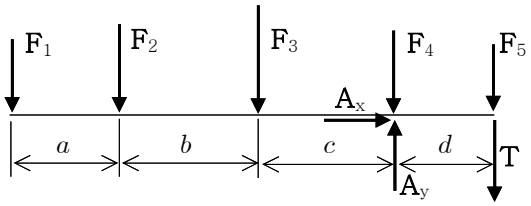
[4.1~4.5절]

4.8 [반력(힌지, 줄), 평형(모멘트, 힘)]　　F.B.D.

$F_1 = F_5 = 60$ N, $F_2 = F_4 = 80$ N, $F_3 = 140$ N

$a = d = 0.15$ m, $b = c = 0.2$ m



(a) $\rightarrow \Sigma F_x = 0$; $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$;

$$(a+b+c+d)\ F_1 + (b+c+d)F_2 + (c+d)\ F_3 + d\ (F_4 - A_y) = 0$$

$$\Rightarrow A_y = \frac{1}{d}[(a+b+c+d)\ F_1 + (b+c+d)F_2 + (c+d)\ F_3 + d\ F_4]$$

$$= \frac{1}{0.15\text{ m}}[(0.7\text{ m})(60\text{ N}) + (0.55\text{ m})(80\text{ N}) + (0.35\text{ m})(140\text{ N}) + (0.15\text{ m})(80\text{ N})]$$

$$= \frac{147\text{ N}\cdot\text{m}}{0.15\text{ m}} = 980\text{ N} \qquad \Rightarrow \qquad \mathbf{A} = 980\text{ N}\uparrow$$

(b) <방법1>

$+\circlearrowleft \Sigma M_A = 0$;

$$(a+b+c)\ F_1 + (b+c)F_2 + c\ F_3 - d\ (F_5 + T) = 0$$

$$\Rightarrow T = \frac{1}{d}[(a+b+c)\ F_1 + (b+c)F_2 + c\ F_3 - d\ F_5]$$

$$= \frac{1}{0.15\text{ m}}[(0.55\text{ m})(60\text{ N}) + (0.4\text{ m})(80\text{ N}) + (0.2\text{ m})(140\text{ N}) - (0.15\text{ m})(60\text{ N})]$$

$$= \frac{84\text{ N}\cdot\text{m}}{0.15\text{ m}} = 560\text{ N} \qquad \Rightarrow \qquad T = 560\text{ N}$$

<방법2>

$\uparrow \Sigma F_y = 0$;

$$-F_1 - F_2 - F_3 - F_4 - F_5 + A_y - T = 0$$

$$\Rightarrow T = A_y - F_1 - F_2 - F_3 - F_4 - F_5$$

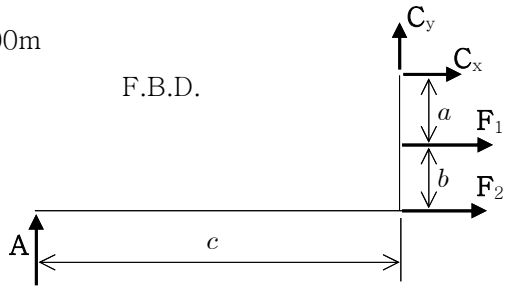
$$= (980\text{ N}) - (60\text{ N}) - (80\text{ N}) - (140\text{ N}) - (80\text{ N}) - (60\text{ N})$$

$$= 560\text{ N} \qquad \Rightarrow \qquad T = 560\text{ N}$$

4.21 [반력(롤러, 힌지), 평형(힘, 모멘트)]

$F_1 = F_2 = 300$ N, $a = b = 200$ mm, $c = 800$m

(a) $\alpha = 0$ F.B.D.



