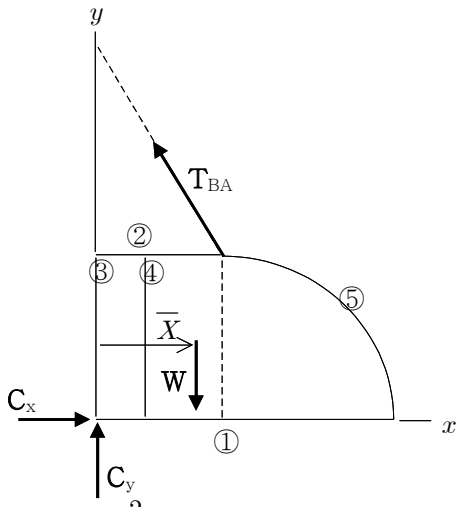
[5.1절]

5.29 S; 단위 길이 당 질량 ρa = 4.73 kg/m　　M;

⇒ 선재의 무게와 무게중심, 강체의 평형



A; 선재의 도심 (= 무게중심)

① L = 1.35 m, $\bar{x} = \frac{1}{2}(1.35\text{ m}) = 0.675$ m

② L = 0.6 m, $\bar{x} = \frac{1}{2}(0.6\text{ m}) = 0.3$ m

③ L = 0.75 m, $\bar{x} = 0$

④ L = 0.75 m, $\bar{x}$ = 0.2 m

⑤ $L = \frac{1}{4}(2\pi R) = \frac{1}{2}\pi\ (0.75\text{ m}) = 1.1781$ m, $\bar{x} = 0.6 + \frac{2}{\pi}(0.75\text{ m}) = 1.0774$ m

ΣL = 1.35 + 0.6 + 0.75 + 0.75 + 1.1781 m = 4.628 m

$\Sigma(\bar{x}L)$ = (0.675)(1.35) + (0.3)(0.6) + (0)(0.75) + (0.2)(0.75) + (1.0774)(1.1781) m^2

= 2.511 m^2

$\bar{X} = \frac{\Sigma(\bar{x}L)}{\Sigma L} = \frac{2.511\text{ m}^2}{4.628\text{ m}} = 0.5424$ m

$W = \rho a\ \Sigma L$ = (4.73 kg/m) (4.628 m) (9.81 m/s^2) = 214.7 N

(a) $\Sigma M_C = 0$;　$-W\ \bar{X} + \frac{3}{5}T_{BA}\ d_y = 0$

⇒　$T_{BA} = W\ \frac{\bar{X}}{\frac{3}{5}d_y} = (214.7\text{ N})\ \frac{0.5424\text{ m}}{0.6(0.75+0.8\text{ m})}$

= (214.7 N) (0.5832) = 125.22 N　　⇒　T_{BA} = 125.2 N

(b) $\Sigma F_x = 0$;　$C_x - \frac{3}{5}T_{BA} = 0$

⇒　$C_x = 0.6\,T_{BA}$ = 0.6 (125.22 N) = 75.13 N

$\Sigma F_y = 0$;　$C_y - W + \frac{4}{5}T_{BA} = 0$

⇒　$C_y = W - 0.8\,T_{BA}$ = (214.7 N) − 0.8 (125.22 N) = 114.52 N

$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(75.13\text{ N})^2 + (114.52\text{ N})^2} = 136.97$ N

$\tan\theta = \frac{C_y}{C_x} = \frac{114.52\text{ N}}{75.13\text{ N}} = 1.5243$　　⇒　$\theta = \tan^{-1}(1.5243) = 56.73°$

⇒　C = 137.0 N ∠56.7°

R; (과정의 타당성 검토)

$\frac{3}{5}T_{BA}\ d_y$ = 0.6 (125.22 N) (0.75+ 0.8 m) = 116.45 N·m

$\frac{3}{5}T_{BA}\ d_{y1} + \frac{4}{5}T_{BA}\ d_{x1}$ = 0.6 (125.22 N) (0.75 m) + 0.8 (125.22 N) (0.6 m)

