

[5.10~5.11절]

5.98 ① 반 원뿔(cone)

$$V = \frac{1}{2} \left( \frac{1}{3} \pi a^2 h \right) = \frac{1}{6} \pi a^2 h$$

$$\bar{y} = -\frac{1}{4} h$$

② 반 원판(disc)

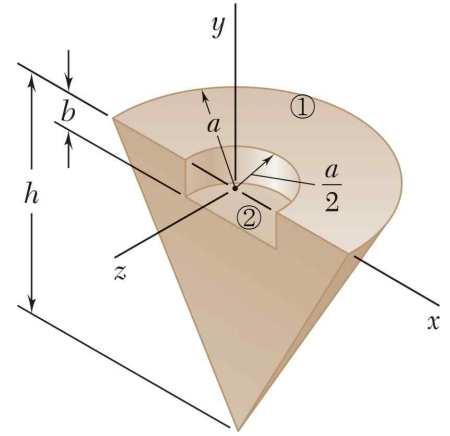
$$V = -\frac{1}{2} \left[ \pi \left( \frac{1}{2} a \right)^2 b \right] = -\frac{1}{8} \pi a^2 b$$

$$\bar{y} = -\frac{1}{2} b$$

$$\Sigma V = \frac{1}{6} \pi a^2 h + \left( -\frac{1}{8} \pi a^2 b \right) = \frac{1}{24} \pi a^2 (4h - 3b)$$

$$\Sigma(\bar{y}V) = \left( -\frac{1}{4} h \right) \left( \frac{1}{6} \pi a^2 h \right) + \left( -\frac{1}{2} b \right) \left( -\frac{1}{8} \pi a^2 b \right) = -\frac{1}{48} \pi a^2 (2h^2 - 3b^2)$$

$$\bar{Y} = \frac{\Sigma(\bar{y}V)}{\Sigma V} = \frac{-\frac{1}{48} \pi a^2 (2h^2 - 3b^2)}{\frac{1}{24} \pi a^2 (4h - 3b)} = -\frac{(2h^2 - 3b^2)}{2(4h - 3b)}$$



5.106 박판은 균질[homogeneous]이어서 무게중심이 도심과 일치한다고 가정함.

$$\text{대칭성에 의해 } \bar{Y} = \frac{1}{2}(160 \text{ mm}) = 80.0 \text{ mm}$$

① 반원

$$A = \frac{1}{2} \pi (80 \text{ mm})^2 = 10,053 \text{ mm}^2$$

$$\bar{x} = 0$$

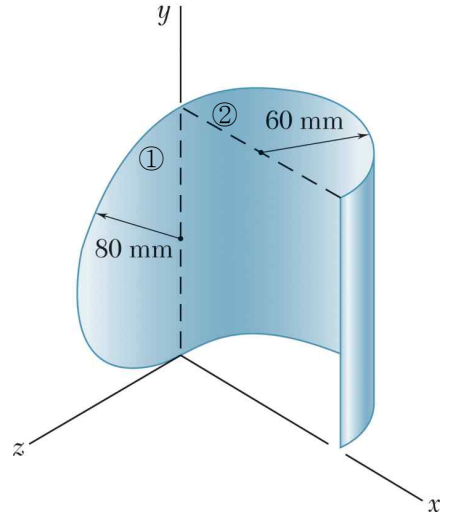
$$\bar{z} = \frac{4}{3\pi} (80 \text{ mm}) = 33.953 \text{ mm}$$

② 반 원기둥 면

$$A = \frac{1}{2} [2\pi(60 \text{ mm})](160 \text{ mm}) = 30,159 \text{ mm}^2$$

$$\bar{x} = 60 \text{ mm}$$

$$\bar{z} = -\frac{2}{\pi} (60 \text{ mm}) = -38.197 \text{ mm}$$



$$\Sigma A = 10,053 + 30,159 = 40,212 \text{ mm}^2$$

$$\Sigma(\bar{x}A) = (0)(10,053) + (60 \text{ mm})(30,159) = 1,809,540 \text{ mm}^3$$

$$\Sigma(\bar{z}A) = (33.953)(10,053) + (-38.197)(30,159) = -810,654 \text{ mm}^3$$

$$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{1,809,540 \text{ mm}^3}{40,212 \text{ mm}^2} = 45.0 \text{ mm}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}A)}{\Sigma A} = \frac{-810,654 \text{ mm}^3}{40,212 \text{ mm}^2} = -20.16 \text{ mm}$$

⇒ center of gravity = (45.0 mm, 80.0 mm, -20.2 mm)

5.114 대칭성에 의해  $\bar{X} = 0$

①  $AB$

$$L = \sqrt{(16 \text{ cm})^2 + (30 \text{ cm})^2} = 34.0 \text{ cm}$$

$$\bar{y} = \frac{1}{2}(30 \text{ cm}) = 15 \text{ cm}$$

$$\bar{z} = 0$$

②  $AD$

$$L = L① = 34.0 \text{ m}$$

$$\bar{y} = \bar{y}① = 15 \text{ cm}$$

$$\bar{z} = \frac{1}{2}(16 \text{ cm}) = 8.0 \text{ cm}$$

③  $AE$

$$L = L① = 34.0 \text{ m}$$

$$\bar{y} = \bar{y}① = 15 \text{ cm}$$

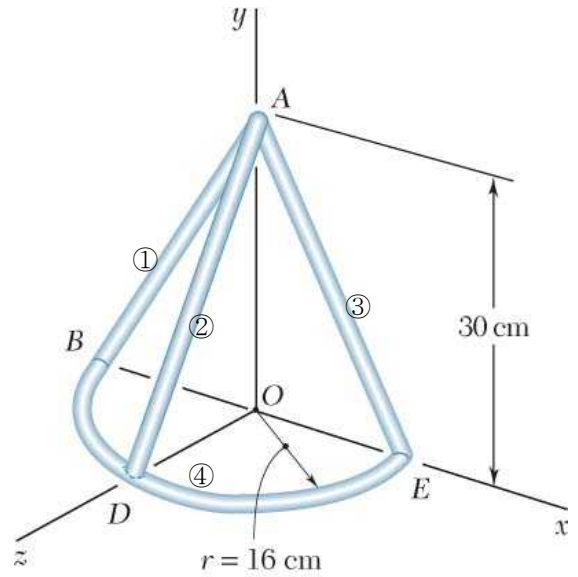
$$\bar{z} = 0$$

④  $BDE$

$$L = \pi (16 \text{ cm}) = 50.27 \text{ cm}$$

$$\bar{y} = 0$$

$$\bar{z} = \frac{2}{\pi} (16 \text{ cm}) = 10.186 \text{ cm}$$



$$\Sigma L = 3(34.0 \text{ cm}) + 50.27 \text{ cm} = 152.27 \text{ cm}$$

$$\Sigma(\bar{y}L) = 3(15)(34.0) + (0)(50.27) \text{ cm}^2 = 1,530 \text{ cm}^2$$

$$\Sigma(\bar{z}L) = (0)(34.0) + (8)(34.0) + (0)(34.0) + (10.186)(50.27) \text{ cm}^2 = 784 \text{ cm}^2$$

$$\bar{Y} = \frac{\Sigma(\bar{y}L)}{\Sigma L} = \frac{1,530 \text{ cm}^2}{152.27 \text{ cm}} = 10.048 \text{ cm}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}L)}{\Sigma L} = \frac{784 \text{ cm}^2}{152.27 \text{ cm}} = 5.149 \text{ cm}$$

center of gravity = (0, 10.05 cm, 5.15 cm)