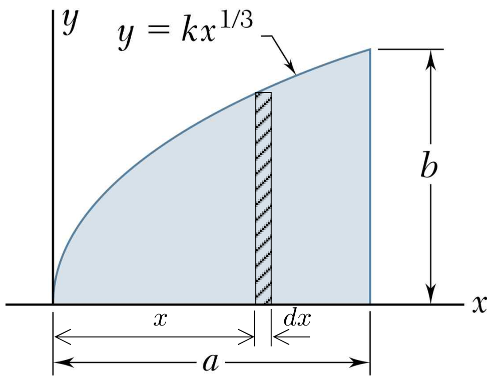
[9.1~9.5절]



9.2&6 $y = k\, x^{\frac{1}{3}}$

$$b = k\, a^{\frac{1}{3}} \quad \Rightarrow \quad k = \frac{b}{a^{1/3}} \quad \Rightarrow \quad y = \frac{b}{a^{1/3}}\, x^{\frac{1}{3}}$$

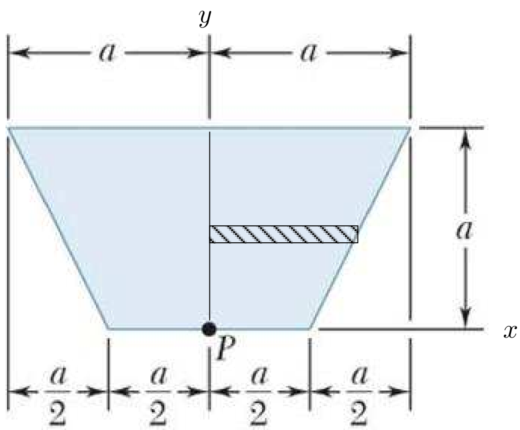
$$dI_y = x^2\, dA = x^2\, y\, dx = x^2 \;\frac{b}{a^{1/3}}\, x^{\frac{1}{3}}\; dx$$

$$= \frac{b}{a^{1/3}}\, x^{\frac{7}{3}}\; dx$$

$$I_y = \int dI_y = \int_0^a \frac{b}{a^{1/3}}\, x^{\frac{7}{3}} dx = \frac{b}{a^{1/3}}\left[\frac{3}{10}\, x^{\frac{10}{3}}\right]_0^a = \frac{b}{a^{1/3}}\left(\frac{3}{10}\, a^{\frac{10}{3}}\right) = \frac{3}{10}\, a^3 b$$

$$dI_x = \frac{1}{3} y^3\, dx = \frac{1}{3}\; \frac{b^3}{a}\; x\; dx$$

$$I_x = \int dI_x = \int_a^b \frac{1}{3}\; \frac{b^3}{a}\; x\; dx = \frac{1}{3}\; \frac{b^3}{a}\left[\frac{1}{2}\, x^2\right]_0^a = \frac{1}{3}\; \frac{b^3}{a}\left(\frac{1}{2}\, a^2\right) = \frac{1}{6}\, a\, b^3$$



9.21 $x = \frac{a}{2} + \frac{1}{2} y$

$$dA = 2\; x\, dy = (a + y)\; dy$$

면적

$$A = \frac{1}{2}\,(2a + a)(a) = \frac{3}{2} a^2$$

또는 $A = \int_0^a dA = \int_0^a (a + y)\; dy$

$$= (a)(a) + \frac{1}{2} a^2 = \frac{3}{2} a^2$$

I_x ;

$$dI_x = y^2\, dA = y^2\; (a + y)\; dy$$

$$I_x = \int_0^a dI_x = \int_0^a y^2\; (a + y)\; dy = a\left[\frac{1}{3} y^3\right]_0^a + \left[\frac{1}{4} y^4\right]_0^a = \frac{7}{12} a^4$$

I_y ;

$$dI_y = 2\left(\frac{1}{3} x^3\, dy\right) = \frac{2}{3}\left(\frac{a}{2} + \frac{1}{2} y\right)^3 dy = \frac{1}{12}\; (a + y)^3\; dy$$

$$I_y = \frac{1}{12}\int_0^a (a + y)^3\; dy = \frac{1}{12}\left[\frac{1}{4}(a + y)^4\right]_0^a = \frac{1}{48}\left[\,(2a)^4 - (a)^4\right] = \frac{5}{16} a^4$$

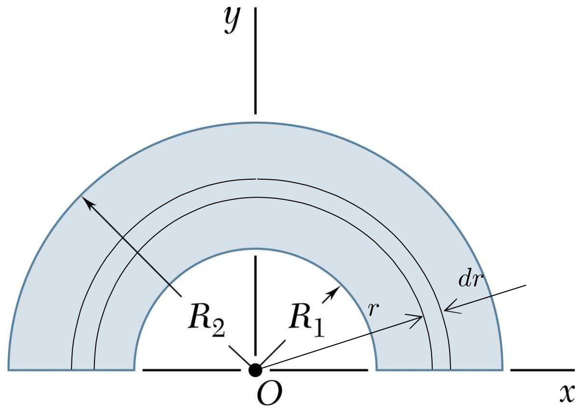
$$J_P = I_x + I_y = \frac{7}{12} a^4 + \frac{5}{16} a^4 = \frac{28 + 15}{48} a^4 = \frac{43}{48} a^4$$

$$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{43}{48} a^4}{\frac{3}{2} a^2}} = \sqrt{\frac{43}{72}}\; a = 0.773\; a$$

9.25 (a) $dJ_O = r^2\,dA = r^2(\pi r\,dr) = \pi r^3\,dr$

$$J_O = \int dJ_O = \int_{R_1}^{R_2} \pi r^3\,dr$$

$$= \left[\frac{1}{4}\pi r^4\right]_{R_1}^{R_2} = \frac{1}{4}\pi\left(R_2^{\,4} - R_1^{\,4}\right)$$



(b) $I_x = I_y$

$$J_O = I_x + I_y = 2\,I_x$$

$$I_x = I_y = \frac{1}{2}J_O = \frac{1}{8}\pi\left(R_2^{\,4} - R_1^{\,4}\right)$$