

[3.1~3.8절]

3.6 [한 점에 관한 모멘트 (2차원, 직각성분)]

$$P = 300 \text{ N}, \quad r = 200 \text{ mm}$$

$$\alpha = 40^\circ, \quad \beta = 30^\circ$$

$$(a) \quad x = r \cos \alpha = (200 \text{ mm}) \cos 40^\circ = 153.21 \text{ mm}$$

$$y = r \sin \alpha = (200 \text{ mm}) \sin 40^\circ = 128.56 \text{ mm}$$

$$\mathbf{r}_{A/O} = x \mathbf{i} + y \mathbf{j} = (153.21 \text{ mm}) \mathbf{i} + (128.56 \text{ mm}) \mathbf{j}$$

$$P_x = P \sin \beta = (300 \text{ N}) \sin 30^\circ = 150 \text{ N}$$

$$P_y = P \cos \beta = (300 \text{ N}) \cos 30^\circ = 260 \text{ N}$$

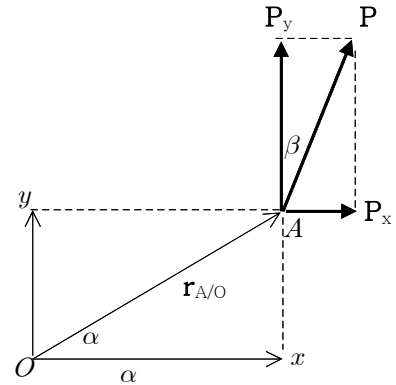
$$\mathbf{P} = P_x \mathbf{i} + P_y \mathbf{j} = (150 \text{ N}) \mathbf{i} + (260 \text{ N}) \mathbf{j}$$

$$\begin{aligned} M_O &= \mathbf{r}_{A/O} \times \mathbf{P} = [(153.21 \text{ mm}) \mathbf{i} + (128.56 \text{ mm}) \mathbf{j}] \times [(150 \text{ N}) \mathbf{i} + (260 \text{ N}) \mathbf{j}] \\ &= (153.21 \text{ mm}) \times (260 \text{ N}) \mathbf{k} - (128.56 \text{ mm}) \times (150 \text{ N}) \mathbf{k} \\ &= 20,551 \text{ N}\cdot\text{mm} \mathbf{k} = 20.551 \text{ N}\cdot\text{m} \mathbf{k} \quad \Rightarrow \quad 20.6 \text{ N}\cdot\text{m} \uparrow \end{aligned}$$

$$(b) \quad M_O = P d$$

$$\Rightarrow \quad d = \frac{M_O}{P} = \frac{20,551 \text{ N}\cdot\text{mm}}{300 \text{ N}} = 68.50 \text{ mm} \quad \Rightarrow \quad 68.5 \text{ mm}$$

F.B.D.



3.10 [모멘트 직각성분 (2차원, 직각성분)]

$$F = 2.5 \text{ kN}, \quad d_1 = 42 \text{ mm},$$

$$d_2 = 56 \text{ mm}, \quad d_3 = 88 \text{ mm}$$

$$\mathbf{r}_{B/C} = -d_1 \mathbf{i} - d_2 \mathbf{j} = -(42 \text{ mm}) \mathbf{i} - (56 \text{ mm}) \mathbf{j}$$

$$\tan \theta = \frac{d_2 + d_3}{d_1} = \frac{56 \text{ mm} + 88 \text{ mm}}{42 \text{ mm}} = 3.429$$

$$\Rightarrow \quad \theta = \tan^{-1}(3.429) = 73.74^\circ$$

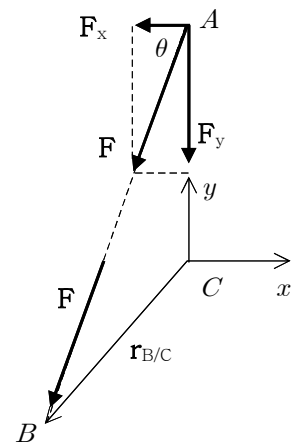
$$F_x = F \cos \theta = (2.5 \text{ kN}) \cos 73.74^\circ = 0.700 \text{ kN}$$

$$F_y = F \sin \theta = (2.5 \text{ kN}) \sin 73.74^\circ = 2.400 \text{ kN}$$

$$\mathbf{F} = -F_x \mathbf{i} - F_y \mathbf{j} = -(0.700 \text{ kN}) \mathbf{i} - (2.400 \text{ kN}) \mathbf{j}$$

