $T(\sin\theta_2 - \sin\theta_1) = P\sin\theta_1$

⇒ $T = P\dfrac{\sin\theta_1}{\sin\theta_2 - \sin\theta_1} = (600\text{ N})\dfrac{\sin 40^\circ}{\sin 60^\circ - \sin 40^\circ}$

$= 1727.6$ N ⇒ $T = 1728$ N

4. $\theta = 35°$



(a) $T_{AD} = 55$ N

$(T_{AD})_y = T_{AD} \cos\theta = (55\text{ N}) \cos 35° = 45.1$ N

$(T_{AD})_h = T_{AD} \sin\theta = (55\text{ N}) \sin 35° = 31.5$ N

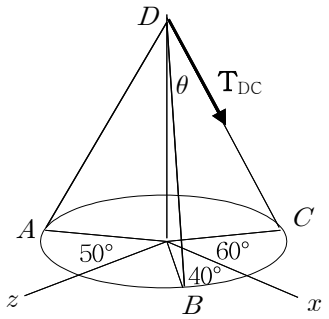
$(T_{AD})_x = (T_{AD})_h \sin 50° = T_{AD} \sin\theta \sin 50°$

$= (55\text{ N}) \sin 35° \sin 50° = 24.2$ N

$(T_{AD})_z = -(T_{AD})_h \cos 50° = -T_{AD} \sin\theta \cos 50°$

$= -(55\text{ N}) \sin 35° \cos 50° = -20.3$ N

$\mathbf{F}_x = (T_{AD})_x\,\mathbf{i} = 24.2\text{ N}\,\mathbf{i}, \quad \mathbf{F}_y = (T_{AD})_y\,\mathbf{j} = 45.1\text{ N}\,\mathbf{j}, \quad \mathbf{F}_z = (T_{AD})_z\,\mathbf{k} = -20.3\text{ N}\,\mathbf{k}$



(b) $(T_{DC})_y = -T_{DC} \cos\theta, \quad (T_{DC})_h = T_{DC} \sin\theta$

$(T_{DC})_x = (T_{DC})_h \cos 60° = T_{DC} \sin\theta \cos 60°$

$(T_{DC})_z = -(T_{DC})_h \sin 60° = -T_{DC} \sin\theta \sin 60°$

$\mathbf{T}_{DC} = (T_{DC})_x\,\mathbf{i} + (T_{DC})_y\,\mathbf{j} + (T_{DC})_z\,\mathbf{k}$

$= T_{DC}\,[(\sin\theta \cos 60°)\,\mathbf{i} + (-\cos\theta)\,\mathbf{j} + (-\sin\theta \sin 60°)\,\mathbf{k}]$

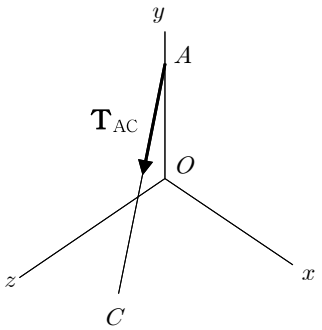
$\mathbf{T}_{DC} = T_{CD}\,\boldsymbol{\lambda}_{DC}$

$\boldsymbol{\lambda}_{DC} = (\sin\theta \cos 60°)\,\mathbf{i} + (-\cos\theta)\,\mathbf{j} + (-\sin\theta \sin 60°)\,\mathbf{k}$

$= (\sin 35° \cos 60°)\,\mathbf{i} + (-\cos 35°)\,\mathbf{j} + (-\sin 35° \sin 60°)\,\mathbf{k}$

$= 0.287\,\mathbf{i} - 0.819\,\mathbf{j} - 0.497\,\mathbf{k}$

5. $T_{AB} = 270$ N, $\quad \mathbf{P} = P\,\mathbf{j}$



(a) $\mathbf{d}_{AC} = (2\text{ m})\,\mathbf{i} + (-5\text{ m})\,\mathbf{j} + (4\text{ m})\,\mathbf{k}$

$d_{AC} = \sqrt{(2\text{ m})^2 + (-5\text{ m})^2 + (4\text{ m})^2} = 6.708$ m

$\boldsymbol{\lambda}_{AC} = \dfrac{1}{6.708\text{ m}}[(2\text{ m})\,\mathbf{i} + (-5\text{ m})\,\mathbf{j} + (4\text{ m})\,\mathbf{k}]$

$= 0.2982\,\mathbf{i} - 0.7454\,\mathbf{j} + 0.5963\,\mathbf{k}$

$\theta_x = \cos^{-1}(0.2982) = 72.7°, \quad \theta_y = \cos^{-1}(-0.7454) = 138.2°$

