

{9.1절}

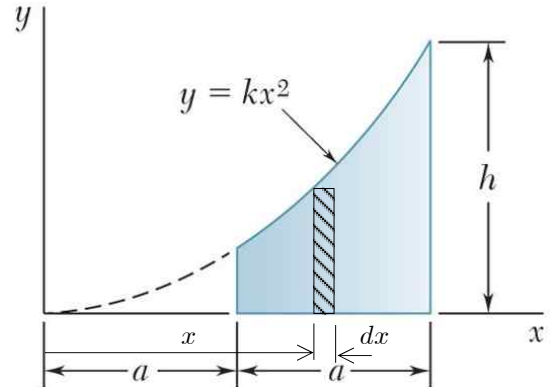
$$9.16\&18 \quad y = kx^2, \quad (2a, h) \Rightarrow h = k(2a)^2$$

$$\Rightarrow k = \frac{h}{4a^2}, \quad y = \frac{h}{4a^2}x^2$$

$$dA = y dx = \frac{h}{4a^2}x^2 dx$$

$$A = \int dA = \int_a^{2a} \frac{h}{4a^2}x^2 dx = \frac{h}{4a^2} \left[\frac{1}{3}x^3 \right]_a^{2a}$$

$$= \frac{h}{12a^2} [(2a)^3 - a^3] = \frac{7}{12}ah$$



$$9.16 \quad dI_x = \frac{1}{3}y^3 dx$$

$$I_x = \int dI_x = \int_a^{2a} \frac{1}{3}y^3 dx = \frac{1}{3} \int_a^{2a} \left(\frac{h}{4a^2}x^2 \right)^3 dx = \frac{1}{3} \left(\frac{h}{4a^2} \right)^3 \int_a^{2a} x^6 dx$$

$$= \frac{1}{3} \frac{h^3}{64a^6} \left[\frac{1}{7}x^7 \right]_a^{2a} = \frac{1}{3} \frac{h^3}{64a^6} \frac{1}{7} [(2a)^7 - a^7] = \frac{127}{(3)(64)(7)} ah^3 = 0.0945 ah^3$$

$$k_x^2 = \frac{I_x}{A} = \frac{\frac{127}{(3)(64)(7)} ah^3}{\frac{7}{12}ah} = \frac{127}{(16)(7)(7)} h^2 = 0.16199 h^2$$

$$k_x = \sqrt{0.16199 h^2} = 0.402 h$$

$$9.18 \quad dI_y = x^2 dA = x^2 \frac{h}{4a^2} x^2 dx = \frac{h}{4a^2} x^4 dx$$

$$I_y = \int dI_y = \int_a^{2a} \frac{h}{4a^2} x^4 dx = \frac{h}{4a^2} \left[\frac{1}{5}x^5 \right]_a^{2a} = \frac{h}{20a^2} [(2a)^5 - a^5] = \frac{31}{20} a^3 h = 1.550 a^3 h$$

$$k_y^2 = \frac{I_y}{A} = \frac{\frac{31}{20} a^3 h}{\frac{7}{12} ah} = \frac{(31)(3)}{(5)(7)} a^2 = \frac{93}{35} a^2 = 2.657 a^2$$

$$k_y = \sqrt{2.657 a^2} = 1.630 a$$

{9.2절}

9.43 Centroid

① 직사각판 $a = 0.9 + 2.0 + 2.1 \text{ cm} = 5.0 \text{ cm}$

$b = 1.8 + 5.0 + 1.2 \text{ cm} = 8.0 \text{ cm}$

$A = (5.0 \text{ cm})(8.0 \text{ cm}) = 40.0 \text{ cm}^2$

$\bar{x} = \frac{1}{2}a = \frac{1}{2}(5.0 \text{ cm}) = 2.5 \text{ cm}$

$\bar{y} = \frac{1}{2}b = \frac{1}{2}(8.0 \text{ cm}) = 4.0 \text{ cm}$

② 직사각형 구멍

$A = -(2.0 \text{ cm})(5.0 \text{ cm}) = -10.0 \text{ cm}^2$

$\bar{x} = (0.9 \text{ cm}) + \frac{1}{2}(2.0 \text{ cm}) = 1.9 \text{ cm}$

$\bar{y} = (1.8 \text{ cm}) + \frac{1}{2}(5.0 \text{ cm}) = 4.3 \text{ cm}$

$\Sigma A = (40.0 \text{ cm}^2) + (-10.0 \text{ cm}^2) = 30.0 \text{ cm}^2$

$\Sigma(\bar{x}A) = (2.5 \text{ cm})(40.0 \text{ cm}^2) + (1.9 \text{ cm})(-10.0 \text{ cm}^2) = 81.0 \text{ cm}^3$