$+\circlearrowleft \Sigma M_C = 0$;

$a\ F_1 + (a+b)\ F_2 - c\ A = 0$

$\Rightarrow \quad A = \frac{1}{c}\ [a\ F_1 + (a+b)\ F_2]$

$= \frac{1}{800\ \text{mm}}\ [(200\ \text{mm})(300\ \text{N}) + (400\ \text{mm})(300\ \text{N})]$

$= 225$ N $\Rightarrow$ **A** = 225 N ↑

$\uparrow \Sigma F_y = 0$;

$A + C_y = 0 \quad \Rightarrow \quad C_y = -A = -225$ N

$\rightarrow \Sigma F_x = 0$;

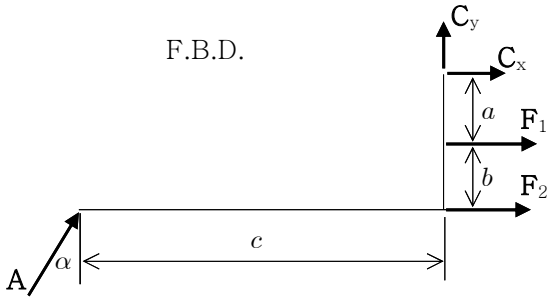
$C_x + F_1 + F_2 = 0 \quad \Rightarrow \quad C_x = -F_1 - F_2 = -600$ N

$C = \sqrt{C_x^{\,2} + C_y^{\,2}} = \sqrt{(-600\ \text{N})^2 + (-225\ \text{N})^2} = 640.8$ N

$\tan\theta = \frac{C_y}{C_x} = \frac{-225\ \text{N}}{-600\ \text{N}} = 0.375$

$\Rightarrow \quad \theta = \tan^{-1}(0.375) = 20.56° \quad \Rightarrow$ **C** = 641 N ↙20.6°

(b) $\alpha = 30°$ F.B.D.



$A_x = A\ \sin\alpha, \quad A_y = A\ \cos\alpha$

$+\circlearrowleft \Sigma M_C = 0$;

$a\ F_1 + (a+b)\ F_2 + (a+b)\ (A\ \sin\alpha) - c\ (A\ \cos\alpha) = 0$

$\Rightarrow \quad A = \frac{a\ F_1 + (a+b)\ F_2}{c\ \cos\alpha - (a+b)\sin\alpha}$

$= \frac{(200\ \text{mm})(300\ \text{N}) + (400\ \text{mm})(300\ \text{N})}{(800\ \text{mm})\cos 30^\circ - (400\ \text{mm})\sin 30^\circ}$

$= 365.2$ N $\Rightarrow$ **A** = 365 N ↗60.0°

$\uparrow \Sigma F_y = 0$;

$A_y + C_y = 0 \quad \Rightarrow \quad C_y = -A\ \cos\alpha = -(365.2\ \text{N})\ \cos 30° = -316.3$ N

$\rightarrow \Sigma F_x = 0$;

$C_x + F_1 + F_2 + A_x = 0$

$\Rightarrow \quad C_x = -F_1 - F_2 - A\ \sin\alpha = -600\ \text{N} - (365.2\ \text{N})\ \sin 30° = -782.6$ N

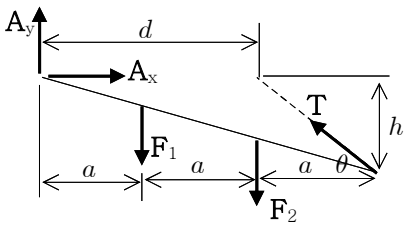
$C = \sqrt{C_x^{\,2} + C_y^{\,2}} = \sqrt{(-782.6\ \text{N})^2 + (-316.3\ \text{N})^2} = 844.1$ N

$\tan\theta = \frac{C_y}{C_x} = \frac{-316.3\ \text{N}}{-782.6\ \text{N}} = 0.4042$

$\Rightarrow \quad \theta = \tan^{-1}(0.4042) = 22.00° \quad \Rightarrow$ **C** = 844 N ↙22.0°

4.27 [반력(힌지, 줄), 평형(힘, 모멘트)]

F.B.D.