$$\begin{aligned} M_C &= \mathbf{r}_{B/C} \times \mathbf{F} = [-(42 \text{ mm}) \mathbf{i} - (56 \text{ mm}) \mathbf{j}] \times [-(0.700 \text{ kN}) \mathbf{i} - (2.400 \text{ kN}) \mathbf{j}] \\ &= (42 \text{ mm})(2.400 \text{ kN}) \mathbf{k} - (56 \text{ mm})(0.700 \text{ kN}) \mathbf{k} \\ &= 61.60 \text{ kN}\cdot\text{mm} \mathbf{k} = 61.60 \text{ N}\cdot\text{m} \mathbf{k} \quad \Rightarrow \quad M_C = 61.6 \text{ N}\cdot\text{m} \uparrow \end{aligned}$$

F.B.D.



3.24 [합력의 모멘트의 직각성분 (3차원, 직각성분)]

$$T = 900 \text{ N}$$

$$\mathbf{r}_{OB} = 2.5 \mathbf{i} + 2.0 \mathbf{j} \text{ (m)}$$

$$(d_{BD})_x = -1 \text{ m}, \quad (d_{BD})_y = -2 \text{ m}, \quad (d_{BD})_z = 2 \text{ m}$$

$$d_{BD} = \sqrt{(-1 \text{ m})^2 + (-2 \text{ m})^2 + (2 \text{ m})^2} = 3 \text{ m}$$

$$\lambda_{BD} = \frac{1}{3} (-1 \mathbf{i} - 2 \mathbf{j} + 2 \mathbf{k})$$

$$\mathbf{F}_{BD} = T \lambda_{BD}$$

$$= (900 \text{ N}) \frac{1}{3} (-1 \mathbf{i} - 2 \mathbf{j} + 2 \mathbf{k})$$

$$= -300 \mathbf{i} - 600 \mathbf{j} + 600 \mathbf{k} \text{ (N)}$$

$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}_{BD}$$

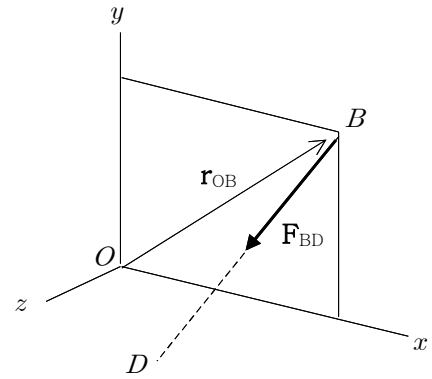
$$= [2.5 \mathbf{i} + 2.0 \mathbf{j} \text{ (m)}] \times [-300 \mathbf{i} - 600 \mathbf{j} + 600 \mathbf{k} \text{ (N)}]$$

$$= [(2.0)(600)] \mathbf{i} + [-(2.5)(600)] \mathbf{j} + [(2.5)(-600) - (2.0)(-300)] \mathbf{k} \text{ (N}\cdot\text{m)}$$

$$= (1,200) \mathbf{i} + (-1,500) \mathbf{j} + (-900) \mathbf{k} \text{ (N}\cdot\text{m)}$$

$$\Rightarrow \mathbf{M}_O = (1,200 \text{ N}\cdot\text{m}) \mathbf{i} + (-1,500 \text{ N}\cdot\text{m}) \mathbf{j} + (-900 \text{ N}\cdot\text{m}) \mathbf{k}$$

F.B.D.



3.32 [점에 관한 모멘트 (3차원)]

$$3.24 \quad T = 900 \text{ N}$$

$$M_O = \sqrt{(1,200)^2 + (-1,500)^2 + (-900)^2} \text{ (N}\cdot\text{m)} = 2,121 \text{ (N}\cdot\text{m)}$$

$$M_O = T d$$

$$\Rightarrow d = \frac{M_O}{T} = \frac{2,121 \text{ N}\cdot\text{m}}{900 \text{ N}} = 2.357 \text{ m} \quad \Rightarrow \quad d = 2.36 \text{ m}$$

[3.9~3.11절]

