

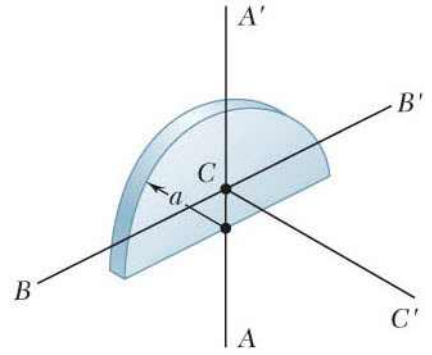
[9.11~9.15절]

$$9.113 \quad m = \rho t A = \rho t \left(\frac{1}{2} \pi a^2 \right)$$

$$I^{mass} = \rho t I^{area} = \frac{2m}{\pi a^2} I^{area}$$

$$(a) \quad I_{BB'}^{area} = I_x^{area} - A \bar{y}^2 \\ = \frac{1}{8} \pi a^4 - \left(\frac{1}{2} \pi a^2 \right) \left(\frac{4a}{3\pi} \right)^2 = \pi a^4 \left(\frac{1}{8} - \frac{8}{9\pi^2} \right) \\ = \frac{\pi}{8} a^4 \left(1 - \frac{64}{9\pi^2} \right)$$

$$I_{BB'}^{mass} = \frac{2m}{\pi a^2} I_{BB'}^{area} \\ = \frac{2m}{\pi a^2} \frac{\pi}{8} a^4 \left(1 - \frac{64}{9\pi^2} \right) = \frac{1}{4} m a^2 \left(1 - \frac{64}{9\pi^2} \right) = 0.0699 m a^2$$



$$(b) \quad \text{Fig. 9.12에} \text{서, } I_{AA'}^{area} = \frac{1}{8} \pi a^4$$

$$I_{AA'}^{mass} = \frac{2m}{\pi a^2} I_{AA'}^{area} \\ = \frac{2m}{\pi a^2} \frac{1}{8} \pi a^4 = \frac{1}{4} m a^2$$

$$\bar{I}_{CC'}^{mass} = I_{AA'}^{mass} + I_{BB'}^{mass} \\ = 0.250 m a^2 + 0.0699 m a^2 = 0.320 m a^2$$

$$9.115 \quad m = \rho t A = \rho t \left[(2a)(a) + \frac{1}{2} (2a)(a) \right] = 3 \rho t a^2 \quad \Rightarrow \quad \rho t = \frac{m}{3a^2}$$

$$I^{mass} = \rho t I^{area} = \frac{m}{3a^2} I^{area}$$

$$(a) \quad \text{사각형 } I_{x1}^{area} = \frac{1}{3} (2a)(a)^3 = \frac{2}{3} a^4, \quad \text{삼각형 } I_{x2}^{area} = \frac{1}{12} (2a)(a)^3 = \frac{1}{6} a^4$$

$$I_x^{area} = I_{x1}^{area} + I_{x2}^{area} = \frac{2}{3} a^4 + \frac{1}{6} a^4 = \frac{5}{6} a^4$$

$$I_x^{mass} = \frac{m}{3a^2} I^{area} = \frac{m}{3a^2} \frac{5}{6} a^4 = \frac{5}{18} m a^2$$

$$(b) \quad \text{사각형 } I_{z1}^{area} = \frac{1}{3} (a)(2a)^3 = \frac{8}{3} a^4$$

$$\text{삼각형 } I_{z2}^{area} = \frac{1}{36} (a)(2a)^3 + \left[\frac{1}{2} (a)(2a) \right] \left(2a + \frac{2}{3} a \right)^2 = \left(\frac{2}{9} + \frac{64}{9} \right) a^4 = \frac{22}{3} a^4$$

$$I_z^{area} = I_{z1}^{area} + I_{z2}^{area} = \frac{8}{3} a^4 + \frac{22}{3} a^4 = 10 a^4$$

$$I_z^{mass} = \frac{m}{3a^2} I^{area} = \frac{m}{3a^2} 10 a^4 = \frac{10}{3} m a^2$$

$$I_y^{mass} = I_x^{mass} + I_z^{mass} = \frac{5}{18} m a^2 + \frac{10}{3} m a^2 = \left(\frac{5}{18} + \frac{10}{3} \right) m a^2 = \frac{65}{18} m a^2 \\ = 3.61 m a^2$$

