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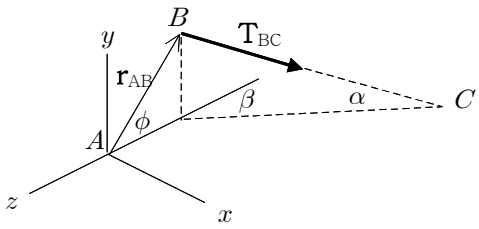
[3.1절]

3.25 l_{AB} = 2 m, T_{BC} = 30 N, α = 8°, β = 30°, ϕ = 45°

S; given l_{AB}, T_{BC}, α, β, ϕ, required $\mathbf{M}_A$　　M; 자유물체도(F.B.D.)

⇒ 점에 관한 모멘트, 위치벡터와 힘벡터의 벡터곱



A; ① 위치벡터

$\mathbf{r}_{AB} = l_{AB} \sin\phi\ \mathbf{j} - l_{AB} \cos\phi\ \mathbf{k}$

$= (2\text{ m}) \sin45°\ \mathbf{j} - (2\text{ m}) \cos45°\ \mathbf{k}$

$= 1.4142\ \mathbf{j} - 1.4142\ \mathbf{k}$ (m)

② 힘벡터

$T_y = -T_{BC} \sin\alpha = -(30\text{ N}) \sin8° = -4.175\text{ N}$

$T_h = T_{BC} \cos\alpha$

$T_x = T_h \sin\beta = T_{BC} \cos\alpha \sin\beta = (30\text{ N}) \cos8° \sin30° = 14.854\text{ N}$

$T_z = -T_h \cos\beta = -T_{BC} \cos\alpha \cos\beta = -(30\text{ N}) \cos8° \cos30° = -25.73\text{ N}$

$\mathbf{T}_{BC} = 14.854\ \mathbf{i} - 4.175\ \mathbf{j} - 25.73\ \mathbf{k}$ (N)

③ 벡터곱

$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{T}_{BC}$

$= [1.4142\ \mathbf{j} - 1.4142\ \mathbf{k}\ (\text{m})] \times [14.854\ \mathbf{i} - 4.175\ \mathbf{j} - 25.73\ \mathbf{k}\ (\text{N})]$

$= [(1.4142)(-25.73) - (-1.4142)(-4.175)]\ \mathbf{i} + [(-1.4142)(14.854) - 0]\ \mathbf{j}$

$+ [0 - (1.4142)(14.854)]\ \mathbf{k}$ (N·m)

$= -42.3\ \mathbf{i} - 21.0\ \mathbf{j} - 21.0\ \mathbf{k}$ (N·m)

R(과정의 타당성) : (가령 T_x>0, T_y<0, T_z<0, 각 힘의 직각성분의 방향)

T(결과의 의미) ; (가령, M_x<0, M_y<0, M_z<0, 각 좌표축에 관한 모멘트의 방향)

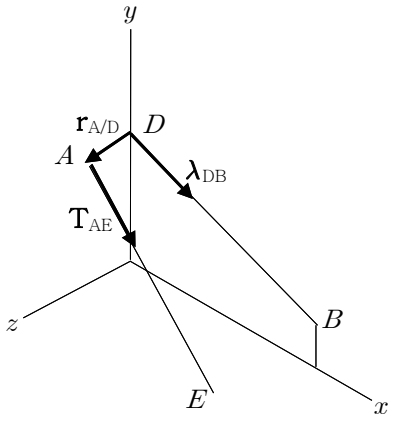
[3.2절]

3.59 S; T_{AE} = 55 N, M_{DB} = ?

⇒ 축 DB에 관한 모멘트

$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{AE})$]

M; 자유물체도(F.B.D.)



A; ① $\mathbf{r}_{A/D} = 0\mathbf{i} + (0.6 - 0.7)\mathbf{j} + (0.2)\mathbf{k}$ (m)

$= -0.1\ \mathbf{j} + 0.2\ \mathbf{k}$ (m)

② $\mathbf{T}_{AE} = T_{AE}\ \boldsymbol{\lambda}_{AE}$

$$= (55\ \text{N})\ \frac{(0.9\ \text{m})\mathbf{i} - (0.6\ \text{m})\mathbf{j} + (0.4\ \text{m} - 0.2\ \text{m})\mathbf{k}}{\sqrt{(0.9\ \text{m})^2 + (-0.6\ \text{m})^2 + (0.2\ \text{m})^2}}$$

$$= \frac{55\ \text{N}}{1.1}(0.9\ \mathbf{i} - 0.6\ \mathbf{j} + 0.2\ \mathbf{k})$$