3.42 [스칼라곱을 응용한 각도 산출, 투영, 좌표로부터 벡터 표현]

$$x_E = \frac{32 \text{ m}}{2} = 16 \text{ m}, \quad y_E = \frac{16.5 \text{ m} + 7.5 \text{ m}}{2} = 12 \text{ m}, \quad z_E = \frac{-24 \text{ m}}{2} = -12 \text{ m}$$

$$T_{EG} = 178 \text{ N}$$

$$(a) \mathbf{r}_{C/B} = 32 \mathbf{i} + (7.5-16.5) \mathbf{j} - 24 \mathbf{k} \text{ (m)}$$

$$= 32 \mathbf{i} - 9 \mathbf{j} - 24 \mathbf{k} \text{ (m)}$$

$$r_{C/B} = \sqrt{(32)^2 + (-9)^2 + (-24)^2} \text{ m} = 41 \text{ m}$$

$$\mathbf{r}_{G/E} = (32-16) \mathbf{i} + (-12) \mathbf{j} + [-2.25-(-12)] \mathbf{k} \text{ (m)}$$

$$= 16 \mathbf{i} - 12 \mathbf{j} + 9.75 \mathbf{k} \text{ (m)}$$

$$r_{G/E} = \sqrt{(16)^2 + (-12)^2 + (9.75)^2} \text{ m} = 22.25 \text{ m}$$

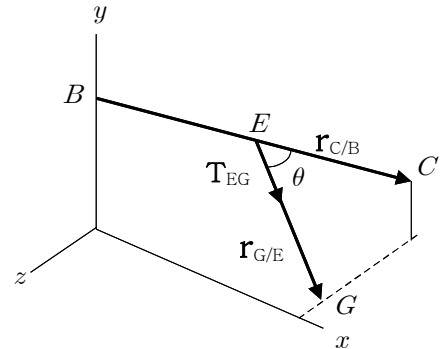
$$\mathbf{r}_{C/B} \cdot \mathbf{r}_{G/E} = (32 \mathbf{i} - 9 \mathbf{j} - 24 \mathbf{k}) \cdot (16 \mathbf{i} - 12 \mathbf{j} + 9.75 \mathbf{k}) \text{ (m}^2)$$

$$= (32)(16) + (-9)(-12) + (-24)(9.75) \text{ (m}^2) = 386 \text{ (m}^2)$$

$$\cos \theta = \frac{\mathbf{r}_{C/B} \cdot \mathbf{r}_{G/E}}{r_{C/B} r_{G/E}} = \frac{386 \text{ m}^2}{(41 \text{ m})(22.25 \text{ m})} = 0.4231$$

$$\Rightarrow \theta = \cos^{-1}(0.423) = 65.0^\circ$$

$$(b) (T_{EG})_{BC} = T_{EG} \cos \theta = (178 \text{ N})(0.4231) = 75.3 \text{ N}$$



3.47 [좌표축에 관한 모멘트]

$$T = 570 \text{ N}$$

$$\mathbf{r}_{OB} = 0.9 \mathbf{i} \text{ (m)}$$

$$(d_{BA})_x = -0.9 \text{ m}, \quad (d_{BA})_y = 0.6 \text{ m}, \quad (d_{BA})_z = 0.36 \text{ m}$$

$$d_{BA} = \sqrt{(-0.9 \text{ m})^2 + (0.6 \text{ m})^2 + (0.36 \text{ m})^2} = 1.14 \text{ m}$$

$$\lambda_{BA} = \frac{1}{1.14} (-0.9 \mathbf{i} + 0.6 \mathbf{j} + 0.36 \mathbf{k})$$

$$\mathbf{F}_{BA} = T \lambda_{BA}$$

$$= (570 \text{ N}) \frac{1}{1.14} (-0.9 \mathbf{i} + 0.6 \mathbf{j} + 0.36 \mathbf{k})$$

$$= -450 \mathbf{i} + 300 \mathbf{j} + 180 \mathbf{k} \text{ (N)}$$

$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}_{BD}$$

$$= [0.9 \mathbf{i} \text{ (m)}] \times [-450 \mathbf{i} + 300 \mathbf{j} + 180 \mathbf{k} \text{ (N)}]$$

$$= [0] \mathbf{i} + [-(0.9)(180)] \mathbf{j} + [(0.9)(300)] \mathbf{k} \text{ (N}\cdot\text{m)}$$

