[2.1절]

2.16 S; P = 900 N, Q = 600 N, α = 30°, β = 45°

known : P, Q, α, β,   unknown : 합력(resultant) R의 크기와 방향

⇒ 삼각법(trigonometry) (sine 공식, cosine 공식)에 의한 힘의 합성
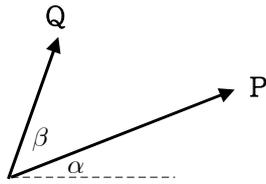
M; 자유물체도 (F.B.D.),   힘 삼각형 (force triangle)
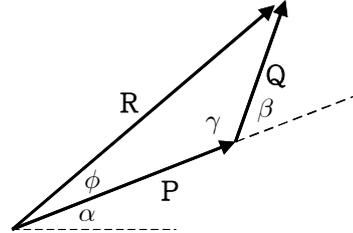
γ = 180° − β = 180° − 45° = 135°

A; 합력의 크기 (cosine 공식)

$R^2 = P^2 + Q^2 - 2PQ\cos\gamma$

$\quad = (900\ \text{N})^2 + (600\ \text{N})^2 - 2(900\ \text{N})(600\ \text{N})\cos135° = 1{,}933{,}675\ \text{N}^2$

$\Rightarrow R = 1{,}390.6\ \text{N}$

합력의 방향 각도 (sine 공식)

$\dfrac{Q}{\sin\phi} = \dfrac{R}{\sin\gamma}$ ⇒ $\sin\phi = \dfrac{Q}{R}\sin\gamma = \dfrac{600\ \text{N}}{1{,}390.6\ \text{N}}\sin135° = 0.3051$

$\Rightarrow \phi = \sin^{-1}(0.3051) = 17.8°$

$\theta = \alpha + \phi = 30° + 17.8° = 47.8°$

합력 $\mathbf{R}$ = 1,391 N ⤢47.8°

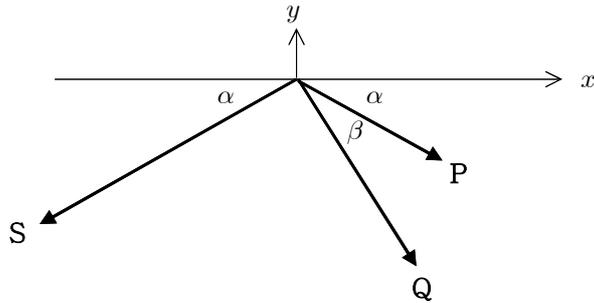R; 과정의 타당성 (가령, 합력의 크기 계산에 sine 공식 대신 cosine 공식을 사용한 사유)

T; 결과 검토 (가령, $\alpha < \theta < \alpha + \beta$, $R < P + Q$.   90° < γ < 180° 일 때 $R > P$, $R > Q$)

[2.2절]

2.35 S; P = 100 N,　Q = 150 N,　S = 200 N,　α = 35°,　β = 30°,
　　known : P, Q, S, α, β,　unknown : 합력(resultant) **R**의 크기와 방향
　　⇒ 직각성분 합에 의한 힘의 합성

　　M; 자유물체도 (F.B.D.)

　　A; $\gamma = \alpha + \beta = 35° + 30° = 65°$
　　　$R_x = P\cos\alpha + Q\cos\gamma - S\cos\alpha$
　　　　$= (100\ \text{N})\cos35° + (150\ \text{N})\cos65° - (200\ \text{N})\cos35°$
　　　　$= (81.92\ \text{N}) + (63.39\ \text{N}) - (163.83\ \text{N}) = -18.52\ \text{N}$
　　　$R_y = -P\sin\alpha - Q\sin\gamma - S\sin\alpha$
　　　　$= -(100\ \text{N})\sin35° - (150\ \text{N})\sin65° - (200\ \text{N})\sin35°$
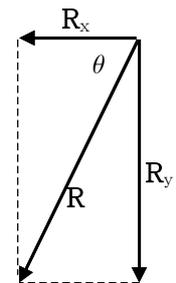　　　　$= -(57.36\ \text{N}) - (135.95\ \text{N}) - (114.72\ \text{N}) = -308.03\ \text{N}$
　　　$\mathbf{R} = R_x\,\mathbf{i} + R_y\,\mathbf{j} = (-18.52\ \text{N})\,\mathbf{i} + (-308.0)\,\mathbf{j}$

　　　$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-18.52\ \text{N})^2 + (-308.0\ \text{N})^2} = 308.6\ \text{N}$
　　　$\tan\theta = \dfrac{|R_y|}{|R_x|} = \dfrac{308.0\ \text{N}}{18.52\ \text{N}} = 16.631$
　　　$\theta = \tan^{-1}(16.631) = 86.56°$
　　　⇒ **R** = 309 N ∠86.6°

　　R; 과정의 타당성 (가령, 삼각법(힘 삼각형 방법)으로 해결하면 어떤지)

　　T; 결과 검토 (가령, 합력의 방향에 관하여)

