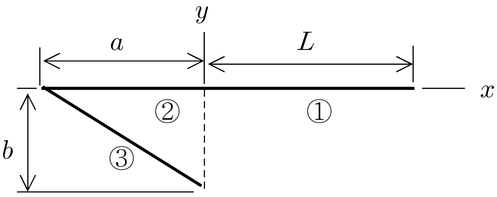
[5.1절]

5.30 S; 선재의 도심 위치

M;



A; a = 80 mm, b = 60 mm

BCD is horizontal ⇒ $\overline{X} = 0$

⇒ $\Sigma(\overline{x}\,l) = 0$

① $l = L$

$\overline{x} = \frac{L}{2}$

② $l = a$ = 80 mm

$\overline{x} = -\frac{1}{2}a$ = -40 mm

③ $l = \sqrt{a^2+b^2}$ = 100 mm

$\overline{x} = -\frac{1}{2}a$ = -40 mm

$\Sigma(\overline{x}\,l) = \frac{L}{2}L + (-40\text{ mm})(80\text{ mm}) + (-40\text{ mm})(100\text{ mm}) = 0$

⇒ $\frac{L^2}{2} = 7{,}200\text{ mm}^2$ ⇒ $L = 120.0\text{ mm}$

R; (과정의 타당성 검토)

T; (결과의 의미 검토)

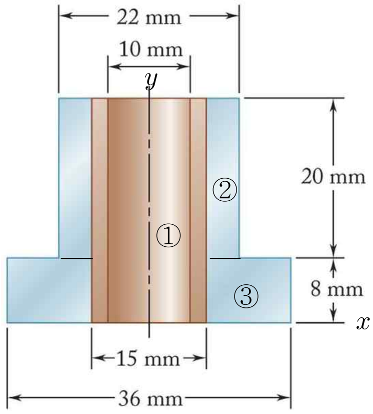
[5.4절]

5.119 $\rho_b = 8{,}800\ \text{kg/m}^3 = 8.80\times10^{-6}\ \text{kg/mm}^3$

$\rho_s = 7{,}860\ \text{kg/m}^3 = 7.86\times10^{-6}\ \text{kg/mm}^3$

$d_1 = 10\ \text{mm},\ d_2 = 15\ \text{mm},\ d_3 = 22\ \text{mm},$

$d_4 = 36\ \text{mm},\ h_1 = 20\ \text{mm},\ h_2 = 8\ \text{mm}$



S; 대칭구조 $\overline{X} = \overline{Z} = 0$ M;

$W = mg = \rho V g \quad \Rightarrow \quad m = \rho V$

$$\overline{Y} = \frac{\Sigma(\overline{y}\,W)}{\Sigma W} = \frac{\Sigma(\overline{y}\,m)}{\Sigma m}$$

A; ① 구리 원통 + ② 철 원통 상부 + ③ 철 원통 하부

① $h = h_1 + h_2 = 28\ \text{mm}$

$$V = \frac{\pi}{4}\left(d_2^2 - d_1^2\right)h = \frac{\pi}{4}\left[(15\ \text{mm})^2 - (10\ \text{mm})^2\right](28\ \text{mm}) = 2{,}749\ \text{mm}^3$$

$$m = \rho_b\,V = (8.80\times10^{-6}\ \text{kg/mm}^3)(2{,}749\ \text{mm}^3) = 24.19\times10^{-3}\ \text{kg}$$

$$\overline{y} = \frac{1}{2}h = \frac{1}{2}(28\ \text{mm}) = 14.0\ \text{mm}$$

② $$V = \frac{\pi}{4}\left(d_3^2 - d_2^2\right)h_1 = \frac{\pi}{4}\left[(22\ \text{mm})^2 - (15\ \text{mm})^2\right](20\ \text{mm}) = 4{,}068\ \text{mm}^3$$

$$m = \rho_s\,V = (7.86\times10^{-6}\ \text{kg/mm}^3)(4{,}068\ \text{mm}^3) = 31.98\times10^{-3}\ \text{kg}$$

$$\overline{y} = h_2 + \frac{1}{2}h_1 = (8\ \text{mm}) + \frac{1}{2}(20\ \text{mm}) = 18.0\ \text{mm}$$

