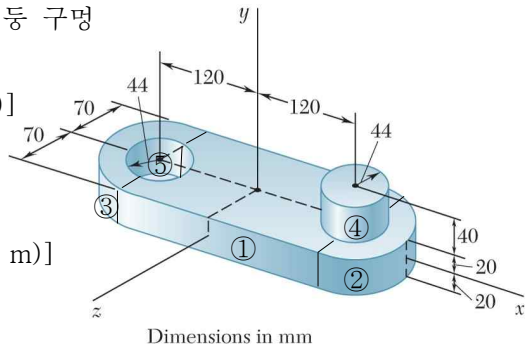
[9.5절]

9.142 $\rho = 7{,}850\ \text{kg/m}^3$

① 사각기둥, ②,③ 반원기둥, ④ 원기둥, ⑤ 원기둥 구멍



Dimensions in mm

$m = \rho V$

$m_1 = (7{,}850\ \text{kg/m}^3)\ [(0.24\ \text{m})(0.14\ \text{m})(0.04\ \text{m})]$

$= (7.85\times10^3\ \text{kg/m}^3)\ (1.344\times10^{-3}\ \text{m}^3)$

$= 10.550\ \text{kg}$

$m_2 = m_3 = (7{,}850\ \text{kg/m}^3)\ [\frac{\pi}{2}\ (0.07\ \text{m})^2\ (0.04\ \text{m})]$

$= (7.85\times10^3\ \text{kg/m}^3)\ (0.3079\times10^{-3}\ \text{m}^3)$

$= 2.417\ \text{kg}$

$m_4 = m_5 = (7{,}850\ \text{kg/m}^3)\ [\pi\ (0.044\ \text{m})^2\ (0.04\ \text{m})]$

$= (7.85\times10^3\ \text{kg/m}^3)\ (0.2433\times10^{-3}\ \text{m}^3) = 1.9098\ \text{kg}$

(a) $I_{x1} = \frac{1}{12} m_1 (b^2 + c^2)$

$= \frac{1}{12}(10.550\ \text{kg})[(0.14\ \text{m})^2 + (0.04\ \text{m})^2] = 18.638\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{x2} = I_{x3} = \frac{1}{12} m_2 (3r_2^2 + L_2^2)$

$= \frac{1}{12}(2.417\ \text{kg})[3\ (0.07\ \text{m})^2 + (0.04\ \text{m})^2] = 3.283\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{x4} = \frac{1}{12} m_4 (3r_4^2 + L_4^2) + m_4 d_4^2$

$= (1.9098\ \text{kg})\ \{\frac{1}{12}[3\ (0.044\ \text{m})^2 + (0.04\ \text{m})^2] + (0.04)^2\} = 4.234\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{x5} = \frac{1}{12} m_5 (3r_5^2 + L_5^2)$

$= \frac{1}{12}(1.9098\ \text{kg})[3\ (0.044\ \text{m})^2 + (0.04\ \text{m})^2] = 1.179\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_x = I_{x1} + I_{x2} + I_{x3} + I_{x4} - I_{x5}$

$= [(18.638) + 2\ (3.283) + (4.234) - (1.179)]\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$= 28.259\times10^{-3}\ \text{kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_x = 28.3\times10^{-3}\ \text{kg}\cdot\text{m}^2$

(b) $I_{y1} = \frac{1}{12} m_1 (a^2 + b^2)$

$= \frac{1}{12}(10.550\ \text{kg})[(0.24\ \text{m})^2 + (0.14\ \text{m})^2] = 67.87\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{y2} = I_{y3} = \frac{1}{2} m_2 r_2^2 - m_2 d_{2a}^2 + m_2 d_{2b}^2$

$= (2.417\ \text{kg})\ \{\frac{1}{2}(0.07\ \text{m})^2 - [\frac{4}{3\pi}(0.07\ \text{m})]^2 + [(0.12\ \text{m}) + \frac{4}{3\pi}(0.07\ \text{m})]^2\}$

$= 57.96\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{y4} = I_{y5} = \frac{1}{2} m_4 r_4^2 + m_4 d_4^2$

$= (1.9098\ \text{kg})\ [\frac{1}{2}(0.044\ \text{m})^2 + (0.12\ \text{m})^2] = 29.34\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4} - I_{y5}$$
$$= [(67.87) + 2\,(57.96) + 0]\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$= 183.79\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_y = 183.8\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

(c) $m = m_1 + m_2 + m_3 + m_4 - m_5$
$$= (10.550\ \text{kg}) + 2\,(2.417\ \text{kg}) + 0 = 15.384\ \text{kg}$$
$$k_x^2 = \frac{I_x}{m} = \frac{28.259\times 10^{-3}\ \text{kg}\cdot\text{m}^2}{15.384\ \text{kg}} = 1.8369\times 10^{-3}\ \text{m}^2$$
$$\Rightarrow \quad k_x = 0.0429\ \text{m} = 42.9\ \text{mm}$$
$$k_y^2 = \frac{I_y}{m} = \frac{183.79\times 10^{-3}\ \text{kg}\cdot\text{m}^2}{15.384\ \text{kg}} = 11.947\times 10^{-3}\ \text{m}^2$$
$$\Rightarrow \quad k_y = 0.1093\ \text{m} = 109.3\ \text{mm}$$