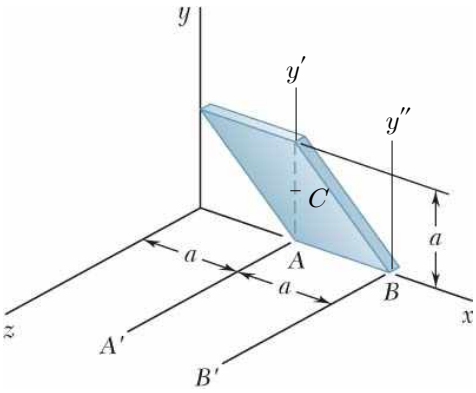
{9.11~9.15절}

9.117 $m = \rho\, t\, A = \rho\, t\, a^2$

$$\Rightarrow \quad \rho\, t = \frac{m}{a^2}$$

$$I = \rho\, t\, I^{area} = \frac{m}{a^2}\, I^{area}$$



(a) 정사각형 $a \times a$에 대한 결과와 동일

$$I_x^{area} = \frac{1}{3}(a)(a)^3 = \frac{1}{3}a^4$$

$$I_x = \frac{m}{a^2}\, I_x^{area} = \frac{m}{a^2}\, \frac{1}{3}a^4 = \frac{1}{3}m a^2$$

(b) 도심 축 y'에 대해서

$$\bar{I}_{y'}^{area} = 2\left[\frac{1}{12}(a)(a)^3\right] = \frac{1}{6}a^4$$

$$\bar{I}_{y'} = \frac{m}{a^2}\, I_{y'}^{area} = \frac{m}{a^2}\left(\frac{1}{6}a^4\right) = \frac{1}{6}m\, a^2$$

$$I_{y''} = \bar{I}_{y'} + m\, a^2 = \frac{1}{6}m\, a^2 + m\, a^2 = \frac{7}{6}m\, a^2$$

$$I_{BB'} = I_x + I_{y''} = \frac{1}{3}m a^2 + \frac{7}{6}m\, a^2 = \frac{3}{2}m a^2$$

9.141 $\rho = 7{,}850 \text{ kg/m}^3$

$m = \rho V$ ① 원판, ② 원기둥, ③ 원판 구멍

$m_1 = \rho V_1$

$= (7{,}850 \text{ kg/m}^3)[\pi (0.08 \text{ m})^2 (0.04 \text{ m})]$

$= 6.313 \text{ kg}$

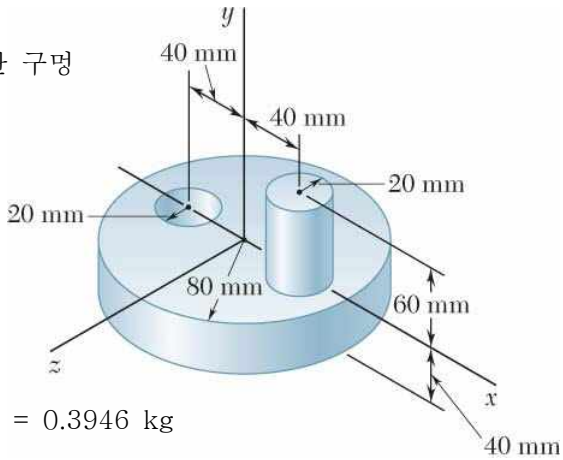
$m_2 = \rho V_2$

$= (7{,}850 \text{ kg/m}^3)[\pi (0.02 \text{ m})^2 (0.06 \text{ m})]$

$= 0.5919 \text{ kg}$

$m_3 = \rho V_3$

$= (7{,}850 \text{ kg/m}^3)[\pi (0.02 \text{ m})^2 (0.04 \text{ m})] = 0.3946 \text{ kg}$



(a) $I_{x1} = \frac{1}{12} m_1 (3r_1^2 + 4L_1^2)$

$= \frac{1}{12}(6.313 \text{ kg})[3 (0.08 \text{ m})^2 + 4 (0.04 \text{ m})^2] = 13.467\times10^{-3} \text{ kg·m}^2$

$I_{x2} = \frac{1}{12} m_2 (3r_2^2 + 4L_2^2)$

$= \frac{1}{12}(0.5919 \text{ kg})[3 (0.02 \text{ m})^2 + 4 (0.06 \text{ m})^2] = 0.7695\times10^{-3} \text{ kg·m}^2$

$I_{x3} = \frac{1}{12} m_3 (3r_3^2 + 4L_3^2)$

$= \frac{1}{12}(0.3946 \text{ kg})[3 (0.02 \text{ m})^2 + 4 (0.04 \text{ m})^2] = 0.2499\times10^{-3} \text{ kg·m}^2$

$I_x = I_{x1} + I_{x2} - I_{x3} = [(13.467) + (0.7695) - (0.2499)]\times10^{-3} \text{ kg·m}^2$

$= 13.9886\times10^{-3} \text{ kg·m}^2 \quad \Rightarrow \quad I_x = 13.99\times10^{-3} \text{ kg·m}^2$