F_1 = 90 N, F_2 = 90 N, d = 200 mm,

h = 100 mm, a = 100 mm

$T_x = T\cos\theta$, $T_y = T\sin\theta$, $\theta = \tan^{-1}\dfrac{100}{100} = 45°$

(a) $+\circlearrowleft\Sigma M_A = 0$;

$(3a)\ T\sin\theta - h\ T\cos\theta - a\ F_1 - (2a)\ F_2 = 0$

$$\Rightarrow\quad T = \frac{a\,F_1 + (2a)\,F_2}{(3a)\sin\theta - h\cos\theta}$$

$$= \frac{(100\text{ mm})(90\text{ N}) + 2(100\text{ mm})(90\text{ N})}{3(100\text{ mm})\sin45^\circ - (100\text{ mm})\cos45^\circ} = 190.92\text{ N}$$

(b) $\rightarrow\Sigma F_x = 0$;

$A_x - T\cos\theta = 0$

$\Rightarrow\quad A_x = T\cos\theta = (190.92\text{ N})\cos45° = 135.0\text{ N}$

$\uparrow\Sigma F_y = 0$;

$A_y - F_1 - F_2 + T\sin\theta = 0$

$\Rightarrow\quad A_y = F_1 + F_2 - T\sin\theta = (90\text{ N}) + (90\text{ N}) - (190.92\text{ N})\sin45° = 45.00\text{ N}$

$$A = \sqrt{{A_x}^2 + {A_y}^2} = \sqrt{(135.0\text{ N})^2 + (45.0\text{ N})^2} = 142.30\text{ N}$$

$$\tan\theta = \frac{A_y}{A_x} = \frac{45.0\text{ N}}{135.0\text{ N}} = 0.3333 \quad\Rightarrow\quad \theta = \tan^{-1}(0.3333) = 18.43°$$

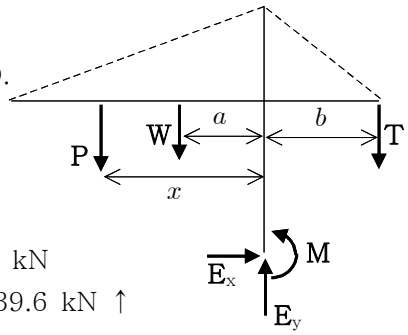
$$\Rightarrow\quad \mathbf{A} = 142.3\text{ N}\ \angle 18.4°$$

4.48 [반력(고정지지, 줄), 평형(힘, 모멘트)]

P = 16.2 kN, W = 5.4 kN, T = 18 kN

x = 4.8 m, a = 2.6 m, b = 1.5 m

(a)



$\rightarrow \Sigma F_x = 0$; $E_x = 0$

$\uparrow \Sigma F_y = 0$; $E_y - P - W - T = 0$

$\Rightarrow$ $E_y = P + W + T$

$= (16.2\ \text{kN}) + (5.4\ \text{kN}) + (18\ \text{kN}) = 39.6\ \text{kN}$

$\Rightarrow$ **E** = 39.6 kN ↑

$+\circlearrowleft \Sigma M_E = 0$; $M + x\ P + a\ W - b\ T = 0$

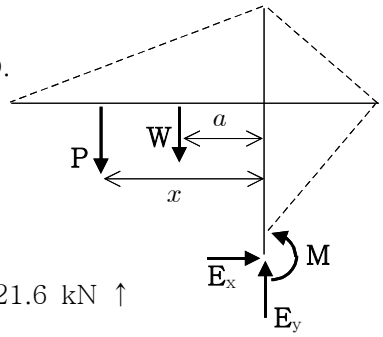
$\Rightarrow$ $M = -x\ P - a\ W + b\ T$

$= -(4.8\ \text{m})(16.2\ \text{kN}) - (2.6\ \text{m})(5.4\ \text{kN}) + (1.5\ \text{m})(18\ \text{kN})$

$= -64.8\ \text{kN·m}$

$\Rightarrow$ **M** = 64.8 kN·m ↺

(b)



$\rightarrow \Sigma F_x = 0$; $E_x = 0$

$\uparrow \Sigma F_y = 0$; $E_y - P - W = 0$

$\Rightarrow$ $E_y = P + W$

$= (16.2\ \text{kN}) + (5.4\ \text{kN}) = 21.6\ \text{kN}$

$\Rightarrow$ **E** = 21.6 kN ↑

$+\circlearrowleft \Sigma M_E = 0$; $M + x\ P + a\ W = 0$

$\Rightarrow$ $M = -x\ P - a\ W$

$= -(4.8\ \text{m})(16.2\ \text{kN}) - (2.6\ \text{m})(5.4\ \text{kN})$

$= -91.8\ \text{kN·m}$

$\Rightarrow$ **M** = 91.8 kN·m ↺