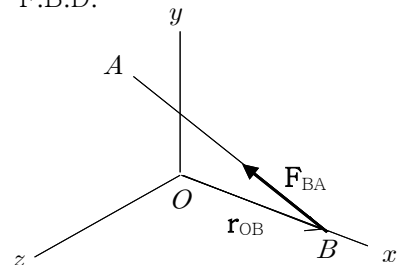
$$= (0) \mathbf{i} + (-162) \mathbf{j} + (270) \mathbf{k} \text{ (N}\cdot\text{m)}$$

$$\Rightarrow M_x = \mathbf{M}_O \cdot \mathbf{i} = [(0) \mathbf{i} + (-162) \mathbf{j} + (270) \mathbf{k} \text{ (N}\cdot\text{m)}] \cdot \mathbf{i} = 0$$

$$M_y = \mathbf{M}_O \cdot \mathbf{j} = [(0) \mathbf{i} + (-162) \mathbf{j} + (270) \mathbf{k} \text{ (N}\cdot\text{m)}] \cdot \mathbf{j} = -162.0 \text{ N}\cdot\text{m}$$

$$M_z = \mathbf{M}_O \cdot \mathbf{k} = [(0) \mathbf{i} + (-162) \mathbf{j} + (270) \mathbf{k} \text{ (N}\cdot\text{m)}] \cdot \mathbf{k} = 270 \text{ N}\cdot\text{m}$$

F.B.D.



3.53 [점에 관한 모멘트 응용]

F.B.D.

$$M_x = 20 \text{ N}\cdot\text{m}, \quad M_y = -8.75 \text{ N}\cdot\text{m}, \quad M_z = -30 \text{ N}\cdot\text{m}$$

$$\mathbf{r}_{OC} = (0.15+0.10 \text{ m})\mathbf{i} + (0.20 \text{ m}) \sin\theta \mathbf{j} + (0.20 \text{ m}) \cos\theta \mathbf{k}$$

$$\mathbf{P} = -P \sin\phi \mathbf{j} + P \cos\phi \mathbf{k}$$

$$\begin{aligned} \mathbf{M}_O &= \mathbf{r}_{OC} \times \mathbf{P} = [0.25 \mathbf{i} + 0.20 \sin\theta \mathbf{j} + 0.20 \cos\theta \mathbf{k} \text{ (m)}] \times [-P \sin\phi \mathbf{j} + P \cos\phi \mathbf{k}] \\ &= [(0.20 \text{ m}) \sin\theta (P \cos\phi) - (0.20 \text{ m}) \cos\theta (-P \sin\phi)]\mathbf{i} \\ &\quad + [-(0.25 \text{ m})(P \cos\phi)]\mathbf{j} + [(0.25 \text{ m})(-P \sin\phi)]\mathbf{k} \\ &= [(0.20 \text{ m}) P (\sin\theta \cos\phi + \cos\theta \sin\phi)] \mathbf{i} \\ &\quad + [-(0.25 \text{ m})(P \cos\phi)] \mathbf{j} + [-(0.25 \text{ m})(P \sin\phi)]\mathbf{k} \end{aligned}$$

$$M_x = (0.20 \text{ m}) P \sin(\theta + \phi) = 20 \text{ N}\cdot\text{m} \quad \dots \text{ ①}$$

$$M_y = -(0.25 \text{ m}) P \cos\phi = -8.75 \text{ N}\cdot\text{m} \quad \dots \text{ ②}$$

$$M_z = -(0.25 \text{ m}) P \sin\phi = -30 \text{ N}\cdot\text{m} \quad \dots \text{ ③}$$

$$\text{②}^2 + \text{③}^2$$

$$(0.25 \text{ m})^2 P^2 = (-8.75 \text{ N}\cdot\text{m})^2 + (-30 \text{ N}\cdot\text{m})^2 = 976.56 (\text{N}\cdot\text{m})^2$$

$$\Rightarrow P^2 = \frac{976.56 (\text{N}\cdot\text{m})^2}{(0.25 \text{ m})^2} = 15,625 \text{ N}^2 \quad \Rightarrow P = 125.0 \text{ N}$$

$$\text{③} \Rightarrow \sin\phi = \frac{-30 \text{ N}\cdot\text{m}}{(-0.25 \text{ m})(125.0 \text{ N})} = 0.96 \quad \Rightarrow \phi = \sin^{-1}(0.96) = 73.7^\circ$$