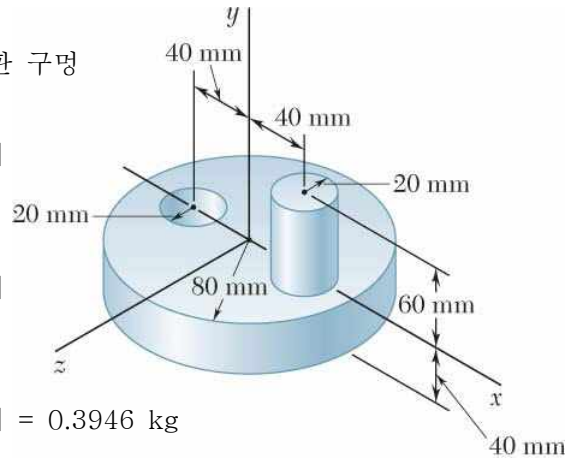
9.141 $\rho = 7,850 \text{ kg/m}^3$

$m = \rho V$ ① 원판, ② 원기둥, ③ 원판 구멍

$m_1 = \rho V_1$
 $= (7,850 \text{ kg/m}^3) [\pi (0.08 \text{ m})^2 (0.04 \text{ m})]$
 $= 6.313 \text{ kg}$

$m_2 = \rho V_2$
 $= (7,850 \text{ kg/m}^3) [\pi (0.02 \text{ m})^2 (0.06 \text{ m})]$
 $= 0.5919 \text{ kg}$

$m_3 = \rho V_3$
 $= (7,850 \text{ kg/m}^3) [\pi (0.02 \text{ m})^2 (0.04 \text{ m})] = 0.3946 \text{ kg}$



(a) $I_{x1} = \frac{1}{12} m_1 (3r_1^2 + 4L_1^2)$
 $= \frac{1}{12} (6.313 \text{ kg}) [3 (0.08 \text{ m})^2 + 4 (0.04 \text{ m})^2] = 13.467 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_{x2} = \frac{1}{12} m_2 (3r_2^2 + 4L_2^2)$
 $= \frac{1}{12} (0.5919 \text{ kg}) [3 (0.02 \text{ m})^2 + 4 (0.06 \text{ m})^2] = 0.7695 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_{x3} = \frac{1}{12} m_3 (3r_3^2 + 4L_3^2)$
 $= \frac{1}{12} (0.3946 \text{ kg}) [3 (0.02 \text{ m})^2 + 4 (0.04 \text{ m})^2] = 0.2499 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_x = I_{x1} + I_{x2} - I_{x3} = [(13.467) + (0.7695) - (0.2499)] \times 10^{-3} \text{ kg}\cdot\text{m}^2$
 $= 13.9886 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_x = 13.99 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

(b) $I_{y1} = \frac{1}{2} m_1 r_1^2$
 $= \frac{1}{2} (6.313 \text{ kg}) (0.08 \text{ m})^2 = 20.20 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_{y2} = \frac{1}{2} m_2 r_2^2 + m_2 d_2^2$
 $= \frac{1}{2} (0.5919 \text{ kg}) (0.02 \text{ m})^2 + (0.5919 \text{ kg}) (0.04 \text{ m})^2 = 1.0654 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_{y3} = \frac{1}{2} m_3 r_3^2 + m_3 d_3^2$
 $= \frac{1}{2} (0.3946 \text{ kg}) (0.02 \text{ m})^2 + (0.3946 \text{ kg}) (0.04 \text{ m})^2 = 0.7103 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_y = I_{y1} + I_{y2} - I_{y3} = [(20.20) + (1.0654) - (0.7103)] \times 10^{-3} \text{ kg}\cdot\text{m}^2$
 $= 20.5551 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_y = 20.6 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

(c) $I_{z1} = I_{x1} = 13.467 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_{z2} = \frac{1}{12} m_2 (3r_2^2 + 4L_2^2) + m_2 d_2^2$
 $= (0.7695 \times 10^{-3} \text{ kg}\cdot\text{m}^2) + (0.5919 \text{ kg}) (0.04 \text{ m})^2 = 1.7165 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_{z3} = \frac{1}{12} m_3 (3r_3^2 + 4L_3^2) + m_3 d_3^2$
 $= (0.2499 \times 10^{-3} \text{ kg}\cdot\text{m}^2) + (0.3946 \text{ kg}) (0.04 \text{ m})^2 = 0.88126 \times 10^{-3} \text{ kg}\cdot\text{m}^2$

$I_z = I_{z1} + I_{z2} - I_{z3} = [(13.467) + (1.7165) - (0.88126)] \times 10^{-3} \text{ kg}\cdot\text{m}^2$
 $= 14.3022 \times 10^{-3} \text{ kg}\cdot\text{m}^2 \quad \Rightarrow \quad I_z = 14.30 \times 10^{-3} \text{ kg}\cdot\text{m}^2$