③ $$V = \frac{\pi}{4}\left(d_4^2 - d_2^2\right)h_2 = \frac{\pi}{4}\left[(36\ \text{mm})^2 - (15\ \text{mm})^2\right](8\ \text{mm}) = 6{,}729\ \text{mm}^3$$

$$m = \rho_s\,V = (7.86\times10^{-6}\ \text{kg/mm}^3)(6{,}729\ \text{mm}^3) = 52.89\times10^{-3}\ \text{kg}$$

$$\overline{y} = \frac{1}{2}h_2 = \frac{1}{2}(8\ \text{mm}) = 4.0\ \text{mm}$$

$$\Sigma m = [24.19 + 31.98 + 52.89]\times10^{-3}\ \text{kg} = 109.06\times10^{-3}\ \text{kg}$$

$$\Sigma(\overline{y}\,m) = [(14.0)(24.19) + (18.0)(31.98) + (4.0)(52.89)]\times10^{-3}\ \text{kg·mm}$$

$$= 1{,}125.5\times10^{-3}\ \text{kg·mm}$$

$$\overline{Y} = \frac{\Sigma(\overline{y}\,m)}{\Sigma m} = \frac{1{,}125.5\times10^{-3}\ \text{kg}\cdot\text{mm}}{109.06\times10^{-3}\ \text{kg}} = 10.320\ \text{mm} \quad \Rightarrow \quad \text{중심} = (0,\ 10.32\ \text{mm},\ 0)$$

R; (과정의 타당성 검토) (가령, 원통의 부피 및 질량 계산 과정)

T; (결과의 의미 검토) (가령, 중심의 위치)

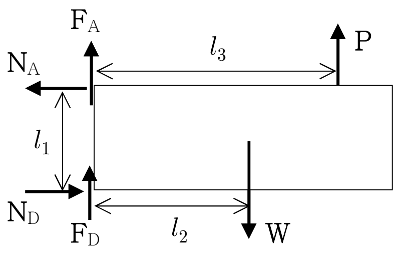
정 역 학　　연습문제 풀이　　제8장　　(2018년)

[8.1절]

8.29　W = 50 N,　l_1 = 0.4 m,　l_2 = 0.6 m, l_3 = 1 m,
μ_s = 0.40

S; 힘과 모멘트의 평형방정식

M;



A;

(a) P = 0

$\circlearrowleft \Sigma M_D = 0$; $l_1 \ N_A - l_2 \ W = 0$

$\Rightarrow \ N_A = \dfrac{l_2}{l_1} W = \dfrac{0.6 \text{ m}}{0.4 \text{ m}} (50 \text{ N}) = 75.0 \text{ N}$

$\rightarrow \Sigma F_x = 0$; $N_D - N_A = 0$

$\Rightarrow \ N_D = N_A = 75.0 \text{ N}$

$(F_A)_{\max} = \mu_s \ N_A = (0.40)\ (75.0 \text{ N}) = 30.0 \text{ N}$

$(F_D)_{\max} = \mu_s \ N_D = (0.40)\ (75.0 \text{ N}) = 30.0 \text{ N}$

$(F_A)_{\max} + (F_D)_{\max} = (30.0 \text{ N}) + (30.0 \text{ N}) = 60.0 \text{ N}$

$\uparrow \Sigma F_y = 0$; $F_A + F_D - W = 0$

$\Rightarrow \ F_A + F_D = W = 50.0 \text{ N} < (F_A)_{\max} + (F_D)_{\max}$: in equilibrium

(b) P = 20 N

$\circlearrowleft \Sigma M_D = 0$; $l_1 \ N_A - l_2 \ W + l_3 \ P = 0$

$\Rightarrow \ N_A = \dfrac{l_2 W - l_3 P}{l_1} = \dfrac{(0.6 \text{ m})(50 \text{ N}) - (1.0 \text{ m})(20 \text{ N})}{0.4 \text{ m}} = 25.0 \text{ N}$

$\rightarrow \Sigma F_x = 0$; $N_D - N_A = 0$

$\Rightarrow \ N_D = N_A = 25.0 \text{ N}$

$(F_A)_{\max} = \mu_s \ N_A = (0.40)\ (25.0 \text{ N}) = 10.0 \text{ N}$

$(F_D)_{\max} = \mu_s \ N_D = (0.40)\ (25.0 \text{ N}) = 10.0 \text{ N}$

$(F_A)_{\max} + (F_D)_{\max} = (10.0 \text{ N}) + (10.0 \text{ N}) = 20.0 \text{ N}$

$\uparrow \Sigma F_y = 0$; $F_A + F_D - W + P = 0$

$\Rightarrow \ F_A + F_D = W - P = (50.0 \text{ N}) - (20.0 \text{ N}) = 30.0 \text{ N} > 0$ (마찰력 방향 타당)

$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$: not in equilibrium

⇒ 미끄러져 내려감

R; (과정의 타당성 검토) (가령, 다른 평형방정식을 선택하면?)