$\Sigma(\bar{y}A) = (4.0 \text{ cm})(40.0 \text{ cm}^2) + (4.3 \text{ cm})(-10.0 \text{ cm}^2) = 117.0 \text{ cm}^3$

$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{81.0 \text{ cm}^3}{30.0 \text{ cm}^2} = 2.70 \text{ cm}$

$\bar{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{117.0 \text{ cm}^3}{30.0 \text{ cm}^2} = 3.90 \text{ cm}$

$I_{x1} = \bar{I}_{x1} + A_1 d_1^2$

$= \frac{1}{12}(5.0 \text{ cm})(8.0 \text{ cm})^3 + (40.0 \text{ cm}^2)(4.0 - 3.9 \text{ cm})^2$

$= 213.73 \text{ cm}^4$

$I_{x2} = \bar{I}_{x2} + A_2 d_2^2$

$= \frac{1}{12}(2.0 \text{ cm})(5.0 \text{ cm})^3 + (10.0 \text{ cm}^2)(4.3 - 3.9 \text{ cm})^2$

$= 22.43 \text{ cm}^4$

$I_x = I_{x1} - I_{x2}$

$= (213.73 \text{ cm}^4) - (22.43 \text{ cm}^4) = 191.3 \text{ cm}^4$

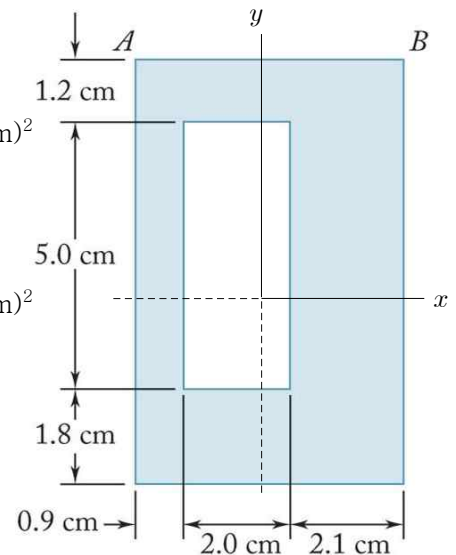
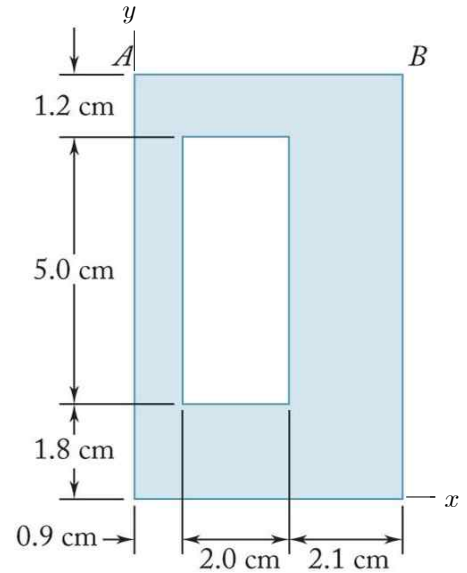
$I_{y1} = \bar{I}_{y1} + A_1 d_1^2$

$= \frac{1}{12}(5.0 \text{ cm})^3(8.0 \text{ cm}) + (40.0 \text{ cm}^2)(2.7 - 2.5 \text{ cm})^2 = 84.93 \text{ cm}^4$

$I_{y2} = \bar{I}_{y2} + A_2 d_2^2 = \frac{1}{12}(2.0 \text{ cm})^3(5.0 \text{ cm}) + (10.0 \text{ cm}^2)(2.7 - 1.9 \text{ cm})^2 = 9.73 \text{ cm}^4$