$= 45.0\ \mathbf{i} - 30.0\ \mathbf{j} + 10.0\ \mathbf{k}$ (N)

①② $\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{T}_{AE} = [-0.1\ \mathbf{j} + 0.2\ \mathbf{k}\ (\text{m})] \times [45.0\ \mathbf{i} - 30.0\ \mathbf{j} + 10.0\ \mathbf{k}\ (\text{N})]$

$= [(-0.1)\times(+10.0) - (0.2)\times(-30.0)]\mathbf{i} + (0.2\times45.0)\mathbf{j} + [-(-0.1)\times45.0]\mathbf{k}$ (N·m)

$= 5.0\ \mathbf{i} + 9.0\ \mathbf{j} + 4.5\ \mathbf{k}$ (N·m)

③ $\boldsymbol{\lambda}_{DB} = \dfrac{(0.9 + 0.3\ \text{m})\mathbf{i} - (0.7 - 0.35\ \text{m})\mathbf{j}}{\sqrt{(1.2\ \text{m})^2 + (-0.35\ \text{m})^2}} = 0.96\ \mathbf{i} - 0.28\ \mathbf{j}$

④ $M_{DB} = \boldsymbol{\lambda}_{DB} \cdot \mathbf{M}_D = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{AE})$

$= (0.96\ \mathbf{i} - 0.28\ \mathbf{j}) \cdot [5.0\ \mathbf{i} + 9.0\ \mathbf{j} + 4.5\ \mathbf{k}\ (\text{N·m})]$

$= (0.96)(5.0) + (-0.28)(9.0) + 0 = 2.28$ (N·m)

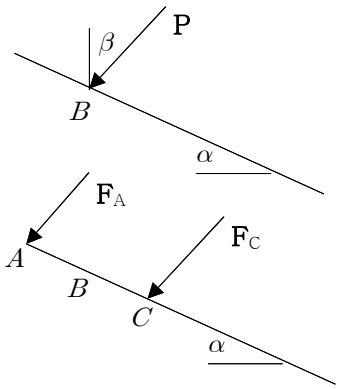
R(과정의 타당성) : (가령 $\mathbf{M}_D$ 계산에 사용될 수 있는 위치벡터 $\mathbf{r}_{A/D}$, $\mathbf{r}_{E/D}$, $\mathbf{r}_{A/B}$, $\mathbf{r}_{E/B}$)

T(결과의 의미) ; (가령, M_{DB}>0, 선 DB에 관한 모멘트의 방향)

[3.3절]

3.86 P = 360 N, α = 30°, β = 40°, d_{AB} = 0.4 m, d_{BC} = 0.35 m

S; $\mathbf{F}_A$ = ?, $\mathbf{F}_C$= ? ⇒ 2차원 등가 힘-우력 계 M; 자유물체도(F.B.D.)



A; $\mathbf{F}_A \parallel \mathbf{F}_C$, $\Sigma\mathbf{F} = \mathbf{F}_A + \mathbf{F}_C = \mathbf{P}$

⇒ $\mathbf{F}_A$의 방향 = $\mathbf{F}_C$의 방향 = $\mathbf{P}$ 의 방향 = ↙50.0°

$F_A + F_C = P$ … ①

$\theta = \beta - \alpha = 40° - 30° = 10°$

↺$\Sigma M_B = 0$

$= d_{AB} F_A \cos\theta - d_{BC} F_C \cos\theta$

$= (d_{AB} F_A - d_{BC} F_C) \cos\theta$

⇒ $d_{AB} F_A - d_{BC} F_C = 0$ … ②

d_{BC}×① + ②

⇒ $d_{BC} F_A + d_{AB} F_A = d_{BC} P$

⇒ $F_A = \dfrac{d_{BC}}{d_{BC}+d_{AB}} P = \dfrac{0.35}{0.35+0.4}$ (360 N) = 168.0 N

① ⇒ $F_C = P - F_A = P - \dfrac{d_{BC}}{d_{BC}+d_{AB}} P$

$= \dfrac{d_{AB}}{d_{BC}+d_{AB}} P = \dfrac{0.4}{0.35+0.4}$ (360 N) = 192.0 N

⇒ $\mathbf{F}_A$ = 168.0 N ↙50.0°, $\mathbf{F}_C$ = 192.0 N ↙50.0°

R(과정의 타당성) ; (가령, ↺ΣM_B 대신 ↺ΣM_A 또는 ↺ΣM_C 을 비교하면?)