$$\text{①} \Rightarrow \sin(\theta + \phi) = \frac{20 \text{ N}\cdot\text{m}}{(0.20 \text{ m})(125.0 \text{ N})} = 0.80$$

$$\Rightarrow \theta + \phi = \sin^{-1}(0.80) = 53.1^\circ \text{ 또는 } 126.9^\circ$$

$$\theta + \phi = 53.1^\circ \text{ 일 때} \quad \theta = 53.1^\circ - 73.7^\circ = 20.6^\circ \text{ 타당하지 않음}$$

$$\theta + \phi = 126.9^\circ \text{ 일 때} \quad \theta = 126.9^\circ - 73.7^\circ = 53.2^\circ \quad \Rightarrow \theta = 53.2^\circ$$

[3.12~3.16절]

3.74 [우력 모멘트, 우력 벡터 합성]

$$F_B = F_C = F = 40 \text{ N},$$

$$\alpha = 55^\circ, \quad \beta = 20^\circ,$$

$$l_1 = 0.390 \text{ m}, \quad l_2 = 0.270 \text{ m}$$

$$(a) \quad d_x = l_2 \cos\alpha = (0.270 \text{ m}) \cos 55^\circ = 0.15486 \text{ m}$$

$$d_y = l_2 \sin\alpha = (0.270 \text{ m}) \sin 55^\circ = 0.2212 \text{ m}$$

$$F_x = F \cos\beta = (40 \text{ N}) \cos 20^\circ = 37.59 \text{ N}$$

$$F_y = F \sin\beta = (40 \text{ N}) \sin 20^\circ = 13.68 \text{ N}$$

$$M = d_x F_y - d_y F_x$$

$$= (0.15486 \text{ m})(13.68 \text{ N}) - (0.2212 \text{ m})(37.59 \text{ N}) = -6.196 \text{ N}\cdot\text{m}$$

$$\Rightarrow \mathbf{M} = 6.20 \text{ N}\cdot\text{m} \uparrow$$

$$(b) \quad \theta = \alpha - \beta = 55^\circ - 20^\circ = 35^\circ$$

$$d = l_2 \sin\theta = (0.270 \text{ m}) \sin 35^\circ = 0.1549 \text{ m}$$

$$M = -d F = -(0.1549 \text{ m})(40 \text{ N}) = -6.194 \text{ N}\cdot\text{m}$$

$$\Rightarrow \mathbf{M} = 6.19 \text{ N}\cdot\text{m} \uparrow$$

$$(c) \quad l = l_1 + l_2 = (0.390 \text{ m}) + (0.270 \text{ m}) = 0.660 \text{ m}$$

$$\mathbf{r}_{AB} = l_1 (\cos\alpha \mathbf{i} + \sin\alpha \mathbf{j})$$

$$= (0.390 \text{ m}) (\cos 55^\circ \mathbf{i} + \sin 55^\circ \mathbf{j}) = 0.2237 \mathbf{i} + 0.3194 \mathbf{j} \text{ (m)}$$

$$\mathbf{r}_{AC} = l (\cos\alpha \mathbf{i} + \sin\alpha \mathbf{j})$$

$$= (0.660 \text{ m}) (\cos 55^\circ \mathbf{i} + \sin 55^\circ \mathbf{j}) = 0.3786 \mathbf{i} + 0.5406 \mathbf{j} \text{ (m)}$$

$$\mathbf{F}_B = -F_x \mathbf{i} - F_y \mathbf{j} = -37.59 \mathbf{i} - 13.68 \mathbf{j} \text{ (N)}$$

$$\mathbf{F}_C = F_x \mathbf{i} + F_y \mathbf{j} = 37.59 \mathbf{i} + 13.68 \mathbf{j} \text{ (N)}$$

$$\Sigma \mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}_B + \mathbf{r}_{AC} \times \mathbf{F}_C$$

$$= [0.2237 \mathbf{i} + 0.3194 \mathbf{j} \text{ (m)}] \times [-37.59 \mathbf{i} - 13.68 \mathbf{j} \text{ (N)}]$$

$$+ [0.3786 \mathbf{i} + 0.5406 \mathbf{j} \text{ (m)}] \times [37.59 \mathbf{i} + 13.68 \mathbf{j} \text{ (N)}]$$

