정 역 학　　연습과제 풀이　　제2장　　(2019년)

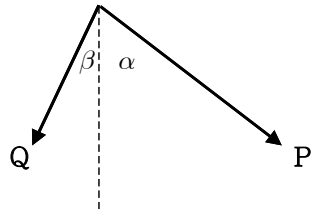
[2.1절]

2.15 S; P = 300 N, Q = 200 N, α = 45°, β = 25°

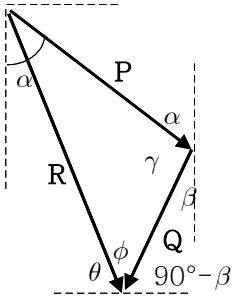
known : P, Q, α, β, unknown : R(크기, 방향)

⇒ 삼각법(trigonometry) sine 공식, cosine 공식 사용하여 힘의 합성

M; 자유물체도 (F.B.D.),



힘 삼각형 (force triangle)



γ = 180° − (α+β) = 180°−(45°+25°) = 110°

A; 합력의 크기 (cosine 공식)

$R^2 = P^2 + Q^2 - 2\,P\,Q\cos\gamma$

$= (300\text{ N})^2 + (200\text{ N})^2 - 2\,(300\text{ N})\,(200\text{ N})\cos 110° = 171{,}042\text{ N}^2$

⇒ R = 413.6 N

합력의 방향 각도 (sine 공식)

$\dfrac{P}{\sin\phi} = \dfrac{R}{\sin\gamma}$ ⇒ $\sin\phi = \dfrac{P}{R}\sin\gamma = \dfrac{300\text{ N}}{413.6\text{ N}}\sin 110° = 0.6816$

⇒ $\phi = \sin^{-1}(0.6816) = 43.0°$

θ = 180°− (90°−β) − ϕ = 180° − (90°−25°) − 43.0° = 71.95°

⇒ **R** = 414 N ↘72.0°

R; 과정의 타당성 (가령, 합력의 크기 계산에 sine 공식 대신 cosine 공식을 사용한 사유)

T; 결과 검토 (가령, $\alpha<\alpha+\beta<\theta$, $R<P+Q$. 90°<γ<180° 일 때 $R>P$, $R>Q$)

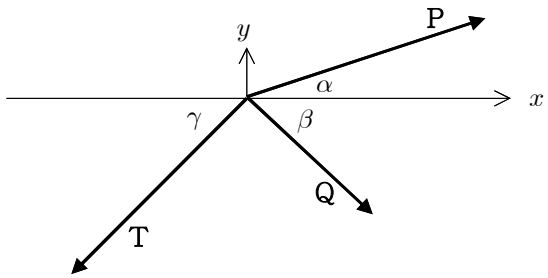
[2.2절]

2.40 S; P = 500 N, Q = 200 N, $\tan\alpha = \frac{7}{24}$, $\beta = \frac{3}{4}$, $\tan\gamma = \frac{110}{96}$

known : P, Q, α, β, 합력 성분 R_y = 0, unknown : 장력 T, 합력 성분 R_x

⇒ 직각성분 합에 의한 힘의 합성

M; 자유물체도 (F.B.D.)



A;

(a) $R_y = P\sin\alpha - Q\sin\beta - T\sin\gamma = 0$

$\Rightarrow T = \frac{1}{\sin\gamma}[P\sin\alpha - Q\sin\beta]$

$= \frac{146}{110}[(500\text{ N})\frac{7}{25} + (200\text{ N})\frac{3}{5}] = 26.54\text{ N} \quad \Rightarrow \quad T = 26.5\text{ N}$

(b) $R_x = P\cos\alpha + Q\cos\beta - T\cos\gamma$

$= (500\text{ N})\frac{24}{25} + (200\text{ N})\frac{4}{5} - (26.54\text{ N})\frac{96}{146} = 622.5\text{ N} \quad \Rightarrow \quad R_x = 623\text{ N}$

R; 과정의 타당성 (가령, 다각형 방법과 비교하면 어떤지)

T; 결과 의미 (가령, 합력의 방향, R_x 방향 →)

[2.3절]

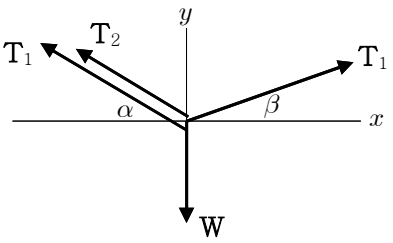
2.55 W = 900N, α = 30°, β = 10°

S; known W, α, β, unknown T_1 , T_2

질점의 평형 문제

⇒ 직각성분 방법 또는 힘 삼각형 방법

M; 자유물체도 (F.B.D.)