T; (결과의 의미 검토) (가령, 힘 P가 가하질 때 오히려 미끄러져 내려감. ∵ 　　)

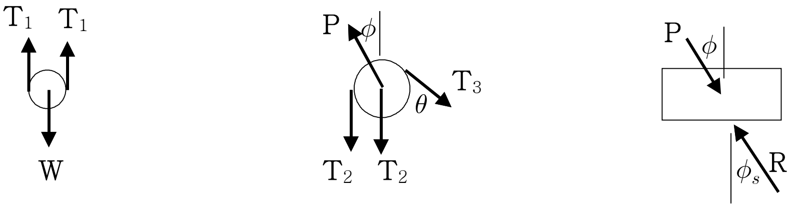
8.6 W = 20 N, μ_s = 0.35

S; 최대 허용 각에서 블록 E가 움직이기 직전

마찰각 $\phi_s = \tan^{-1}(0.35) = 19.29°$, 블록 E에서 힘 삼각형

M;

움직 도르래 B　　　고정 도르래 C　　　블록 E



$T_1 = T_2 = T_3 = T$

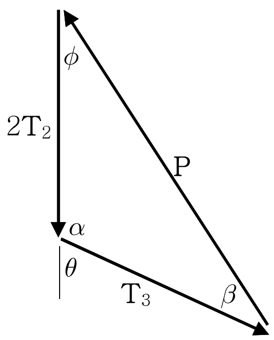
A;

도르래 B 에서,

$$\uparrow \Sigma F_y = 0 \; ; \; 2\,T - W = 0 \quad \Rightarrow \quad T = \frac{1}{2}W = \frac{1}{2}(20\text{ N}) = 10\text{ N}$$

블록 E 에서, $\phi = \phi_s = 19.29°$

도르래 C 에서,



$\alpha = 180° - \theta$

$\theta = \phi + \beta = 19.29° + \beta$

sine공식 $\dfrac{2\,T}{\sin\beta} = \dfrac{T}{\sin\phi}$

$\Rightarrow \sin\beta = 2\sin\phi = 2\sin 19.29° = 0.6607$

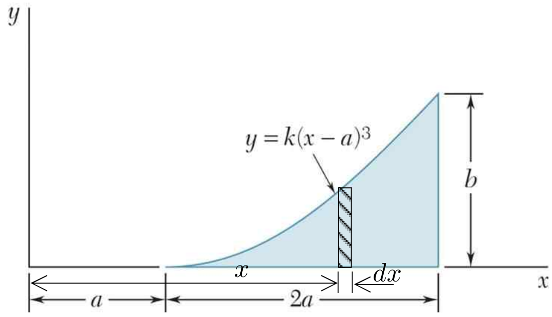
$\Rightarrow \beta = \sin^{-1}(0.6607) = 41.35°$

$\theta = 19.29° + \beta = 19.29° + 41.35° = 60.64° \quad \Rightarrow \quad \theta = 60.6°$

R; (과정의 타당성 검토) (가령, 블록 E 에서 힘 P의 방향, 도르래 C에서 직각성분 방법)

T; (결과의 의미 검토) (가령, $\theta > \phi$, 무게 W에 무관)

[9.1절]



9.9&12 $y = k(x-a)^3, \quad (3a,\ b) \quad \Rightarrow \quad b = k(2a)^3$

$$\Rightarrow \quad k = \frac{b}{8a^3}\ , \qquad y = \frac{b}{8a^3}(x-a)^3$$

$$dA = y\,dx = \frac{b}{8a^3}(x-a)^3\ dx$$

(문제 외 $A = \int dA = \int_a^{3a} \frac{b}{8a^3}(x-a)^3\,dx$

$$= \frac{b}{8a^3}\left[\frac{1}{4}(x-a)^4\right]_a^{3a} = \frac{b}{32a^3}\left[(2a)^4 - 0\right] = \frac{1}{2}ab$$)