(b) $\mathbf{T}_{AC} = T_{AC}\,(0.2982\,\mathbf{i} - 0.7454\,\mathbf{j} + 0.5963\,\mathbf{k})$

$\mathbf{d}_{AB} = (-4\text{ m})\,\mathbf{i} + (-5\text{ m})\,\mathbf{j}, \quad d_{AB} = \sqrt{(-4\text{ m})^2 + (-5\text{ m})^2 + 0} = 6.403$ m

$\boldsymbol{\lambda}_{AB} = \dfrac{1}{6.403\text{ m}}[(-4\text{ m})\,\mathbf{i} + (-5\text{ m})\,\mathbf{j}] = -0.6247\,\mathbf{i} - 0.7809\,\mathbf{j}$

$\mathbf{T}_{AB} = (270\text{ N})\,(-0.6247\,\mathbf{i} - 0.7809\,\mathbf{j}) = (-168.7\text{ N})\,\mathbf{i} + (-210.8\text{ N})\,\mathbf{j}$

$\mathbf{d}_{AD} = (-5\text{ m})\,\mathbf{j} + (-3\text{ m})\,\mathbf{k}, \quad d_{AD} = \sqrt{0 + (-5\text{ m})^2 + (-3\text{ m})^2} = 5.831$ m

$\boldsymbol{\lambda}_{AD} = \dfrac{1}{5.831\text{ m}}[(-5\text{ m})\,\mathbf{j} + (-3\text{ m})\,\mathbf{k}] = -0.8574\,\mathbf{j} - 0.5144\,\mathbf{k}$

$\mathbf{T}_{AD} = T_{AD}\,(-0.8574\,\mathbf{j} - 0.5144\,\mathbf{k})$

$\Sigma\mathbf{F} = 0 \quad \Rightarrow \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0$

$\Sigma F_x = 0\ ; \quad (-168.7\text{ N}) + 0.2982\,T_{AC} + 0 + 0 = 0$

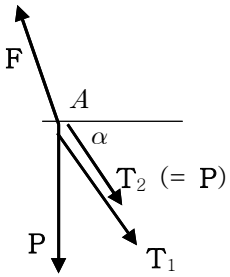
$\Rightarrow \quad T_{AC} = \dfrac{168.7\text{ N}}{0.2982} = 565.7\text{ N} \qquad \Rightarrow \quad T_{AC} = 566\text{ N}$

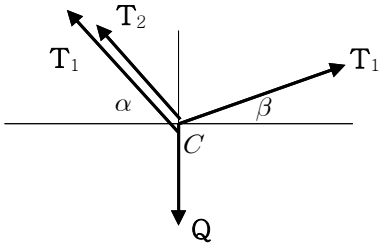
정 역 학　　2011년 시험1 (나반)　해 답

1. (a) 정역학이란 정지하고 있는 물체에 작용하는 힘의 평형이며, 평형이란 작용하는 힘의 합력이 0인 상황이다. 마술사가 공중에 떠서 정지해 있으려면, 몸무게 중력과 크기가 같고 방향이 반대인 힘이 가해지고 있어야 한다.

(b) $300\ \text{N} = 300\ \text{kg·m/s}^2 \times \dfrac{1\ \text{kgf}}{(1\ \text{kg})(9.81\ \text{m/s}^2)} \times \dfrac{1\ \text{lb}}{0.4536\ \text{kgf}} = 67.4\ \text{lb}$

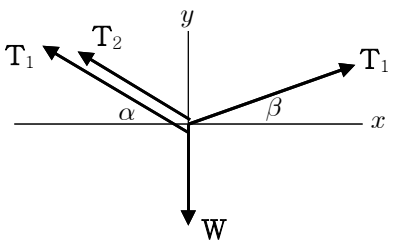
2. 자유물체도 (free-body diagram)





3. $\alpha = 25°$,　$\beta = 15°$

(a) $T_2 = 80$ N



$\Sigma F_x = 0$; $T_1 \cos\beta - (T_1 + T_2)\cos\alpha = 0$

$\Rightarrow \quad T_1(\cos\beta - \cos\alpha) - T_2\cos\alpha = 0 \quad \cdots ①$

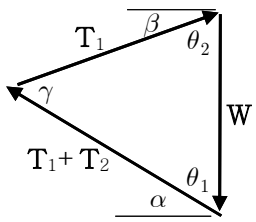
$\Sigma F_y = 0$; $T_1 \sin\beta + (T_1 + T_2)\sin\alpha - W = 0$

