{5.10~5.11절}

5.104 ① 사각판

V = (90 mm)(38 mm)(10 mm)

  = 34,200 mm^3

$\bar{x}$ = 19 mm

② 사각기둥

V = (30 mm)(10 mm)(24 mm)

  = 7,200 mm^3

$\bar{x}$ = $\frac{1}{2}$(10 mm) = 5 mm

③ 삼각기둥

V = $\frac{1}{2}$(30 mm)(9 mm)(24 mm)

  = 3,240 mm^3

$\bar{x}$ = (10 mm) + $\frac{1}{3}$(9 mm) = 13 mm

④ 반원기둥

V = $\frac{1}{2}\pi$ (20 mm)^2 (10 mm) = 6,283 mm^3

$\bar{x}$ = (38 mm) + $\frac{4}{3\pi}$(20 mm) = 46.49 mm

⑤ 원기둥 구멍

V = $-\pi$ (12 mm)^2 (10 mm) = $-4,524$ mm^3

$\bar{x}$ = 38 mm
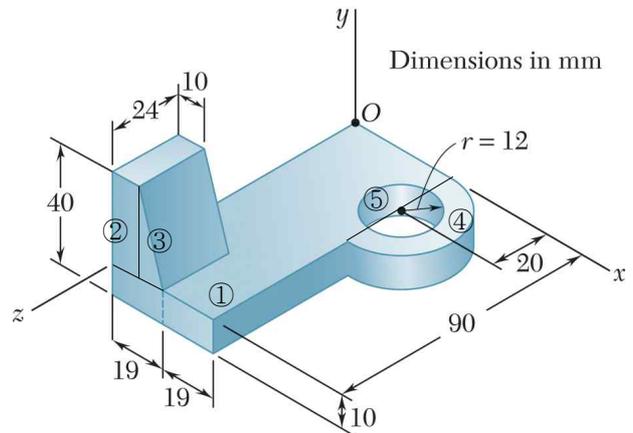
ΣV = 34,200 + 7,200 + 3,240 + 6,283 + $(-4,524)$ (mm^3) = 46,399 mm^3

$\Sigma(\bar{x} V)$ = (19)(34,200) + (5)(7,200) + (13)(3,240) + (46.49)(6,283)

    + (38)($-4,524$) (mm^4) = 848,104 mm^4

$\overline{X}$ = $\dfrac{\Sigma(\bar{x} V)}{\Sigma V}$ = $\dfrac{848,104 \text{ mm}^4}{46,399 \text{ mm}^3}$ =18.278 mm $\Rightarrow$ $\overline{X}$ = 18.28 mm

5.110   박판은 균질[homogeneous]이어서 무게중심이 도심과 일치한다고 가정함.

대칭성 의해 $\overline{X} = \overline{Z}$

① 사각판

$A = (10 \text{ cm})(16 \text{ cm}) = 160 \text{ cm}^2$

$\overline{x} = 5 \text{ cm}$

$\overline{y} = 8 \text{ cm}$

② 사각판

$A = 160 \text{ cm}^2$

$\overline{x} = 0$

$\overline{y} = 8 \text{ cm}$

③ $\frac{1}{4}$원주면

$A = \frac{1}{4}[2\pi(10 \text{ cm})](16 \text{ cm}) = 251.3 \text{ cm}^2$

$\overline{x} = \frac{2}{\pi}(10 \text{ cm}) = 6.366 \text{ cm}$

$\overline{y} = 8 \text{ cm}$

④ $\frac{1}{4}$원판

$A = \frac{1}{4}\pi(10 \text{ cm})^2 = 78.54 \text{ cm}^2$

$\overline{x} = \frac{4}{3\pi}(10 \text{ cm}) = 4.244 \text{ cm}$

$\overline{y} = 0$
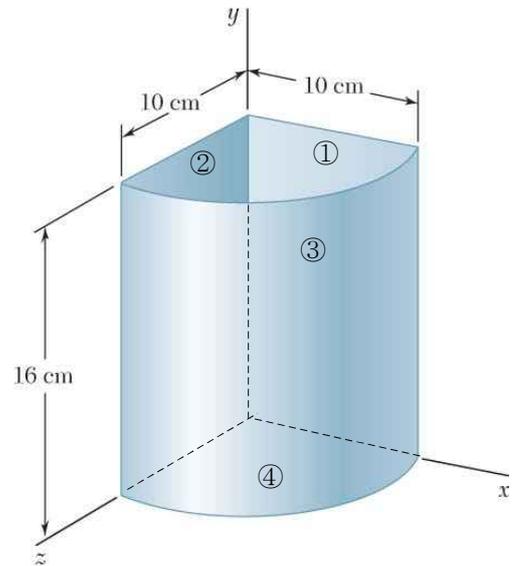
$\Sigma A = 160 + 160 + 251.3 + 78.54 \text{ (cm}^2) = 649.8 \text{ cm}^2$