= 116.45 N·m

T; (결과의 의미 검토) 반력 방향 (C_x →,　C_y ↑) 과　T_{BA}, W의 관계

[5.4절]

5.121 ① 유리 원판 + ② 철 다리 상부 + ③ 철 다리 하부

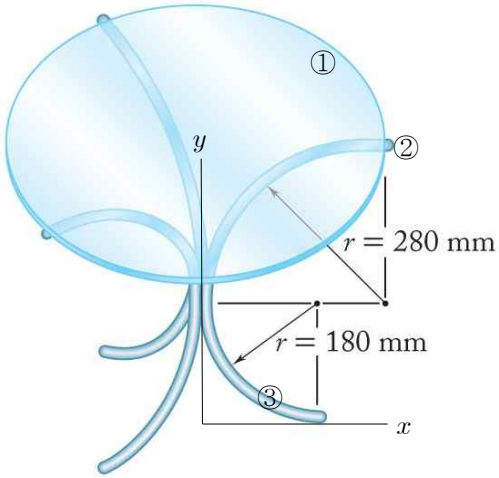
d_1 = 600 mm, t_1 = 10 mm

ρ_g = 2,190 kg/m^3 = 2.19×10^{-6} kg/mm^3

r_2 = 280 mm, r_3 = 180 mm

d_s = 24 mm, a_s = 150 mm^2

ρ_s = 7,860 kg/m^3 = 7.86×10^{-6} kg/mm^3



대칭구조 ⇒ $\overline{X} = \overline{Z} = 0$

$W = mg = \rho V g \quad \Rightarrow \quad m = \rho V$

$$\overline{Y} = \frac{\Sigma(\overline{y}\,W)}{\Sigma W} = \frac{\Sigma(\overline{y}\,m)}{\Sigma m}$$

① $V = \frac{\pi}{4} d_1^2\, t_1 = \frac{\pi}{4}(600\text{ mm})^2\,(10\text{ mm}) = 2{,}827{,}433\text{ mm}^3 = 2.827\times10^6\text{ mm}^3$

$m = \rho_g\, V = (2.19\times10^{-6}\text{ kg/mm}^3)(2.827\times10^6\text{ mm}^3) = 6.192\text{ kg}$

$\overline{y} = \frac{1}{2}t_1 + r_2 + r_3 + d_s = \frac{1}{2}(10) + 280 + 180 + 24\text{ mm} = 489\text{ mm}$

② $V = \frac{\pi}{2} r_2\, a_s = \frac{\pi}{2}(280\text{ mm})(150\text{ mm}^2) = 65{,}973\text{ mm}^3 = 65.97\times10^3\text{ mm}^3$

$m = \rho_s\, V = (7.86\times10^{-6}\text{ kg/mm}^3)(65.97\times10^3\text{ mm}^3) = 518.6\times10^{-3}\text{ kg}$

$\overline{y} = \frac{1}{2}d_s + r_3 + \frac{2}{\pi}r_2 = \frac{1}{2}(24) + 180 + \frac{2}{\pi}(280)\text{ mm} = 370.3\text{ mm}$

③ $V = \frac{\pi}{2} r_3\, a_s = \frac{\pi}{2}(180\text{ mm})(150\text{ mm}^2) = 42{,}412\text{ mm}^3 = 42.41\times10^3\text{ mm}^3$

$m = \rho_s\, V = (7.86\times10^{-6}\text{ kg/mm}^3)(42.41\times10^3\text{ mm}^3) = 333.3\times10^{-3}\text{ kg}$

$\overline{y} = \frac{1}{2}d_s + r_3 - \frac{2}{\pi}r_3 = \frac{1}{2}(24) + 180 - \frac{2}{\pi}(180)\text{ mm} = 77.41\text{ mm}$

$\Sigma m = [6{,}192 + 3\,(518.6) + 3\,(333.3)]\times10^{-3}\text{ kg} = 8{,}748\times10^{-3}\text{ kg}$