T(결과의 의미) ; (가령, $F_C > F_A$ 의 의미)

[4.1절]

4.24　P = 40 N,　Q = 50 N

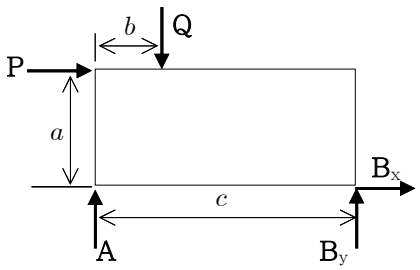
a = 0.10 m,　b = 0.04 m,　c = 0.20 m

S; 모멘트 평형, 힘의 평형, 반력 유형 1&2

$+\circlearrowleft\Sigma M_B = 0$, $\rightarrow\Sigma F_x = 0$, $\uparrow\Sigma F_y = 0$

(a) A;

M; 자유물체도(F.B.D.)



$+\circlearrowleft\Sigma M_B = 0$;　$-c\,A - a\,P + (c-b)\,Q = 0$

$$\Rightarrow\ A = \frac{-a\,P + (c-b)\,Q}{c}$$

$$= \frac{-(0.10\text{ m})(40\text{ N}) + (0.2-0.04\text{ m})(50\text{ N})}{0.20\text{ m}}$$

$$= 20.0\text{ N}$$

$\rightarrow\Sigma F_x = 0$;　$B_x + P = 0$　$\Rightarrow$　$B_x = -P = -40.0$ N

$\uparrow\Sigma F_y = 0$;　$B_y + A - Q = 0$

$\Rightarrow$　$B_y = -A + Q = -(20.0\text{ N}) + (50.0\text{ N}) = 30.0$ N

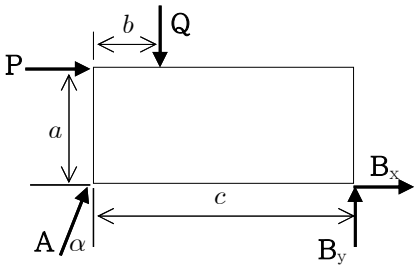
$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-40.0\text{ N})^2 + (30.0\text{ N})^2} = 50.0\text{ N}$$

$$\theta = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\frac{30.0\text{ N}}{-40.0\text{ N}} = \tan^{-1}(-0.750) = -36.9^\circ$$

$\Rightarrow$　**A** = 20.0 N ↑, **B** = 50.0 N ↖36.9°,

(b)　$\alpha = 30^\circ$

M; 자유물체도(F.B.D.)



A;$+\circlearrowleft\Sigma M_B = 0$;　$-c\,A\cos\alpha - a\,P + (c-b)\,Q = 0$

$$\Rightarrow\ A = \frac{-a\,P + (c-b)\,Q}{c\cos\alpha}$$

$$= \frac{-(0.10\text{ m})(40\text{ N}) + (0.20-0.04\text{ m})(50\text{ N})}{(0.20\text{ m})\cos30^\circ}$$

$$= 23.09\text{ N}$$

$\rightarrow\Sigma F_x = 0$;　$B_x + P + A\sin\alpha = 0$

$\Rightarrow$　$B_x = -P - A\sin\alpha = -(40.0\text{ N}) - (23.09\text{ N})\sin30^\circ = -51.54$ N

$\uparrow\Sigma F_y = 0$;　$B_y + A\cos\alpha - Q = 0$

$\Rightarrow$　$B_y = -A\cos\alpha + Q = -(23.09\text{ N})\cos30^\circ + (50.0\text{ N}) = 30.0$ N

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-51.54\text{ N})^2 + (30.0\text{ N})^2} = 59.63\text{ N}$$

$$\theta = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\frac{30.0\text{ N}}{-51.54\text{ N}} = \tan^{-1}(-0.582) = -30.2^\circ$$

$\Rightarrow$　**A** = 23.1 N ↗60.0°,　**B** = 59.6 N ↖30.2°

R; (예: $+\circlearrowleft\Sigma M_A = 0$ 을 사용하면?)

T; (예: B_x가 음수인 의미는?)

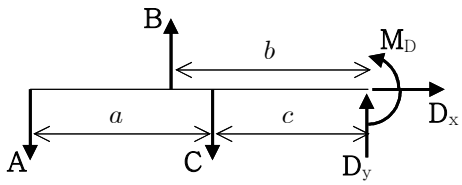
4.48 A = 200 N, C = 200 N, $B = W$

a = 4 m, c = 4 m, b = 5 m

S; 힘의 평형, 모멘트 평형, 반력 유형 1&3

$\rightarrow \Sigma F_x = 0$, $\uparrow \Sigma F_y = 0$, $+\circlearrowleft \Sigma M_D = 0$

M; 자유물체도(F.B.D.)