$\Sigma(\overline{x}A) = (5)(160) + (0)(160) + (6.366)(251.3) + (4.244)(78.54) \text{ (cm}^3) = 2{,}733 \text{ cm}^3$

$\Sigma(\overline{y}A) = (8)(160) + (8)(160) + (8)(251.3) + (0)(78.54) \text{ (cm}^3) = 4{,}570 \text{ cm}^3$

$\overline{X} = \dfrac{\Sigma(\overline{x}A)}{\Sigma A} = \dfrac{2{,}733 \text{ cm}^3}{649.8 \text{ cm}^2} = 4.206 \text{ cm}$

$\overline{Y} = \dfrac{\Sigma(\overline{z}A)}{\Sigma A} = \dfrac{4{,}570 \text{ cm}^3}{649.8 \text{ cm}^2} = 7.033 \text{ cm}$

⇒   center of gravity = (4.21 cm, 7.03 cm, 4.21 cm)

5.117 ① $L = 0.4$ m

$\bar{x} = 0.6$ m, $\bar{y} = 0$

$\bar{z} = \dfrac{1}{2}(0.4$ m$) = 0.2$ m

② $L = 0.6$ m

$\bar{x} = \dfrac{1}{2}(0.6$ m$) = 0.3$ m

$\bar{y} = 0$, $\bar{z} = 0.4$ m

③ $L = 1.0$ m

$\bar{x} = 0.6$ m

$\bar{y} = \dfrac{1}{2}(1.0$ m$) = 0.5$ m, $\bar{z} = 0$

④ $L = 1.0$ m

$\bar{x} = 0.6$ m, $\bar{y} = 0.5$ m, $\bar{z} = 0.4$ m

⑤ $L = (1+0.6)$ m $= 1.6$ m

$\bar{x} = 0$, $\bar{y} = \dfrac{1}{2}(1.6$ m$) = 0.8$ m

$\bar{z} = 0.4$ m

⑥ $L = 0.4$ m

$\bar{x} = 0.6$ m, $\bar{y} = 1.0$ m, $\bar{z} = 0.2$ m

⑦ $L = 0.6$ m

$\bar{x} = 0.3$ m, $\bar{y} = 1.0$ m, $\bar{z} = 0.4$ m

⑧ $L = 0.4$ m

$\bar{x} = 0$, $\bar{y} = 1.6$ m, $\bar{z} = 0.2$ m

⑨ $L = \dfrac{1}{4}[2\pi(0.6$ m$)] = 0.9424$ m

$\bar{x} = \dfrac{2}{\pi}(0.6$ m$) = 0.382$ m, $\bar{y} = (1+0.3820)$ m $= 1.382$ m, $\bar{z} = 0$

⑩ $L = L⑨ = 0.9424$ m

$\bar{x} = \bar{x}⑨ = 0.382$ m, $\bar{y} = \bar{y}⑨ = 1.382$ m, $\bar{z} = 0.4$ m

$\Sigma L = 0.4 + 0.6 + 1.0 + 1.0 + 1.6 + 0.4 + 0.6 + 0.4 + 0.943 + 0.943$ m
$= 7.886$ m

$\Sigma(\bar{x}L) = (0.4)(0.6) + (0.6)(0.3) + (1.0)(0.6) + (1.0)(0.6) + (1.6)(0)\, + (0.4)(0.6)$
$+ (0.6)(0.3) + (0.4)(0) + (0.943)(0.382) + (0.943)(0.382)$ m$^2 = 2.760$ m^2

$\Sigma(\bar{y}L) = (0.4)(0) + (0.6)(0) + (1.0)(0.5) + (1.0)(0.5) + (1.6)(0.8) + (0.4)(1.0)$
$+ (0.6)(1.0) + (0.4)(1.6) + (0.943)(1.382) + (0.943)(1.382)$ m$^2 = 6.526$ m^2

$\Sigma(\bar{z}L) = (0.4)(0.2) + (0.6)(0.4) + (1.0)(0) + (1.0)(0.4) + (1.6)(0.4) + (0.4)(0.2)$
$+ (0.6)(0.4) + (0.4)(0.2) + (0.943)(0) + (0.943)(0.4)$ m$^2 = 2.137$ m^2

$\overline{X} = \dfrac{\Sigma(\bar{x}L)}{\Sigma L} = \dfrac{2.760 \text{ m}^2}{7.886 \text{ m}} = 0.350$ m $\Rightarrow$ $\overline{X} = 0.350$ m

$\overline{Y} = \dfrac{\Sigma(\bar{y}L)}{\Sigma L} = \dfrac{6.526 \text{ m}^2}{7.886 \text{ m}} = 0.828$ m $\Rightarrow$ $\overline{Y} = 0.828$ m

$\overline{Z} = \dfrac{\Sigma(\bar{z}L)}{\Sigma L} = \dfrac{2.137 \text{ m}^2}{7.886 \text{ m}} = 0.271$ m $\Rightarrow$ $\overline{Z} = 0.271$ m

center of gravity $= (0.350$ m, 0.828 m, 0.271 m$)$