$\Sigma(\overline{y}\,m) = [(489)\,(6{,}192) + (370.3)\,3\,(518.6) + (77.41)\,3\,(333.3)]\times10^{-3}\text{ kg}\cdot\text{mm}$

$= 3{,}681\text{ kg}\cdot\text{mm}$

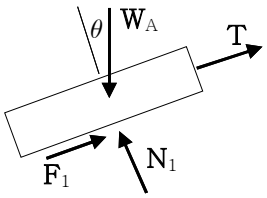
$$\overline{Y} = \frac{\Sigma(\overline{y}\,m)}{\Sigma m} = \frac{3{,}681\text{ kg}\cdot\text{mm}}{8{,}748\times10^{-3}\text{ kg}} = 420.8\text{ mm} \quad \Rightarrow \quad \text{중심} = (0,\ 421\text{ mm},\ 0)$$

[8.1절]

8.12　W_A = 50 N,　W_B = 25 N,　μ_s = 0.15

S; 힘의 평형방정식, 최대 정지마찰력　　M;

A;



블럭 A

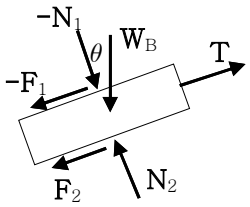
$\nwarrow \Sigma F_n = 0$;　$N_1 - W_A \cos\theta = 0$

$\Rightarrow$　$N_1 = W_A \cos\theta$

$F_1 = \mu_s N_1 = \mu_s W_A \cos\theta$

$\nearrow \Sigma F_t = 0$;　$T - W_A \sin\theta + F_1 = 0$

$\Rightarrow$　$T = W_A \sin\theta - \mu_s W_A \cos\theta$



블록 B

$\nwarrow \Sigma F_n = 0$;　$N_2 - N_1 - W_B \cos\theta = 0$

$\Rightarrow$　$N_2 = N_1 + W_B \cos\theta$

$= W_A \cos\theta + W_B \cos\theta$

$\nearrow \Sigma F_t = 0$;　$T - F_1 - F_2 - W_B \sin\theta = 0$

$\Rightarrow$　$T = W_B \sin\theta + F_1 + F_2$

$= W_B \sin\theta + \mu_s W_A \cos\theta + \mu_s (W_A + W_B) \cos\theta$

$W_A \sin\theta - \mu_s W_A \cos\theta = W_B \sin\theta + \mu_s (2W_A + W_B) \cos\theta$

$\Rightarrow$　$(W_A - W_B) \sin\theta = \mu_s (3W_A + W_B) \cos\theta$

$$\Rightarrow \tan\theta = \frac{\mu_s (3W_A + W_B)}{W_A - W_B} = \frac{(0.15)[3(50\text{ N}) + (25\text{ N})]}{(50\text{ N}) - (25\text{ N})} = 1.05$$

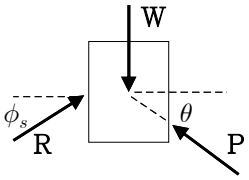
$\Rightarrow$　$\theta = \tan^{-1}(1.05) = 46.40°$　$\Rightarrow$　$\theta = 46.4°$

R; (과정의 타당성 검토) (가령, 마찰력 방향)

T; (결과의 의미 검토) (가령, θ가 커짐에 따라 A가 ↙방향으로 움직이려 하고 B가 ↗방향으로 움직이려 함)

8.10 P = 100 N, m = 7.5 kg, μ_s = 0.45, μ_k = 0.35

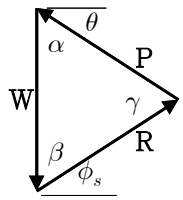
S; 최대 마찰력의 마찰각, 힘 삼각형 M;