A;

<방법 1 : 직각 성분>

$\Sigma F_x = 0$; $T_1 \cos\beta - (T_1 + T_2)\cos\alpha = 0$

$\Rightarrow \quad T_1(\cos\beta - \cos\alpha) - T_2 \cos\alpha = 0 \quad \cdots ①$

$\Sigma F_y = 0$; $T_1 \sin\beta + (T_1 + T_2)\sin\alpha - W = 0$

$\Rightarrow \quad T_1(\sin\beta + \sin\alpha) + T_2 \sin\alpha = W \quad \cdots ②$

(a) ①×sinα + ②×cosα

$T_1(\cos\beta - \cos\alpha)\sin\alpha + T_1(\sin\beta + \sin\alpha)\cos\alpha = W\cos\alpha$

$$\Rightarrow \quad T_1 = W\frac{\cos\alpha}{\sin\alpha\cos\beta + \cos\alpha\sin\beta} = W\frac{\cos\alpha}{\sin(\alpha+\beta)}$$

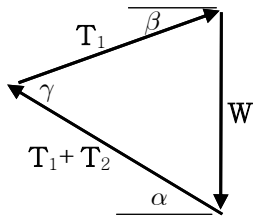
$$= (900\text{ N})\frac{\cos 30^\circ}{\sin(30^\circ + 10^\circ)} = 1{,}212.6\text{ N} \qquad \Rightarrow \qquad T_1 = 1{,}213\text{ N}$$

(b) ① ⇒ $$T_2 = T_1\frac{\cos\beta - \cos\alpha}{\cos\alpha} = (1{,}212.6\text{ N})\frac{\cos 10^\circ - \cos 30^\circ}{\cos 30^\circ} = 166.32\text{ N}$$

$\Rightarrow \qquad T_2 = 166.3$ N

<방법 2 : 힘 삼각형>

$\gamma = \alpha + \beta = 30° + 10° = 40°$



(a) $$\frac{T_1}{\sin(90^\circ - \alpha)} = \frac{W}{\sin\gamma}$$

$$\Rightarrow \quad T_1 = W\frac{\sin(90^\circ - \alpha)}{\sin\gamma} = (900\text{ N})\frac{\sin(90^\circ - 30^\circ)}{\sin 40^\circ}$$

$= 1{,}212.6$ N $\qquad \Rightarrow \qquad T_1 = 1{,}213$ N

(b) $$\frac{T_1 + T_2}{\sin(90^\circ - \beta)} = \frac{W}{\sin\gamma}$$

$$\Rightarrow \quad T_1 + T_2 = W\frac{\sin(90^\circ - \beta)}{\sin\gamma} = (900\text{ N})\frac{\sin(90^\circ - 10^\circ)}{\sin 40^\circ} = 1{,}378.9\text{ N}$$

$\Rightarrow \quad T_2 = 1{,}378.9\text{ N} - T_1 = 1{,}378.9\text{ N} - 1{,}212.6\text{ N} = 166.3\text{ N}$

R; 과정의 타당성 (가령, 두 가지 방법 비교)

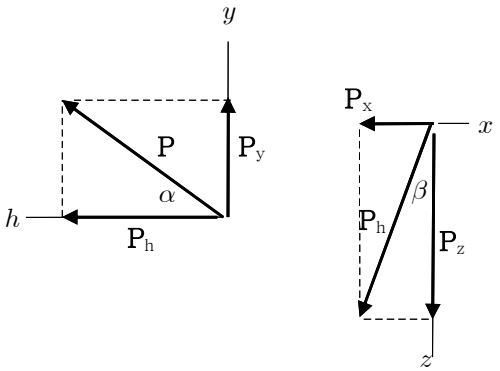
T; 결과 검토 (가령, $T_1 + T_2 > T_1 \quad \Rightarrow \quad \alpha > \beta$)

[2.4절]

2.72 P = 450 N, α = 35°, β = 40°

S; known P, α, β,
unknown $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{F}_z$, θ_x, θ_y, θ_z
⇒ 공간에서 힘의 직각성분 (각도 이용), 방향여현

M;



A;

(a) $P_y = P \sin\alpha$ = (450 N) sin35° = 258.1 N

$P_h = P \cos\alpha$

$P_x = -P_h \sin\beta = -P \cos\alpha \sin\beta$

= -(450 N) cos35° sin40° = -236.9 N

$P_z = P_h \cos\beta = P \cos\alpha \cos\beta$

= (450 N) cos35° cos40° = 282.4 N

⇒ $\mathbf{P}$ = (-237 N) $\mathbf{i}$ + (258 N) $\mathbf{j}$ + (282 N) $\mathbf{k}$

(b) $\cos\theta_x = \dfrac{P_x}{P} = \dfrac{-236.9\text{ N}}{450\text{ N}} = -0.5264$ ⇒ $\theta_x = \cos^{-1}(-0.5264) = 121.7°$

$\cos\theta_y = \dfrac{P_y}{P} = \dfrac{258.1\text{ N}}{450\text{ N}} = 0.5736$ ⇒ $\theta_x = \cos^{-1}(0.5736) = 55.0°$