2.63 W = 200 N, h = 400 mm,   줄의 장력은 동일 T = W, $\theta = \tan^{-1}\dfrac{h}{x}$

S; known W, T, h, x,   unknown P M;
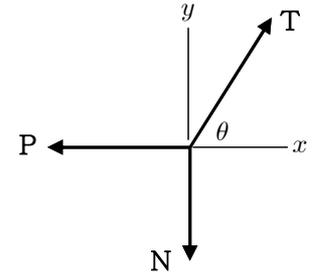
질점의 평형 문제 ⇒ 직각성분 방법 또는 힘 삼각형 방법

A;

<방법 1 : 직각성분>

$\Sigma F_x = 0$; $-P + T\cos\theta = 0$ ⇒ $P = T\cos\theta$

(a) x = 90 mm, $\theta = \tan^{-1}\dfrac{400\ mm}{90\ mm} = 77.3°$

$P = (200\ N)\cos 77.3° = 43.90\ N$ ⇒ $P = 43.9\ N$

(b) x = 300 mm, $\theta = \tan^{-1}\dfrac{400\ mm}{300\ mm} = \tan^{-1}\dfrac{4}{3}$

$P = (200\ N)\dfrac{3}{5} = 120\ N$ ⇒ $P = 120.0\ N$

<방법 2 : 힘 삼각형>

$P = T\cos\theta$

(a) x = 90 mm, $\theta = \tan^{-1}\dfrac{400\ mm}{90\ mm} = 77.3°$

$P = (200\ N)\cos 77.3° = 43.90\ N$ ⇒ $P = 43.9\ N$

(b) x = 300 mm, $\theta = \tan^{-1}\dfrac{400\ mm}{300\ mm} = \tan^{-1}\dfrac{4}{3}$

$P = (200\ N)\dfrac{3}{5} = 120\ N$ ⇒ $P = 120.0\ N$
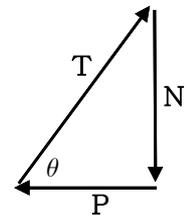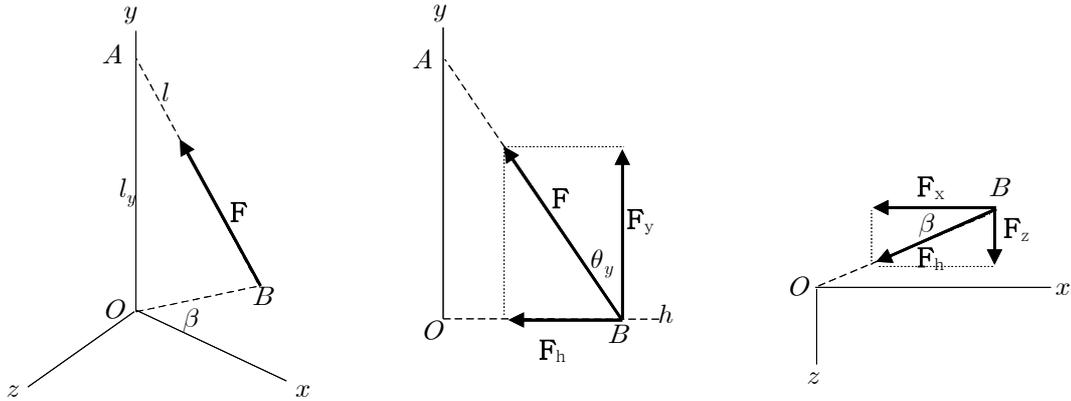
R; 과정의 타당성 (가령, 두 가지 방법 비교)

T; 결과 검토 (가령, x값에 따른 차이)

2.77 $T = F = 15$ kN, $l = 32.5$ m, $l_y = 28$ m, $\beta = 20°$

S; known F, l, l_y, β,   unknown $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{F}_z$, θ_x, θ_y, θ_z

⇒ 공간에서 힘의 직각성분 (각도 이용)

M;

A; 방향여현 $\cos\theta_y = \dfrac{l_y}{l} = \dfrac{28\ \text{m}}{32.5\ \text{m}} = 0.8615$

⇒ $\theta_y = \cos^{-1}(0.8615) = 30.51°$

(a) $F_y = F\cos\theta_y = (15\ \text{kN})(0.8615) = 12.923$ kN   ⇒ $\mathbf{F}_y = 12.92$ kN $\mathbf{j}$

$F_h = F\sin\theta_y = (15\ \text{kN})\sin30.51° = 7.615$ kN

$F_x = -F_h\cos\beta = -(7.615\ \text{kN})\cos20° = -7.156$ kN ⇒ $\mathbf{F}_x = -7.16$ kN $\mathbf{i}$

$F_z = F_h\sin\beta = (7.615\ \text{kN})\sin20° = 2.604$ kN   ⇒ $\mathbf{F}_z = 2.60$ kN $\mathbf{k}$

(b) $\theta_y = 30.5°$

$\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-7.16\ \text{kN}}{15\ \text{kN}} = -0.4773$ ⇒ $\theta_x = \cos^{-1}(-0.4773) = 118.5°$

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{2.60\ \text{kN}}{15\ \text{kN}} = 0.17333$ ⇒ $\theta_x = \cos^{-1}(0.17333) = 80.0°$