9.9 $dI_x = \frac{1}{3}y^3\,dx$

$$I_x = \int dI_x = \int_a^{3a}\frac{1}{3}y^3\,dx = \frac{1}{3}\int_a^{3a}\left[\frac{b}{8a^3}(x-a)^3\right]^3 dx = \frac{1}{3}\left(\frac{b}{8a^3}\right)^3 \int_a^{3a}(x-a)^9\,dx$$

$$= \frac{1}{3}\frac{b^3}{2^9a^9}\left[\frac{1}{10}(x-a)^{10}\right]_a^{3a} = \frac{1}{3}\frac{b^3}{2^9a^9}\ \frac{1}{10}\left[(2a)^{10} - 0\right] = \frac{1}{15}ab^3$$

$$\Rightarrow \qquad I_x = 0.0667\ ab^3$$

(문제 외 $k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{15}ab^3}{\frac{1}{2}ab} = \frac{2}{15}b^2 \qquad \Rightarrow \qquad k_x = \sqrt{\frac{2}{15}b^2} = 0.365\ b$)

9.12 $dI_y = x^2\,dA = x^2\ \frac{b}{8a^3}(x-a)^3\ dx = \frac{b}{8a^3}x^2\ (x^3 - 3ax^2 + 3a^2x - a^3)\ dx$

$$I_y = \int dI_y = \int_a^{3a}\frac{b}{8a^3}(x^5 - 3ax^4 + 3a^2x^3 - a^3x^2)\,dx$$

$$= \frac{b}{8a^3}\left[\frac{1}{6}x^6 - \frac{3a}{5}x^5 + \frac{3a^2}{4}x^4 - \frac{a^3}{3}x^3\right]_a^{3a}$$

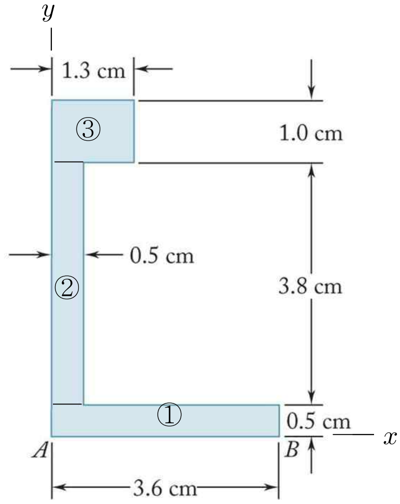
$$= \frac{b}{8a^3}\left\{\left[\frac{1}{6}(3a)^6 - \frac{3a}{5}(3a)^5 + \frac{3a^2}{4}(3a)^4 - \frac{a^3}{3}(3a)^3\right]\right.$$

$$\left. - \left[\frac{1}{6}a^6 - \frac{3a}{5}a^5 + \frac{3a^2}{4}a^4 - \frac{a^3}{3}a^3\right]\right\} = 3.433\ a^3b \qquad \Rightarrow \qquad I_y = 3.43\ a^3b$$

(문제 외 $k_y^2 = \frac{I_y}{A} = \frac{3.433\,a^3b}{0.500\,ab} = 6.866\ a^2 \quad \Rightarrow \quad k_y = \sqrt{6.866\,a^2} = 2.62\ a$)

[9.2절]

9.44 Centroid



① $A = (3.6\ \text{cm})(0.5\ \text{cm}) = 1.80\ \text{cm}^2$

$\bar{x} = \frac{1}{2}(3.6\ \text{cm}) = 1.80\ \text{cm}$

$\bar{y} = \frac{1}{2}(0.5\ \text{cm}) = 0.25\ \text{cm}$

② $A = (0.5\ \text{cm})(3.8\ \text{cm}) = 1.90\ \text{cm}^2$

$\bar{x} = \frac{1}{2}(0.5\ \text{cm}) = 0.25\ \text{cm}$

$\bar{y} = (0.5\ \text{cm}) + \frac{1}{2}(3.8\ \text{cm}) = 2.40\ \text{cm}$

③ $A = (1.3\ \text{cm})(1.0\ \text{cm}) = 1.30\ \text{cm}^2$

$\bar{x} = \frac{1}{2}(1.3\ \text{cm}) = 0.65\ \text{cm}$

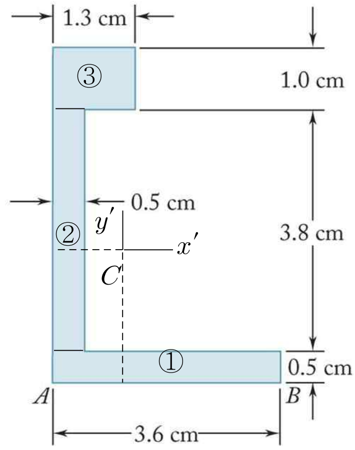
$\bar{y} = (0.5\ \text{cm}) + (3.8\ \text{cm}) + \frac{1}{2}(1.0\ \text{cm}) = 4.80\ \text{cm}$

