

[9.6~9.7절]

$$9.32\&34 \quad A_1 = (5 \text{ cm})(6 \text{ cm}) = 30 \text{ cm}^2$$

$$A_2 = (4 \text{ cm})(2 \text{ cm}) = 8 \text{ cm}^2$$

$$A_3 = (4 \text{ cm})(1 \text{ cm}) = 4 \text{ cm}^2$$

$$A = A_1 - A_2 - A_3 \\ = 30 - 8 - 4 \text{ cm}^2 = 18 \text{ cm}^2$$

$$I_{x1} = \frac{1}{12}(5 \text{ cm})(6 \text{ cm})^3 = 90 \text{ cm}^4$$

$$I_{x2} = \bar{I}_{x2} + A_2 d_2^2 \\ = \frac{1}{12}(4 \text{ cm})(2 \text{ cm})^3 + (8 \text{ cm}^2)(2 \text{ cm})^2 \\ = \left(\frac{8}{3} + 32\right) \text{ cm}^4 = 34\frac{2}{3} \text{ cm}^4$$

$$I_{x3} = \bar{I}_{x3} + A_3 d_3^2 = \frac{1}{12}(4 \text{ cm})(1 \text{ cm})^3 + (4 \text{ cm}^2)\left(\frac{3}{2} \text{ cm}\right)^2 \\ = \frac{1}{3} + 9 \text{ cm}^4 = 9\frac{1}{3} \text{ cm}^4$$

$$I_x = I_{x1} - I_{x2} - I_{x3} \\ = 90 - 34\frac{2}{3} - 9\frac{1}{3} \text{ cm}^4 = 46.0 \text{ cm}^4$$

$$\Rightarrow I_x = 46.0 \text{ cm}^4 (= 46.0 \times 10^4 \text{ mm}^4)$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{46.0 \text{ cm}^4}{18 \text{ cm}^2}} = 1.599 \text{ cm} \quad \Rightarrow \quad k_x = 1.599 \text{ cm} (= 15.99 \text{ mm})$$

$$I_{y1} = \frac{1}{12}(6 \text{ cm})(5 \text{ cm})^3 = 62\frac{1}{2} \text{ cm}^4$$

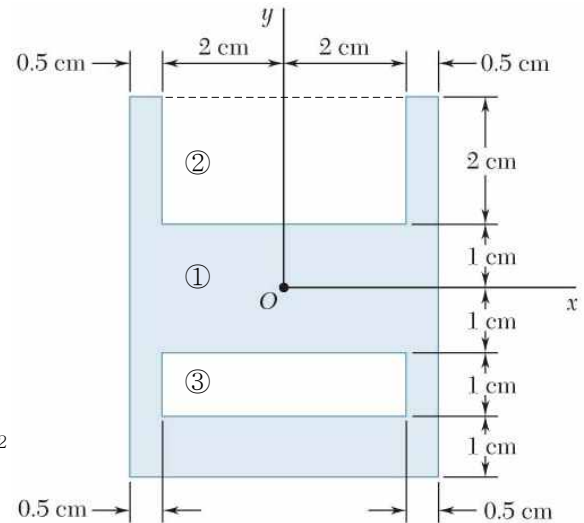
$$I_{y2} = \frac{1}{12}(2 \text{ cm})(4 \text{ cm})^3 = 10\frac{2}{3} \text{ cm}^4$$

$$I_{y3} = \frac{1}{12}(1 \text{ mm})(4 \text{ cm})^3 = 5\frac{1}{3} \text{ cm}^4$$

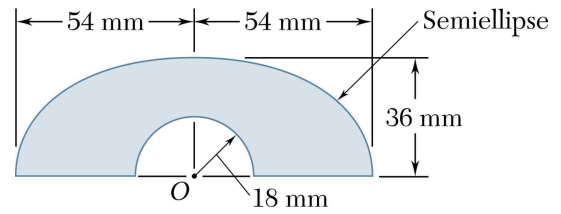
$$I_y = I_{y1} - I_{y2} - I_{y3} \\ = 62\frac{1}{2} - 10\frac{2}{3} - 5\frac{1}{3} \text{ cm}^4 = 46.5 \text{ cm}^4$$

$$\Rightarrow I_y = 46.5 \text{ cm}^4 (= 46.5 \times 10^4 \text{ mm}^4)$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{46.5 \text{ cm}^4}{18 \text{ cm}^2}} = 1.607 \text{ cm} \quad \Rightarrow \quad k_y = 1.607 \text{ cm} (= 16.07 \text{ mm})$$



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$$(a) (J_O)_1 = \frac{1}{8} \pi a^3 b + \frac{1}{8} \pi a b^3$$

$$= \frac{1}{8} \pi a b (a^2 + b^2)$$

$$= \frac{1}{8} \pi (54 \text{ mm})(36 \text{ mm})[(54 \text{ mm})^2 + (36 \text{ mm})^2]$$

$$= 3.215 \times 10^6 \text{ mm}^4$$

$$(J_O)_2 = \frac{1}{4} \pi r^4$$

$$= \frac{1}{4} \pi (18 \text{ mm})^4 = 0.0824 \times 10^6 \text{ mm}^4$$

$$J_O = (J_O)_1 - (J_O)_2$$

$$= (3.215 \times 10^6 \text{ mm}^4) - (0.0824 \times 10^6 \text{ mm}^4) = 3.1326 \times 10^6 \text{ mm}^4$$

$$\Rightarrow J_O = 3.13 \times 10^6 \text{ mm}^4$$

(b) y 축 대칭 $\Rightarrow \bar{X} = 0$

$$A_1 = \frac{1}{2} \pi a b = \frac{1}{2} \pi (54 \text{ mm})(36 \text{ mm}) = 3,053 \text{ mm}^2$$

$$\bar{y}_1 = \frac{4}{3\pi} b = \frac{4}{3\pi} (36 \text{ mm}) = 15.279 \text{ mm}$$

$$A_2 = -\frac{1}{2} \pi r^2 = -\frac{1}{2} \pi (18 \text{ mm})^2 = -508.9 \text{ mm}^2$$

$$\bar{y}_2 = \frac{4}{3\pi} r = \frac{4}{3\pi} (18 \text{ mm}) = 7.639 \text{ mm}$$

$$\Sigma A = 3,053 + (-508.9) \text{ mm}^2 = 2,544 \text{ mm}^2$$

$$\Sigma(\bar{y}A) = (15.279)(3,053) + (7.639)(-508.9) \text{ mm}^3 = 42,759 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{42,759 \text{ mm}^3}{2,544 \text{ mm}^2} = 16.807 \text{ mm}$$

$$\bar{J}_C = J_O - A \bar{Y}^2$$

$$= (3.13 \times 10^6 \text{ mm}^4) - (2,544 \text{ mm}^2) (16.807 \text{ mm})^2 = 2.411 \times 10^6 \text{ mm}^4$$

$$\Rightarrow \bar{J}_C = 2.41 \times 10^6 \text{ mm}^4$$