A; (a) W = 500 N

$\rightarrow \Sigma F_x = 0$; $D_x = 0$

$\uparrow \Sigma F_y = 0$; $D_y + B - A - C = 0$

$\Rightarrow D_y = -B + A + C = -(500\text{ N}) + (200\text{ N}) + (200\text{ N}) = -100\text{ N}$

$\Rightarrow$ **D** = 100.0 N ↓

$+\circlearrowleft \Sigma M_D = 0$; $M_D + (a+c)A + c\,C - b\,B = 0$

$\Rightarrow M_D = -(a+c)A - c\,C + b\,B$

$= -(4+4\text{ m})(200\text{ N}) - (4\text{ m})(200\text{ N}) + (5\text{ m})(500\text{ N}) = 100\text{ N·m}$

$\Rightarrow$ $\mathbf{M}_D$ = 100.0 N·m ↺

(b) W = 450 N

$\rightarrow \Sigma F_x = 0$; $D_x = 0$

$\uparrow \Sigma F_y = 0$; $D_y + B - A - C = 0$

$\Rightarrow D_y = -B + A + C = -(450\text{ N}) + (200\text{ N}) + (200\text{ N}) = -50\text{ N}$

$\Rightarrow$ **D** = 50.0 N ↓

$+\circlearrowleft \Sigma M_D = 0$; $M_D + (a+c)A + c\,C - b\,B = 0$

$\Rightarrow M_D = -(a+c)A - c\,C + b\,B$

$= -(4+4\text{ m})(200\text{ N}) - (4\text{ m})(200\text{ N}) + (5\text{ m})(450\text{ N}) = -150\text{ N·m}$

$\Rightarrow$ $\mathbf{M}_D$ = −150.0 N·m ↻

R; (예: ΣM_A 또는 ΣM_C 를 사용하는 경우)

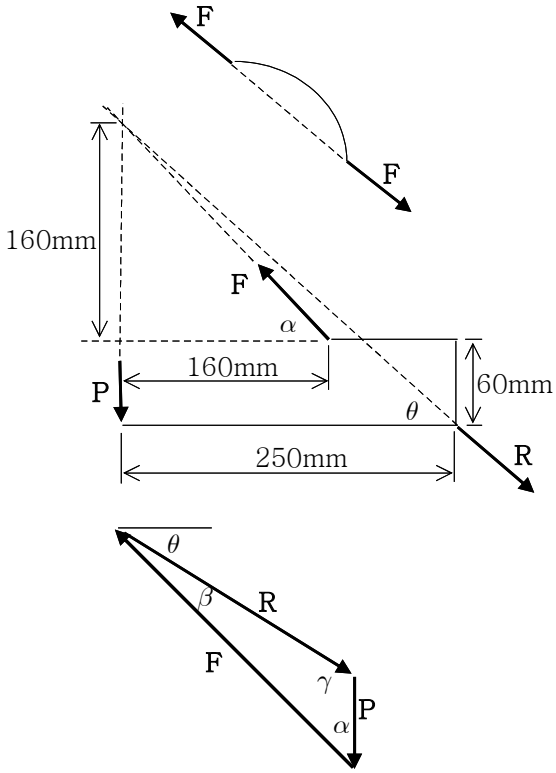
T; (예: a와 b에서 반력 모멘트 방향이 다른 이유)

[4.2절]

4.67 P = 80 N, α = 45°

S; 두 힘의 평형, 세 힘의 평형, 반력 유형2
세 힘의 작용선이 한 점에서 만남

M; 자유물체도(F.B.D.)



A; $\tan\theta = \dfrac{160+60\ \text{mm}}{250\ \text{mm}} = 0.880$

$\Rightarrow \quad \theta = \tan^{-1}(0.880) = 41.348°$

$\gamma = 90° + \theta = 90° + 41.35° = 131.35°$

$\beta = 180° - \gamma - \alpha$

$= 180° - 131.35° - 45° = 3.65°$

$$\frac{R}{\sin\alpha} = \frac{P}{\sin\beta}$$

$$\Rightarrow \quad R = P\frac{\sin\alpha}{\sin\beta} = (80\ \text{N})\ \frac{\sin 45^\circ}{\sin 3.65^\circ}$$

$= 888.58$ N

$\Rightarrow$ **R** = 889 N ↘41.3°

$$\frac{F}{\sin\gamma} = \frac{P}{\sin\beta}$$

$$\Rightarrow \quad F = P\frac{\sin\gamma}{\sin\beta} = (80\ \text{N})\ \frac{\sin 131.35^\circ}{\sin 3.65^\circ}$$

$= 943.35$ N

$\Rightarrow$ **F** = 943 N ↖45.0°

R; (예: 직각성분 방법으로 풀이 한다면?)

T; (예: B의 반력의 방향, D의 반력의 방향)