$\Sigma A = (1.80\ \text{cm}^2) + (1.90\ \text{cm}^2) + (1.30\ \text{cm}^2) = 5.00\ \text{cm}^2$

$\Sigma(\bar{x}A) = (1.80\ \text{cm})(1.80\ \text{cm}^2) + (0.25\ \text{cm})(1.90\ \text{cm}^2) + (0.65\ \text{cm})(1.30\ \text{cm}^2) = 4.56\ \text{cm}^3$

$\Sigma(\bar{y}A) = (0.25\ \text{cm})(1.80\ \text{cm}^2) + (2.40\ \text{cm})(1.90\ \text{cm}^2) + (4.80\ \text{cm})(1.30\ \text{cm}^2) = 11.25\ \text{cm}^3$

$\overline{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{4.56\ \text{cm}^3}{5.00\ \text{cm}^2} = 0.912\ \text{cm},\quad \overline{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{11.25\ \text{cm}^3}{5.00\ \text{cm}^2} = 2.25\ \text{cm}$



$I_{x1} = \bar{I}_{x1} + A_1 d_1^2$

$= \frac{1}{12}(3.6\ \text{cm})(0.5\ \text{cm})^3 + (1.8\ \text{cm}^2)(2.25\text{−}0.25\ \text{cm})^2$

$= 7.238\ \text{cm}^4$

$I_{x2} = \bar{I}_{x2} + A_2 d_2^2$

$= \frac{1}{12}(0.5\ \text{cm})(3.8\ \text{cm})^3 + (1.9\ \text{cm}^2)(2.25\text{−}1.9\text{−}0.5\ \text{cm})^2$

$= 2.329\ \text{cm}^4$

$I_{x3} = \bar{I}_{x3} + A_3 d_3^2$

$= \frac{1}{12}(1.3\ \text{cm})(1.0\ \text{cm})^3 + (1.3\ \text{cm}^2)(0.5+3.8+0.5\text{−}2.25\ \text{cm})^2$

$= 8.562\ \text{cm}^4$

$I_x = I_{x1} + I_{x2} + I_{x3} = (7.238\ \text{cm}^4) + (2.329\ \text{cm}^4) + (8.562\ \text{cm}^4) = 18.13\ \text{cm}^4$

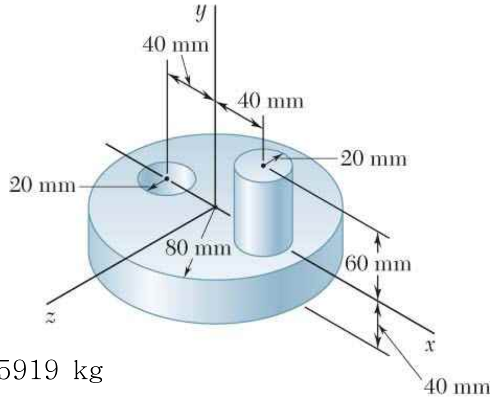
$I_{y1} = \bar{I}_{y1} + A_1 d_1^2 = \frac{1}{12}(3.6\ \text{cm})^3(0.5\ \text{cm}) + (1.8\ \text{cm}^2)(1.8\text{−}0.912\ \text{cm})^2 = 3.363\ \text{cm}^4$

$I_{y2} = \bar{I}_{y2} + A_2 d_2^2 = \frac{1}{12}(0.5\ \text{cm})^3(3.8\ \text{cm}) + (1.9\ \text{cm}^2)(0.912\text{−}0.25\ \text{cm})^2 = 0.872\ \text{cm}^4$

$I_{y3} = \bar{I}_{y3} + A_3 d_3^2 = \frac{1}{12}(1.3\ \text{cm})^3(1.0\ \text{cm}) + (1.3\ \text{cm}^2)(0.912\text{−}0.65\ \text{cm})^2 = 0.273\ \text{cm}^4$

$I_y = I_{y1} + I_{y2} + I_{y3} = (3.363\ \text{cm}^4) + (0.872\ \text{cm}^4) + (0.273\ \text{cm}^4) = 4.51\ \text{cm}^4$

[9.5절]