$\Rightarrow \quad T_1(\sin\beta + \sin\alpha) + T_2\sin\alpha = W \quad \cdots ②$

① $\Rightarrow \quad T_1 = T_2 \dfrac{\cos\alpha}{\cos\beta - \cos\alpha} = (80\ \text{N})\ \dfrac{\cos25^\circ}{\cos15^\circ - \cos25^\circ} = 1216.2\ \text{N}$

② $\Rightarrow \quad W = (1216.2\ \text{N})(\sin15° + \sin25°) + (80\ \text{N})\sin25° = 862.6\ \text{N} \quad \Rightarrow \quad W = 863\ \text{N}$

(b) $T_2 = 100$ N



$\theta_1 = 90° - \alpha = 90° - 25° = 65°$,　$\theta_2 = 90° - \beta = 90° - 15° = 75°$

$\gamma = \alpha + \beta = 25° + 15° = 40°$

$\dfrac{T_1}{\sin\theta_1} = \dfrac{T_1 + T_2}{\sin\theta_2}$

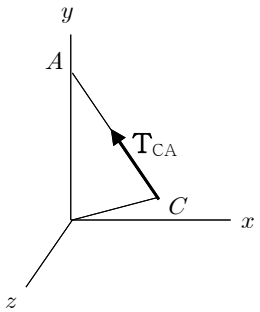
$\Rightarrow \quad T_1\sin\theta_2 = (T_1 + T_2)\sin\theta_1 \quad \Rightarrow \quad T_1(\sin\theta_2 - \sin\theta_1) = T_2\sin\theta_1$

$\Rightarrow \quad T_1 = T_2\ \dfrac{\sin\theta_1}{\sin\theta_2 - \sin\theta_1} = (100\ \text{N})\ \dfrac{\sin65^\circ}{\sin75^\circ - \sin65^\circ} = 1520.2\ \text{N}$

$\dfrac{W}{\sin\gamma} = \dfrac{T_1}{\sin\theta_1}$

$\Rightarrow \quad W = T_1\ \dfrac{\sin\gamma}{\sin\theta_1} = (1520.2\ \text{N})\ \dfrac{\sin40^\circ}{\sin65^\circ} = 1078.2\ \text{N} \quad \Rightarrow \quad W = 1078\ \text{N}$

4. (a) T_{CA} = 290 N



$(T_{CA})_y = T_{CA} \sin 50° = (290\text{ N}) \sin 50° = 222.2\text{ N}$

$(T_{CA})_h = T_{CA} \cos 50°$ (= (290 N) cos50° = 186.4 N)

$(T_{CA})_x = -(T_{CA})_h \cos 20° = -T_{CA} \cos 50° \cos 20°$

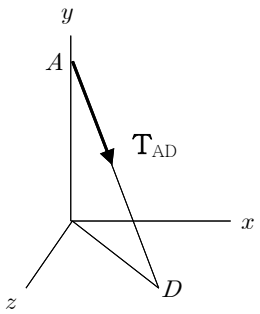
$= -(290\text{ N}) \cos 50° \cos 20° = -175.2\text{ N}$

$(T_{CA})_z = (T_{CA})_h \sin 20° = T_{CA} \cos 50° \sin 20°$

$= (290\text{ N}) \cos 50° \sin 20° = 63.76\text{ N}$

$\mathbf{F}_x = (T_{CA})_x\,\mathbf{i} = -175.2\text{ N}\,\mathbf{i}, \quad \mathbf{F}_y = (T_{CA})_y\,\mathbf{j} = 222\text{ N}\,\mathbf{j}, \quad \mathbf{F}_z = (T_{CA})_z\,\mathbf{k} = 63.8\text{ N}\,\mathbf{k}$

(b) $(T_{AD})_y = -T_{AD} \cos 30°, \quad (T_{AD})_h = T_{AD} \sin 30°$



$(T_{AD})_x = (T_{AD})_h \sin 40° = T_{AD} \sin 30° \sin 40°$

$(T_{AD})_z = (T_{AD})_h \cos 40° = T_{AD} \sin 30° \cos 40°$

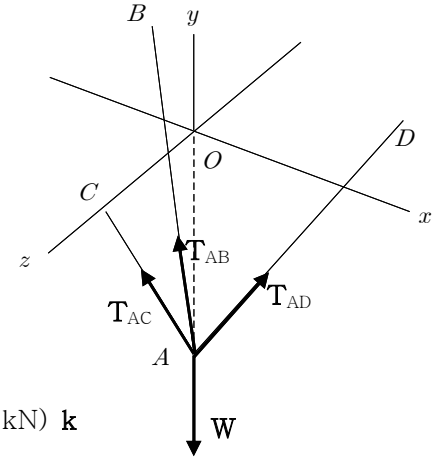
$\mathbf{T}_{AD} = (T_{AD})_x\,\mathbf{i} + (T_{AD})_y\,\mathbf{j} + (T_{AD})_z\,\mathbf{k}$