A; $W = m\,g$ = (7.5 kg)(9.806 m/s^2) = 73.54 N

마찰각 $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.23°$

내려가려 할 때



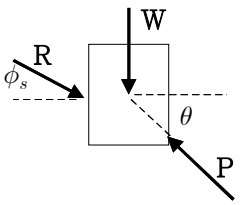
$\beta = 90° - \phi_s = 90° - 24.23° = 65.77°$

$$\frac{W}{\sin\gamma} = \frac{P}{\sin\beta}$$

$$\Rightarrow \sin\gamma = \frac{W}{P}\sin\beta = \frac{73.54\text{ N}}{100\text{ N}}\sin 65.77° = 0.6706$$

$$\Rightarrow \gamma = \sin^{-1}0.6706 = 42.11°$$

$\theta = \gamma - \phi_s = 42.11° - 24.23° = 17.88°$



올라가려 할 때

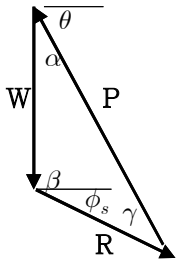
$\beta = 90° + \phi_s = 90° + 24.23° = 114.23°$

$$\frac{W}{\sin\gamma} = \frac{P}{\sin\beta}$$

$$\Rightarrow \sin\gamma = \frac{W}{P}\sin\beta = \frac{73.54\text{ N}}{100\text{ N}}\sin 114.23° = 0.6706$$

$$\Rightarrow \gamma = \sin^{-1}0.6706 = 42.11°$$

$\theta = \gamma + \phi_s = 42.11° + 24.23° = 66.34°$



$\Rightarrow$ $17.88° \le \theta \le 66.3°$

R; (과정의 타당성 검토) (가령, 마찰력 방향)

T; (결과의 의미 검토) (가령, ϕ=0 일 때 θ = 42.11°)

[9.1절]

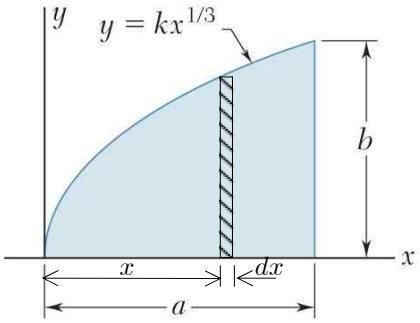
9.2&6　$y = k\,x^{1/3}$,　　$(a,\ b)$　$\Rightarrow$　$b = k\,a^{1/3}$

$\Rightarrow$　$k = \dfrac{b}{a^{1/3}}$,　　$y = \dfrac{b}{a^{1/3}}\,x^{1/3}$

$dA = y\,dx = \dfrac{b}{a^{1/3}}\,x^{1/3}\,dx$

(문제 외 $A = \int dA = \int_0^a \dfrac{b}{a^{1/3}}\,x^{1/3}\,dx$

$= \dfrac{b}{a^{1/3}}\left[\dfrac{3}{4}x^{4/3}\right]_0^a = \dfrac{b}{a^{1/3}}\left[\dfrac{3}{4}(a^{4/3}-0)\right] = \dfrac{3}{4}ab$)



9.2　$dI_y = x^2\,dA = x^2\,\dfrac{b}{a^{1/3}}\,x^{1/3}\,dx = \dfrac{b}{a^{1/3}}\,x^{7/3}\,dx$

$I_y = \int dI_y = \int_0^a \dfrac{b}{a^{1/3}}\,x^{7/3}\,dx = \dfrac{b}{a^{1/3}}\left[\dfrac{3}{10}x^{10/3}\right]_0^a = \dfrac{b}{a^{1/3}}\left[\dfrac{3}{10}(a^{10/3}-0)\right] = \dfrac{3}{10}a^3 b$

$\Rightarrow$　$I_y = 0.300\ a^3 b$

(문제 외 $k_y^2 = \dfrac{I_y}{A} = \dfrac{\frac{3}{10}a^3 b}{\frac{3}{4}ab} = \dfrac{2}{5}a^2$　$\Rightarrow$　$k_y = \sqrt{\dfrac{2}{5}a^2} = 0.632\ a$)