9.141 ρ = 7,850 kg/m^3

$m = \rho V$ ① 원판, ② 원기둥, ③ 원판 구멍

$m_1 = \rho V_1$

$= (7{,}850 \text{ kg/m}^3)\ [\pi\ (0.08 \text{ m})^2\ (0.04 \text{ m})]$

$= 6.313$ kg

$m_2 = \rho V_2$

$= (7{,}850 \text{ kg/m}^3)\ [\pi\ (0.02 \text{ m})^2\ (0.06 \text{ m})] = 0.5919$ kg

$m_3 = \rho V_3 = (7{,}850 \text{ kg/m}^3)\ [\pi\ (0.02 \text{ m})^2\ (0.04 \text{ m})] = 0.3946$ kg

(a) $I_{x1} = \dfrac{1}{12} m_1 (3r_1^2 + 4L_1^2)$

$= \dfrac{1}{12}(6.313 \text{ kg})[3\ (0.08 \text{ m})^2 + 4\ (0.04 \text{ m})^2] = 13.467\times10^{-3} \text{ kg·m}^2$

$I_{x2} = \dfrac{1}{12} m_2 (3r_2^2 + 4L_2^2)$

$= \dfrac{1}{12}(0.5919 \text{ kg})[3\ (0.02 \text{ m})^2 + 4\ (0.06 \text{ m})^2] = 0.7695\times10^{-3} \text{ kg·m}^2$

$I_{x3} = \dfrac{1}{12} m_3 (3r_3^2 + 4L_3^2)$

$= \dfrac{1}{12}(0.3946 \text{ kg})[3\ (0.02 \text{ m})^2 + 4\ (0.04 \text{ m})^2] = 0.2499\times10^{-3} \text{ kg·m}^2$

$I_x = I_{x1} + I_{x2} - I_{x3} = [(13.467) + (0.7695) - (0.2499)]\times10^{-3} \text{ kg·m}^2$

$= 13.9886\times10^{-3} \text{ kg·m}^2 \quad \Rightarrow \quad I_x = 13.99\times10^{-3} \text{ kg·m}^2$

(b) $I_{y1} = \dfrac{1}{2} m_1 r_1^2$

$= \dfrac{1}{2}(6.313 \text{ kg})\ (0.08 \text{ m})^2 = 20.20\times10^{-3} \text{ kg·m}^2$

$I_{y2} = \dfrac{1}{2} m_2 r_2^2 + m_2 d_2^2$

$= \dfrac{1}{2}(0.5919 \text{ kg})\ (0.02 \text{ m})^2 + (0.5919 \text{ kg})\ (0.04 \text{ m})^2 = 1.0654\times10^{-3} \text{ kg·m}^2$

$I_{y3} = \dfrac{1}{2} m_3 r_3^2 + m_3 d_3^2$

$= \dfrac{1}{2}(0.3946 \text{ kg})\ (0.02 \text{ m})^2 + (0.3946 \text{ kg})\ (0.04 \text{ m})^2 = 0.7103\times10^{-3} \text{ kg·m}^2$

$I_y = I_{y1} + I_{y2} - I_{y3} = [(20.20) + (1.0654) - (0.7103)]\times10^{-3} \text{ kg·m}^2$

$= 20.5551\times10^{-3} \text{ kg·m}^2 \quad \Rightarrow \quad I_y = 20.6\times10^{-3} \text{ kg·m}^2$

(c) $I_{z1} = I_{x1} = 13.467\times10^{-3} \text{ kg·m}^2$

$I_{z2} = \dfrac{1}{12} m_2 (3r_2^2 + 4L_2^2) + m_2 d_2^2$

$= (0.7695\times10^{-3} \text{ kg·m}^2) + (0.5919 \text{ kg})\ (0.04 \text{ m})^2 = 1.7165\times10^{-3} \text{ kg·m}^2$

$I_{z3} = \dfrac{1}{12} m_3 (3r_3^2 + 4L_3^2) + m_3 d_3^2$

$= (0.2499\times10^{-3} \text{ kg·m}^2) + (0.3946 \text{ kg})\ (0.04 \text{ m})^2 = 0.88126\times10^{-3} \text{ kg·m}^2$

$I_z = I_{z1} + I_{z2} - I_{z3} = [(13.467) + (1.7165) - (0.88126)]\times10^{-3} \text{ kg·m}^2$

$= 14.3022\times10^{-3} \text{ kg·m}^2 \quad \Rightarrow \quad I_z = 14.30\times10^{-3} \text{ kg·m}^2$