$= T_{AD}\,[\sin 30° \sin 40°\,\mathbf{i} - \cos 30°\,\mathbf{j} + \sin 30° \cos 40°\,\mathbf{k}]$

$\mathbf{T}_{AD} = T_{AD}\,\boldsymbol{\lambda}_{AD}$

$\boldsymbol{\lambda}_{AD} = \sin 30° \sin 40°\,\mathbf{i} - \cos 30°\,\mathbf{j} + \sin 30° \cos 40°\,\mathbf{k} = 0.321\,\mathbf{i} - 0.866\,\mathbf{j} + 0.383\,\mathbf{k}$

5. T_{AB} = 3.00 kN, $\quad \mathbf{W} = -W\,\mathbf{j}$



(a) $\mathbf{d}_{AB} = (-0.72\text{ m})\,\mathbf{i} + (1.20\text{ m})\,\mathbf{j} + (-0.54\text{ m})\,\mathbf{k}$

$d_{AB} = \sqrt{(-0.72\text{ m})^2 + (1.20\text{ m})^2 + (-0.54\text{ m})^2} = 1.50\text{ m}$

$\boldsymbol{\lambda}_{AB} = \dfrac{1}{1.50\text{ m}}[(-0.72\text{ m})\,\mathbf{i} + (1.20\text{ m})\,\mathbf{j} + (-0.54\text{ m})\,\mathbf{k}]$

$= -0.48\,\mathbf{i} + 0.80\,\mathbf{j} - 0.36\,\mathbf{k}$

$\theta_y = \cos^{-1}(0.80) = 36.9°, \quad \theta_z = \cos^{-1}(-0.36) = 111.1°$

(b) $\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = (3.00\text{ kN})(-0.48\,\mathbf{i} + 0.80\,\mathbf{j} - 0.36\,\mathbf{k})$

$= (-1.440\text{ kN})\,\mathbf{i} + (2.40\text{ kN})\,\mathbf{j} + (-1.080\text{ kN})\,\mathbf{k}$

$d_{AC} = \sqrt{0^2 + (1.2\text{ m})^2 + (0.64\text{ m})^2} = 1.36\text{ m}$

$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = \dfrac{T_{AC}}{1.36\text{ m}}[(1.2\text{ m})\mathbf{j} + (0.64\text{ m})\mathbf{k}] = T_{AC}\,(0.88235\,\mathbf{j} + 0.47059\,\mathbf{k})$

$d_{AD} = \sqrt{(0.8\text{ m})^2 + (1.2\text{ m})^2 + (-0.54\text{ m})^2} = 1.54\text{ m}$

$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = \dfrac{T_{AD}}{1.54\text{ m}}[(0.8\text{ m})\mathbf{i} + (1.2\text{ m})\mathbf{j} + (-0.54\text{ m})\mathbf{k}]$

$= T_{AD}\,(0.51948\,\mathbf{i} + 0.77922\,\mathbf{j} - 0.35065\,\mathbf{k})$

$\Sigma\mathbf{F} = 0 \quad \Rightarrow \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

$\Sigma F_x = 0$; $(-1.440\text{ kN}) + 0 + 0.51948\,T_{AD} = 0 \quad \cdots ①$

$\Sigma F_y = 0$; $(2.40\text{ kN}) + 0.88235\,T_{AC} + 0.77922\,T_{AD} - W = 0 \quad \cdots ②$

$\Sigma F_z = 0$; $(-1.080\text{ kN}) + 0.47059\,T_{AC} - 0.35065\,T_{AD} = 0 \quad \cdots ③$

① $\Rightarrow \quad T_{AD} = \dfrac{1.440\text{ kN}}{0.51948} = 2.772\text{ kN}$

③ $\Rightarrow \quad T_{AC} = \dfrac{1}{0.47059}[(1.08\text{ kN}) + 0.35065\,(2.772\text{ kN})] = 4.360\text{ kN}$

② $\Rightarrow \quad W = (2.40\text{ kN}) + 0.88235\,(4.360\text{ kN}) + 0.77922\,(2.772\text{ kN}) = 8.407\text{ kN}$

$\Rightarrow \quad W = 8.41\text{ kN}$