$I_y = I_{y1} - I_{y2}$

$= (84.93 \text{ cm}^4) - (9.73 \text{ cm}^4) = 75.2 \text{ cm}^4$



{9.5절}

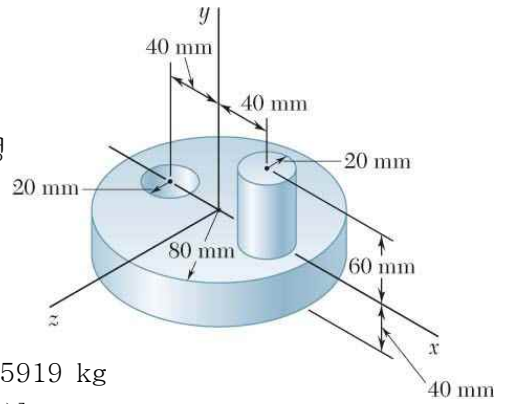
9.141 $\rho = 7,850 \text{ kg/m}^3$

$m = \rho V$ ① 원판, ② 원기둥, ③ 원판 구멍

$$m_1 = \rho V_1 = (7,850 \text{ kg/m}^3) [\pi (0.08 \text{ m})^2 (0.04 \text{ m})] = 6.313 \text{ kg}$$

$$m_2 = \rho V_2 = (7,850 \text{ kg/m}^3) [\pi (0.02 \text{ m})^2 (0.06 \text{ m})] = 0.5919 \text{ kg}$$

$$m_3 = \rho V_3 = (7,850 \text{ kg/m}^3) [\pi (0.02 \text{ m})^2 (0.04 \text{ m})] = 0.3946 \text{ kg}$$



(a) $I_{x1} = \frac{1}{12} m_1 (3r_1^2 + 4L_1^2)$
 $= \frac{1}{12} (6.313 \text{ kg}) [3 (0.08 \text{ m})^2 + 4 (0.04 \text{ m})^2] = 13.467 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$$I_{x2} = \frac{1}{12} m_2 (3r_2^2 + 4L_2^2) = \frac{1}{12} (0.5919 \text{ kg}) [3 (0.02 \text{ m})^2 + 4 (0.06 \text{ m})^2] = 0.7695 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{x3} = \frac{1}{12} m_3 (3r_3^2 + 4L_3^2) = \frac{1}{12} (0.3946 \text{ kg}) [3 (0.02 \text{ m})^2 + 4 (0.04 \text{ m})^2] = 0.2499 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_x = I_{x1} + I_{x2} - I_{x3} = [(13.467) + (0.7695) - (0.2499)] \times 10^{-3} \text{ kg}\cdot\text{m}^2 = 13.9886 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \Rightarrow I_x = 13.99 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

(b) $I_{y1} = \frac{1}{2} m_1 r_1^2 = \frac{1}{2} (6.313 \text{ kg}) (0.08 \text{ m})^2 = 20.20 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$$I_{y2} = \frac{1}{2} m_2 r_2^2 + m_2 d_2^2 = \frac{1}{2} (0.5919 \text{ kg}) (0.02 \text{ m})^2 + (0.5919 \text{ kg}) (0.04 \text{ m})^2 = 1.0654 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{y3} = \frac{1}{2} m_3 r_3^2 + m_3 d_3^2 = \frac{1}{2} (0.3946 \text{ kg}) (0.02 \text{ m})^2 + (0.3946 \text{ kg}) (0.04 \text{ m})^2 = 0.7103 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_y = I_{y1} + I_{y2} - I_{y3} = [(20.20) + (1.0654) - (0.7103)] \times 10^{-3} \text{ kg}\cdot\text{m}^2 = 20.5551 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \Rightarrow I_y = 20.6 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

(c) $I_{z1} = I_{x1} = 13.467 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$$I_{z2} = \frac{1}{12} m_2 (3r_2^2 + 4L_2^2) + m_2 d_2^2 = (0.7695 \times 10^{-3} \text{ kg}\cdot\text{m}^2) + (0.5919 \text{ kg}) (0.04 \text{ m})^2 = 1.7165 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{z3} = \frac{1}{12} m_3 (3r_3^2 + 4L_3^2) + m_3 d_3^2 = (0.2499 \times 10^{-3} \text{ kg}\cdot\text{m}^2) + (0.3946 \text{ kg}) (0.04 \text{ m})^2 = 0.88126 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_z = I_{z1} + I_{z2} - I_{z3} = [(13.467) + (1.7165) - (0.88126)] \times 10^{-3} \text{ kg}\cdot\text{m}^2 = 14.3022 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \Rightarrow I_z = 14.30 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$