

[9.1절]

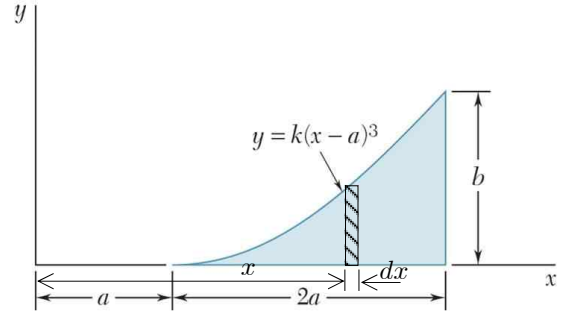
$$9.9\&12 \quad y = k(x-a)^3, \quad (3a, b) \Rightarrow b = k(2a)^3$$

$$\Rightarrow k = \frac{b}{8a^3}, \quad y = \frac{b}{8a^3}(x-a)^3$$

$$dA = y dx = \frac{b}{8a^3}(x-a)^3 dx$$

$$(\text{문제 외}) \quad A = \int dA = \int_a^{3a} \frac{b}{8a^3}(x-a)^3 dx$$

$$= \frac{b}{8a^3} \left[\frac{1}{4}(x-a)^4 \right]_a^{3a} = \frac{b}{32a^3} [(2a)^4 - 0] = \frac{1}{2} ab$$



$$9.9 \quad dI_x = \frac{1}{3} y^3 dx$$

$$I_x = \int dI_x = \int_a^{3a} \frac{1}{3} y^3 dx = \frac{1}{3} \int_a^{3a} \left[\frac{b}{8a^3}(x-a)^3 \right]^3 dx = \frac{1}{3} \left(\frac{b}{8a^3} \right)^3 \int_a^{3a} (x-a)^9 dx$$

$$= \frac{1}{3} \frac{b^3}{2^9 a^9} \left[\frac{1}{10}(x-a)^{10} \right]_a^{3a} = \frac{1}{3} \frac{b^3}{2^9 a^9} \frac{1}{10} [(2a)^{10} - 0] = \frac{1}{15} ab^3$$

$$\Rightarrow I_x = 0.0667 ab^3$$

$$(\text{문제 외}) \quad k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{15} ab^3}{\frac{1}{2} ab} = \frac{2}{15} b^2 \Rightarrow k_x = \sqrt{\frac{2}{15} b^2} = 0.365 b$$

$$9.12 \quad dI_y = x^2 dA = x^2 \frac{b}{8a^3} (x-a)^3 dx = \frac{b}{8a^3} x^2 (x^3 - 3ax^2 + 3a^2x - a^3) dx$$

$$I_y = \int dI_y = \int_a^{3a} \frac{b}{8a^3} (x^5 - 3ax^4 + 3a^2x^3 - a^3x^2) dx$$

$$= \frac{b}{8a^3} \left[\frac{1}{6}x^6 - \frac{3a}{5}x^5 + \frac{3a^2}{4}x^4 - \frac{a^3}{3}x^3 \right]_a^{3a}$$

$$= \frac{b}{8a^3} \left\{ \left[\frac{1}{6}(3a)^6 - \frac{3a}{5}(3a)^5 + \frac{3a^2}{4}(3a)^4 - \frac{a^3}{3}(3a)^3 \right] \right.$$

$$\left. - \left[\frac{1}{6}a^6 - \frac{3a}{5}a^5 + \frac{3a^2}{4}a^4 - \frac{a^3}{3}a^3 \right] \right\} = 3.433 a^3 b \Rightarrow I_y = 3.43 a^3 b$$

$$(\text{문제 외}) \quad k_y^2 = \frac{I_y}{A} = \frac{3.433 a^3 b}{0.500 ab} = 6.866 a^2 \Rightarrow k_y = \sqrt{6.866 a^2} = 2.62 a$$