9.6　$dI_x = \dfrac{1}{3}y^3\,dx$

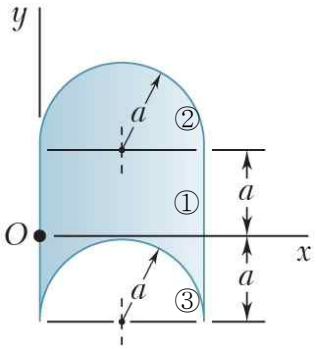
$I_x = \int dI_x = \int_0^a \dfrac{1}{3}y^3\,dx = \dfrac{1}{3}\int_0^a \dfrac{b^3}{a}x\,dx = \dfrac{1}{3}\dfrac{b^3}{a}\left[\dfrac{1}{2}x^2\right]_0^a = \dfrac{1}{6}ab^3$

$\Rightarrow$　$I_x = 0.0667\ ab^3$

(문제 외 $k_x^2 = \dfrac{I_x}{A} = \dfrac{\frac{1}{6}ab^3}{\frac{3}{4}ab} = \dfrac{2}{9}b^2$　$\Rightarrow$　$k_x = \sqrt{\dfrac{2}{9}b^2} = 0.471\ b$)

[9.2절]

9.35 ① 직사각형, ② 위 반원, ③ 아래 반원 구멍



$$I_{y1} = \bar{I}_{y1} + A_1 d_1^2$$

$$= \frac{1}{12}(2a)(2a)^3 + (2a)^2 a^2 = \left(\frac{4}{3}+4\right)a^4 = \frac{16}{3}a^4$$

$$I_{y2} - I_{y3} = 0$$

$$I_y = I_{y1} + I_{y2} - I_{y3} = \frac{16}{3}a^4 = 5.33\ a^4$$

$$I_{x1} = \frac{1}{12}(2a)(2a)^3 = \frac{4}{3}a^4 = 1.333\ a^4$$

$$I_{x2} = \frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4}{3\pi}a\right)^2 + \frac{\pi}{2}a^2\left(a+\frac{4}{3\pi}a\right)^2 = \frac{\pi}{8}a^4 - \frac{8}{9\pi}a^4 + \frac{\pi}{2}\left(1+\frac{4}{3\pi}\right)^2 a^4$$

$$= 3.30\ a^4$$

$$I_{x3} = \frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4}{3\pi}a\right)^2 + \frac{\pi}{2}a^2\left(a-\frac{4}{3\pi}a\right)^2 = \frac{\pi}{8}a^4 - \frac{8}{9\pi}a^4 + \frac{\pi}{2}\left(1-\frac{4}{3\pi}\right)^2 a^4$$

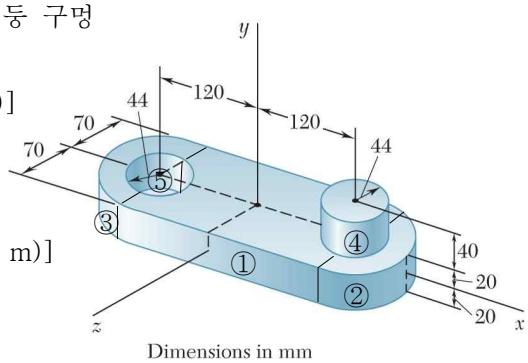
$$= 0.630\ a^4$$

$$I_x = I_{x1} + I_{x2} - I_{x3} = 1.333\ a^4 + 3.30\ a^4 - 0.630\ a^4 = 4.00\ a^4$$

[9.5절]

9.142 $\rho = 7{,}850\ \text{kg/m}^3$

① 사각기둥, ②,③ 반원기둥, ④ 원기둥, ⑤ 원기둥 구멍



Dimensions in mm

$$m = \rho V$$

$$m_1 = (7{,}850\ \text{kg/m}^3)\ [(0.24\ \text{m})(0.14\ \text{m})(0.04\ \text{m})]$$

$$= (7.85\times10^3\ \text{kg/m}^3)\ (1.344\times10^{-3}\ \text{m}^3)$$

$$= 10.550\ \text{kg}$$

$$m_2 = m_3 = (7{,}850\ \text{kg/m}^3)\ \left[\frac{\pi}{2}\ (0.07\ \text{m})^2\ (0.04\ \text{m})\right]$$

$$= (7.85\times10^3\ \text{kg/m}^3)\ (0.3079\times10^{-3}\ \text{m}^3)$$

$$= 2.417\ \text{kg}$$

$$m_4 = m_5 = (7{,}850\ \text{kg/m}^3)\ [\pi\ (0.044\ \text{m})^2\ (0.04\ \text{m})]$$

$$= (7.85\times10^3\ \text{kg/m}^3)\ (0.2433\times10^{-3}\ \text{m}^3) = 1.9098\ \text{kg}$$

