

{9.6~9.7절}

9.31&33

$$A_1 = (12 + 12 \text{ mm})(6 \text{ mm}) = 144 \text{ mm}^2$$

$$A_2 = (8 \text{ mm})(24 + 24 \text{ mm}) = 384 \text{ mm}^2$$

$$A_3 = (24 + 24 \text{ mm})(6 \text{ mm}) = 288 \text{ mm}^2$$

$$\begin{aligned} A &= A_1 + A_2 + A_3 \\ &= 144 + 384 + 288 \text{ mm}^2 = 816 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} I_{x1} &= \bar{I}_{x1} + A_1 d_1^2 \\ &= \frac{1}{12}(12 + 12 \text{ mm})(6 \text{ mm})^3 + (144 \text{ mm}^2)(3 + 24 \text{ mm})^2 \\ &= 432 + 104,976 \text{ mm}^4 = 105,408 \text{ mm}^4 \end{aligned}$$

$$I_{x2} = \frac{1}{12}(8 \text{ mm})(24 + 24 \text{ mm})^3 = 73,728 \text{ mm}^4$$

$$\begin{aligned} I_{x3} &= \bar{I}_{x3} + A_3 d_3^2 = \frac{1}{12}(24 + 24 \text{ mm})(6 \text{ mm})^3 + (288 \text{ mm}^2)(3 + 24 \text{ mm})^2 \\ &= 864 + 209,952 \text{ mm}^4 = 210,816 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_x &= I_{x1} + I_{x2} + I_{x3} \\ &= 105,408 + 73,728 + 210,816 \text{ mm}^4 = 389,952 \text{ mm}^4 \\ &\Rightarrow I_x = 390,000 \text{ mm}^4 = 390 \times 10^3 \text{ mm}^4 = 0.390 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{389,952 \text{ mm}^4}{816 \text{ mm}^2}} = 21.86 \text{ mm} \quad \Rightarrow \quad k_x = 21.9 \text{ mm}$$

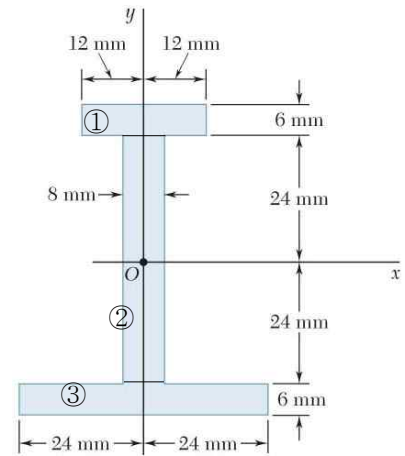
$$I_{y1} = \frac{1}{12}(6 \text{ mm})(12 + 12 \text{ mm})^3 = 6,912 \text{ mm}^4$$

$$I_{y2} = \frac{1}{12}(24 + 24 \text{ mm})(8 \text{ mm})^3 = 2,048 \text{ mm}^4$$

$$I_{y3} = \frac{1}{12}(6 \text{ mm})(24 + 24 \text{ mm})^3 = 55,296 \text{ mm}^4$$

$$\begin{aligned} I_y &= I_{y1} + I_{y2} + I_{y3} \\ &= 6,912 + 2,048 + 55,296 \text{ mm}^4 = 64,256 \text{ mm}^4 \\ &\Rightarrow I_y = 64,300 \text{ mm}^4 = 64.3 \times 10^3 \text{ mm}^4 \end{aligned}$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{64,256 \text{ mm}^4}{816 \text{ mm}^2}} = 8.874 \text{ mm} \quad \Rightarrow \quad k_y = 8.87 \text{ mm}$$



9.36  $a = 20 \text{ mm}$

① 정사각형, ② 반원 2개

$$(I_x)_1 = \frac{1}{12}(2a)(2a)^3 = \frac{4}{3}a^4 = \frac{4}{3}(20 \text{ mm})^4$$

$$= 0.2133 \times 10^6 \text{ mm}^4$$

$$(I_x)_2 = \left[ \frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2 \right] + \frac{\pi}{2}a^2\left(a + \frac{4a}{3\pi}\right)^2$$

$$= \left( \frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} + \frac{4}{3} + \frac{8}{9\pi} \right) a^4$$

$$= \left( \frac{5\pi}{8} + \frac{4}{3} \right) a^4 = 3.297 a^4$$

$$= 3.297 (20 \text{ mm})^4 = 0.5275 \times 10^6 \text{ mm}^4$$

$$I_x = (I_x)_1 + 2 (I_x)_2$$

$$= [0.2133 + 2 (0.5275)] \times 10^6 \text{ mm}^4$$

$$= 1.268 \times 10^6 \text{ mm}^4$$

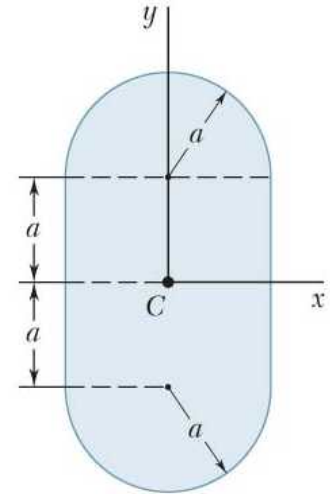
$$(I_y)_1 = \frac{1}{12}(2a)(2a)^3 = \frac{4}{3}a^4 = \frac{4}{3}(20 \text{ mm})^4 = 0.2133 \times 10^6 \text{ mm}^4$$