(b) $I_{y1} = \frac{1}{2} m_1 r_1^2$

$= \frac{1}{2}(6.313 \text{ kg}) (0.08 \text{ m})^2 = 20.20\times10^{-3} \text{ kg·m}^2$

$I_{y2} = \frac{1}{2} m_2 r_2^2 + m_2 d_2^2$

$= \frac{1}{2}(0.5919 \text{ kg}) (0.02 \text{ m})^2 + (0.5919 \text{ kg}) (0.04 \text{ m})^2 = 1.0654\times10^{-3} \text{ kg·m}^2$

$I_{y3} = \frac{1}{2} m_3 r_3^2 + m_3 d_3^2$

$= \frac{1}{2}(0.3946 \text{ kg}) (0.02 \text{ m})^2 + (0.3946 \text{ kg}) (0.04 \text{ m})^2 = 0.7103\times10^{-3} \text{ kg·m}^2$

$I_y = I_{y1} + I_{y2} - I_{y3} = [(20.20) + (1.0654) - (0.7103)]\times10^{-3} \text{ kg·m}^2$

$= 20.5551\times10^{-3} \text{ kg·m}^2 \quad \Rightarrow \quad I_y = 20.6\times10^{-3} \text{ kg·m}^2$

(c) $I_{z1} = I_{x1} = 13.467\times10^{-3} \text{ kg·m}^2$

$I_{z2} = \frac{1}{12} m_2 (3r_2^2 + 4L_2^2) + m_2 d_2^2$

$= (0.7695\times10^{-3} \text{ kg·m}^2) + (0.5919 \text{ kg}) (0.04 \text{ m})^2 = 1.7165\times10^{-3} \text{ kg·m}^2$

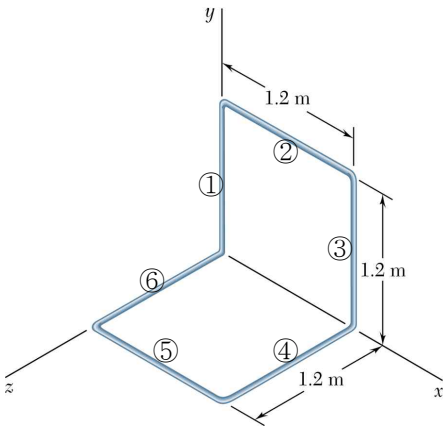
$I_{z3} = \frac{1}{12} m_3 (3r_3^2 + 4L_3^2) + m_3 d_3^2$

$= (0.2499\times10^{-3} \text{ kg·m}^2) + (0.3946 \text{ kg}) (0.04 \text{ m})^2 = 0.88126\times10^{-3} \text{ kg·m}^2$

$I_z = I_{z1} + I_{z2} - I_{z3} = [(13.467) + (1.7165) - (0.88126)]\times10^{-3} \text{ kg·m}^2$

$= 14.3022\times10^{-3} \text{ kg·m}^2 \quad \Rightarrow \quad I_z = 14.30\times10^{-3} \text{ kg·m}^2$

9.148 $\frac{m}{L} = 0.056$ kg/m, $L = 1.2$ m



$m = \frac{m}{L} L = (0.056 \text{ kg/m})\ (1.2 \text{ m}) = 0.0672$ kg

I_x ;

$I_{x1} = I_{x3} = I_{x4} = I_{x6}$

$= \frac{1}{3} m L^2 = \frac{1}{3} (0.0672 \text{ kg})\ (1.2 \text{ m})^2$

$= 0.032256$ kg·m^2

$I_{x2} = I_{x5} = m L^2 = (0.0672 \text{ kg})\ (1.2 \text{ m})^2$

$= 0.096768$ kg·m^2

$I_x = I_{x1} + I_{x2} + I_{x3} + I_{x4} + I_{x5} + I_{x6}$

$= 4\ I_{x1} + 2\ I_{x2}$

$= 4\ (0.032256 \text{ kg·m}^2) + 2\ (0.096768 \text{ kg·m}^2) = 0.32256 \text{ kg·m}^2$

I_y ;

$I_{y1} = 0$

$I_{y2} = I_{y6} = \frac{1}{3} m L^2 = 0.032256$ kg·m^2

$I_{y3} = m L^2 = 0.096768$ kg·m^2

$I_{y4} = I_{y5} = \frac{1}{12} m L^2 + m d^2$

$= \frac{1}{12} (0.0672 \text{ kg})\ (1.2 \text{ m})^2 + (0.0672 \text{ kg})\ [(1.2 \text{ m})^2 + (0.6 \text{ m})^2]$

$= 0.129024$ kg·m^2

$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4} + I_{y5} + I_{y6}$

$= I_{y1} + 2\ I_{y2} + I_{y3} + 2\ I_{y4}$

$= 0 + 2\ (0.032256 \text{ kg·m}^2) + (0.096768 \text{ kg·m}^2) + 2\ (0.129024 \text{ kg·m}^2)$

$= 0.419328$ kg·m^2

$I_z = I_y$ (by symmetry)

⇒ $I_x = 0.323$ kg·m^2 , $I_y = 0.419$ kg·m^2 , $I_z = 0.419$ kg·m^2