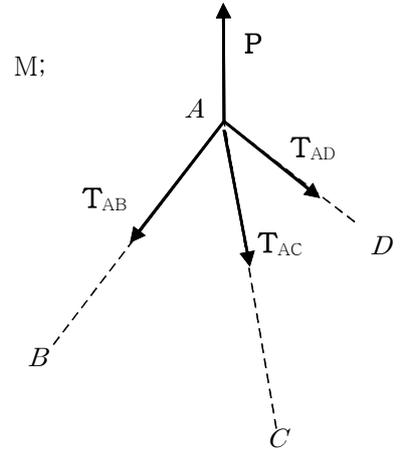R; 과정의 타당성 (가령, B 지점에서 힘의 방향)

T; 결과 검토 (가령, 힘 성분의 부호와 방향 각도의 범위)

2.109 T_{AC} = 60 N

S; known T_{AC}, 좌표들,   unknown T_{AB}, T_{AD}, W M;

⇒ 공간에서 힘의 직각성분 (좌표 이용), 평형

A; $\mathbf{P} = -\,P\,\mathbf{j}$, $\mathbf{W} = W\,\mathbf{j}$

$\mathbf{P} + \mathbf{W} = 0$ ⇒ $-\,P\,\mathbf{j} + W\,\mathbf{j} = 0$ ⇒ $W = P$

$$d_{AB} = \sqrt{(-320\,\text{mm})^2 + (-480\,\text{mm})^2 + (360\,\text{mm})^2}$$
$$= 680\ \text{mm}$$

$$\boldsymbol{\lambda}_{AB} = \frac{1}{680}(-320\,\mathbf{i} - 480\,\mathbf{j} + 360\,\mathbf{k})$$
$$= -0.4706\,\mathbf{i} - 0.7059\,\mathbf{j} + 0.5294\,\mathbf{k}$$

$\mathbf{T}_{AB} = T_{AB}\,\boldsymbol{\lambda}_{AB} = T_{AB}\,(-0.4706\,\mathbf{i} - 0.7059\,\mathbf{j} + 0.5294\,\mathbf{k})$

$$d_{AC} = \sqrt{(450\,\text{mm})^2 + (-480\,\text{mm})^2 + (360\,\text{mm})^2}\ = 750\ \text{mm}$$

$$\boldsymbol{\lambda}_{AC} = \frac{1}{750}(450\,\mathbf{i} - 480\,\mathbf{j} + 360\,\mathbf{k}) = 0.6000\,\mathbf{i} - 0.6400\,\mathbf{j} + 0.4800\,\mathbf{k}$$

$\mathbf{T}_{AC} = T_{AC}\,\boldsymbol{\lambda}_{AC} = (60\ \text{N})\,(0.600\,\mathbf{i} - 0.6400\,\mathbf{j} + 0.4800\,\mathbf{k})$

$= (36.0\ \text{N})\,\mathbf{i} - (38.4\ \text{N})\,\mathbf{j} + (28.8\ \text{N})\,\mathbf{k}$

$$d_{AD} = \sqrt{(250\,\text{mm})^2 + (-480\,\text{mm})^2 + (-360\,\text{mm})^2}\ = 650\ \text{mm}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{650}(250\,\mathbf{i} - 480\,\mathbf{j} - 360\,\mathbf{k}) = 0.3846\,\mathbf{i} - 0.7384\,\mathbf{j} - 0.5538\,\mathbf{k}$$

$\mathbf{T}_{AD} = T_{AD}\,\boldsymbol{\lambda}_{AD} = T_{AD}\,(0.3846\,\mathbf{i} - 0.7384\,\mathbf{j} - 0.5538\,\mathbf{k})$

$\Sigma\mathbf{F} = 0$ ⇒ $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0$

$\Sigma F_x = 0$; $-0.4706\,T_{AB} + (36.0\ \text{N}) + 0.3846\,T_{AD} = 0$ ⋯ ①

$\Sigma F_y = 0$; $-0.7059\,T_{AB} - (38.4\ \text{N}) - 0.7384\,T_{AD} + P = 0$ ⋯ ②

$\Sigma F_z = 0$; $0.5294\,T_{AB} + (28.8\ \text{N}) - 0.5538\,T_{AD} = 0$ ⋯ ③

①×0.5538 + ③×0.3846

$[(-0.4706)(0.5538) + (0.5294)(0.3846)]\,T_{AB} + [(36.0\ \text{N})(0.5538) + (28.8\ \text{N})(0.3846)] = 0$

⇒ $T_{AB} = 544.0\ \text{N}$

① ⇒ $T_{AD} = \dfrac{1}{0.3846}\,[(0.4706)(544.0\ \text{N}) - (36.0\ \text{N})] = 572.0\ \text{N}$

② ⇒ $P = (0.7059)(544.0\ \text{N}) + (38.4\ \text{N}) + (0.7384)(572.0\ \text{N}) = 844.4\ \text{N}$

⇒ $P = 844\ \text{N}$

R; 과정의 타당성 (가령, 공간에서 힘의 직각성분)

T; 결과 검토 (가령, 세 힘의 수평성분의 합)