$$(I_y)_2 = \frac{\pi}{8}a^4 = \frac{\pi}{8}(20 \text{ mm})^4 = 0.0628 \times 10^6 \text{ mm}^4$$

$$I_y = (I_y)_1 + 2 (I_y)_2$$

$$= [0.2133 + 2 (0.0628)] \times 10^6 \text{ mm}^4$$

$$= 0.339 \times 10^6 \text{ mm}^4$$



9.46 ① 타원, ② 큰 반원 구멍, ③ 작은 반원 구멍

Centroid  $\bar{X}$ ,  $\bar{Y}$  (대칭  $\Rightarrow \bar{Y} = 0$ )

① 타원 :  $a = 100 \text{ mm}$ ,  $b = 60 \text{ mm}$

$$A = \pi a b = \pi (100 \text{ mm})(60 \text{ mm}) = 6,000 \pi \text{ mm}^2$$

$$\bar{x} = 0$$

② 큰 반원 구멍 :  $r_1 = 45 \text{ mm}$

$$A = -\frac{1}{2}\pi r_1^2 = -\frac{1}{2}\pi (45 \text{ mm})^2 = -1,012.5 \pi \text{ mm}^2$$

$$\bar{x} = \frac{4}{3\pi} r_1 = \frac{4}{3\pi} (45 \text{ mm}) = \frac{60}{\pi} \text{ mm}^2$$

③ 작은 반원 구멍 :  $r_2 = 30 \text{ mm}$

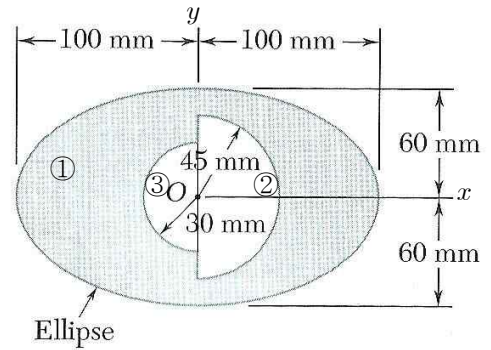
$$A = -\frac{1}{2}\pi r_2^2 = -\frac{1}{2}\pi (30 \text{ mm})^2 = -450 \pi \text{ mm}^2$$

$$\bar{x} = -\frac{4}{3\pi} r_2 = -\frac{4}{3\pi} (30 \text{ mm}) = -\frac{40}{\pi} \text{ mm}^2$$

$$\Sigma A = (6,000 \pi) + (-1,012.5 \pi) + (-450 \pi) \text{ mm}^2 = 14,255 \text{ mm}^2$$

$$\Sigma(\bar{x}A) = (0)(6,000 \pi) + \left(\frac{60}{\pi}\right)(-1,012.5 \pi) + \left(-\frac{40}{\pi}\right)(-450 \pi) \text{ mm}^3 = -42,750 \text{ mm}^3$$

$$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{-42,750 \text{ mm}^3}{14,255 \text{ mm}^2} = -2.999 \text{ mm} = -3.00 \text{ mm}$$



$$(a) (J_O)_1 = \frac{1}{4}\pi a b (a^2 + b^2) = \frac{1}{4}\pi (100)(60)(100^2 + 60^2) \text{ mm}^4 = 64.09 \times 10^6 \text{ mm}^4$$

$$(J_O)_2 = \frac{1}{4}\pi r_1^4 = \frac{1}{4}\pi (45^4) \text{ mm}^4 = 3.22 \times 10^6 \text{ mm}^4$$

$$(J_O)_3 = \frac{1}{4}\pi r_2^4 = \frac{1}{4}\pi (30^4) \text{ mm}^4 = 0.636 \times 10^6 \text{ mm}^4$$

$$J_O = (J_O)_1 - (J_O)_2 - (J_O)_3 = (64.09 \times 10^6) - (3.22 \times 10^6) - (0.636 \times 10^6) \text{ mm}^4 = 60.2 \times 10^6 \text{ mm}^4$$

$$(b) \bar{J}_C = J_O - A \bar{d}^2$$

$$= (60.2 \times 10^6 \text{ mm}^4) - (14,255 \text{ mm}^2) (-3.00 \text{ mm})^2 = 60.1 \times 10^6 \text{ mm}^4$$