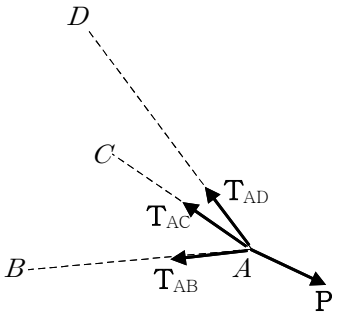
$\cos\theta_z = \dfrac{P_z}{P} = \dfrac{282.4\text{ N}}{450\text{ N}} = 0.6276$ ⇒ $\theta_z = \cos^{-1}(0.6276) = 51.1°$

R; 과정의 타당성 (가령,)

T; 결과 검토 (가령, 힘 성분의 부호와 방향 각도의 범위)

[2.5절]

2.107 T_{AD} = 305 N



S; known T_{AD}, 좌표들, unknown T_{AB}, T_{AC}, P M;

⇒ 공간에서 힘의 직각성분 (좌표 이용), 평형

A; $\mathbf{P} = P\,\mathbf{i}$

$$d_{AB} = \sqrt{(-960\text{ mm})^2 + (-240\text{ mm})^2 + (380\text{ mm})^2}$$
$$= 1{,}060\text{ mm}$$
$$\boldsymbol{\lambda}_{AB} = \frac{1}{1{,}060}(-960\,\mathbf{i} - 240\,\mathbf{j} + 380\,\mathbf{k})$$
$$= -0.9057\,\mathbf{i} - 0.2264\,\mathbf{j} + 0.3585\,\mathbf{k}$$
$$\mathbf{T}_{AB} = T_{AB}\,\boldsymbol{\lambda}_{AB} = T_{AB}\,(-0.9057\,\mathbf{i} - 0.2264\,\mathbf{j} + 0.3585\,\mathbf{k})$$

$$d_{AC} = \sqrt{(-960\text{ mm})^2 + (-240\text{ mm})^2 + (-320\text{ mm})^2} = 1{,}040\text{ mm}$$
$$\boldsymbol{\lambda}_{AC} = \frac{1}{1{,}040}(-960\,\mathbf{i} - 240\,\mathbf{j} - 320\,\mathbf{k}) = -0.9231\,\mathbf{i} - 0.2308\,\mathbf{j} - 0.3077\,\mathbf{k}$$
$$\mathbf{T}_{AC} = T_{AC}\,\boldsymbol{\lambda}_{AC} = T_{AC}\,(-0.9231\,\mathbf{i} - 0.2308\,\mathbf{j} - 0.3077\,\mathbf{k})$$

$$d_{AD} = \sqrt{(-960\text{ mm})^2 + (960-240\text{ mm})^2 + (-220\text{ mm})^2} = 1{,}220\text{ mm}$$
$$\boldsymbol{\lambda}_{AD} = \frac{1}{1{,}220}(-960\,\mathbf{i} + 720\,\mathbf{j} - 220\,\mathbf{k}) = -0.7869\,\mathbf{i} + 0.5902\,\mathbf{j} - 0.1803\,\mathbf{k}$$
$$\mathbf{T}_{AD} = T_{AD}\,\boldsymbol{\lambda}_{AD} = (305\text{ N})\,(-0.7869\,\mathbf{i} + 0.5902\,\mathbf{j} - 0.1803\,\mathbf{k})$$
$$= -(240\text{ N})\,\mathbf{i} + (180.0\text{ N})\,\mathbf{j} - (55.0\text{ N})\,\mathbf{k}$$

$\Sigma\mathbf{F} = 0 \Rightarrow \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0$

$\Sigma F_x = 0$; $-0.9057\ T_{AB} - 0.9231\ T_{AC} + (-240\text{ N}) + P = 0$ … ①

$\Sigma F_y = 0$; $-0.2264\ T_{AB} - 0.2308\ T_{AC} + (180.0\text{ N}) = 0$ … ②

$\Sigma F_z = 0$; $0.3585\ T_{AB} - 0.3077\ T_{AC} + (-55.0\text{ N}) = 0$ … ③

②×0.3077 − ③×0.2308

$$[(-0.2264)(0.3077)-(0.3585)(0.2308)]\,T_{AB} + [(180.0\text{ N})(0.3077)-(-55.0\text{ N})(0.2308)] = 0$$

$\Rightarrow \quad T_{AB} = 446.7$ N

② $\Rightarrow \quad T_{AC} = \frac{1}{0.2308}[(-0.2264)(446.7\text{ N}) + (180.0\text{ N})] = 341.7$ N

① $\Rightarrow \quad P = (0.9057)(446.7\text{ N}) + (0.9231)(341.7\text{ N}) + (240\text{ N}) = 960.0$ N

$\Rightarrow \quad P = 960$ N

R; 과정의 타당성 (가령, 공간에서 힘의 직각성분)

T; 결과 검토 (가령, 세 힘의 y성분의 합 또는 z성분의 합)