(a) $$I_{x1} = \frac{1}{12}m_1(b^2+c^2)$$

$$= \frac{1}{12}(10.550\ \text{kg})[(0.14\ \text{m})^2 + (0.04\ \text{m})^2] = 18.638\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_{x2} = I_{x3} = \frac{1}{12}m_2(3r_2^2 + L_2^2)$$

$$= \frac{1}{12}(2.417\ \text{kg})[3\ (0.07\ \text{m})^2 + (0.04\ \text{m})^2] = 3.283\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_{x4} = \frac{1}{12} m_4 (3r_4^2 + L_4^2) + m_4 d_4^2$$

$$= (1.9098 \text{ kg}) \{\frac{1}{12}[3 (0.044 \text{ m})^2 + (0.04 \text{ m})^2] + (0.04)^2\} = 4.234\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{x5} = \frac{1}{12} m_5 (3r_5^2 + L_5^2)$$

$$= \frac{1}{12}(1.9098 \text{ kg})[3 (0.044 \text{ m})^2 + (0.04 \text{ m})^2] = 1.179\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_x = I_{x1} + I_{x2} + I_{x3} + I_{x4} - I_{x5}$$

$$= [(18.638) + 2 (3.283) + (4.234) - (1.179)]\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$= 28.259\times10^{-3} \text{ kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_x = 28.3\times10^{-3} \text{ kg}\cdot\text{m}^2$$

(b) $$I_{y1} = \frac{1}{12} m_1 (a^2 + b^2)$$

$$= \frac{1}{12}(10.550 \text{ kg})[(0.24 \text{ m})^2 + (0.14 \text{ m})^2] = 67.87\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{y2} = I_{y3} = \frac{1}{2} m_2 r_2^2 - m_2 d_{2a}^2 + m_2 d_{2b}^2$$

$$= (2.417 \text{ kg}) \{\frac{1}{2}(0.07 \text{ m})^2 - [\frac{4}{3\pi}(0.07 \text{ m})]^2 + [(0.12 \text{ m}) + \frac{4}{3\pi}(0.07 \text{ m})]^2\}$$

$$= 57.96\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{y4} = I_{y5} = \frac{1}{2} m_4 r_4^2 + m_4 d_4^2$$

$$= (1.9098 \text{ kg}) [\frac{1}{2}(0.044 \text{ m})^2 + (0.12 \text{ m})^2] = 29.34\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4} - I_{y5}$$

$$= [(67.87) + 2 (57.96) + 0]\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$= 183.79\times10^{-3} \text{ kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_y = 183.8\times10^{-3} \text{ kg}\cdot\text{m}^2$$

(c) $$m = m_1 + m_2 + m_3 + m_4 - m_5$$

$$= (10.550 \text{ kg}) + 2 (2.417 \text{ kg}) + 0 = 15.384 \text{ kg}$$

$$k_x^2 = \frac{I_x}{m} = \frac{28.259\times10^{-3} \text{ kg}\cdot\text{m}^2}{15.384 \text{ kg}} = 1.8369\times10^{-3} \text{ m}^2$$

$$\Rightarrow \quad k_x = 0.0429 \text{ m} = 42.9 \text{ mm}$$

$$k_y^2 = \frac{I_y}{m} = \frac{183.79\times10^{-3} \text{ kg}\cdot\text{m}^2}{15.384 \text{ kg}} = 11.947\times10^{-3} \text{ m}^2$$

$$\Rightarrow \quad k_y = 0.1093 \text{ m} = 109.3 \text{ mm}$$