$$= \{[(0.2237)(-13.68) - (0.3194)(-37.59)]$$

$$+ [(0.3786)(13.68) - (0.5406)(37.59)]\} \mathbf{k} \text{ (N}\cdot\text{m)}$$

$$= (8.946 - 15.142) \mathbf{k} \text{ (N}\cdot\text{m)} = -6.196 \mathbf{k} \text{ (N}\cdot\text{m)}$$

$$\Rightarrow \mathbf{M} = 6.20 \text{ N}\cdot\text{m} \uparrow$$

<다른 방법>

$$\Sigma \mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}_B + \mathbf{r}_{AC} \times \mathbf{F}_C$$

$$= [l_1 (\cos\alpha \mathbf{i} + \sin\alpha \mathbf{j})] \times [-F_x \mathbf{i} - F_y \mathbf{j}]$$

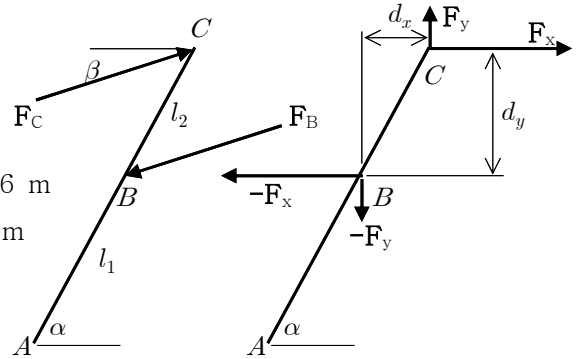
$$+ [l (\cos\alpha \mathbf{i} + \sin\alpha \mathbf{j})] \times [F_x \mathbf{i} + F_y \mathbf{j}]$$

$$= [(l-l_1) (\cos\alpha \mathbf{i} + \sin\alpha \mathbf{j})] \times [F_x \mathbf{i} + F_y \mathbf{j}]$$

$$= (l_2 \cos\alpha F_y - l_2 \sin\alpha F_x) \mathbf{k} = (d_x F_y - d_y F_x) \mathbf{k}$$

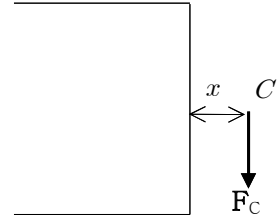
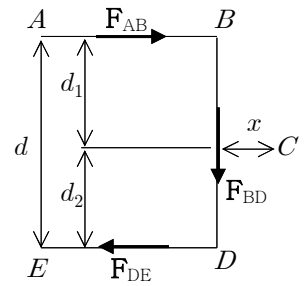
$$= [(0.1549 \text{ m})(13.68 \text{ N}) - (0.2212 \text{ m})(37.59 \text{ N})] \mathbf{k} = -6.196 \text{ N}\cdot\text{m} \mathbf{k}$$

$$\Rightarrow \mathbf{M} = 6.20 \text{ N}\cdot\text{m} \uparrow$$



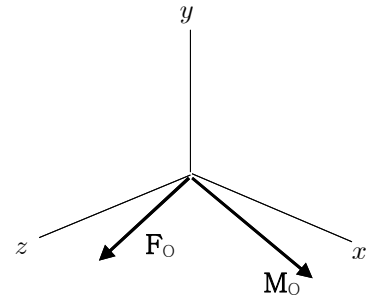
3.91 [2차원 등가 힘-우력]

$$\begin{aligned}
 F_{AB} &= F_{DE} = F = 250 \text{ N}, \\
 d_1 &= d_2 = 0.090 \text{ m}, \quad d = d_1 + d_2 = 0.180 \text{ m} \\
 \mathbf{F}_{AB} &= 250 \text{ N } \rightarrow, \quad \mathbf{F}_{DE} = 250 \text{ N } \leftarrow, \quad \mathbf{F}_{BD} = 900 \text{ N } \downarrow \\
 \Sigma \mathbf{F} : \quad \mathbf{F}_C &= \mathbf{F}_{AB} + \mathbf{F}_{DE} + \mathbf{F}_{BD} \\
 &= (250 \text{ N } \rightarrow) + (250 \text{ N } \leftarrow) + (900 \text{ N } \downarrow) \\
 &\Rightarrow \mathbf{F}_C = 900 \text{ N } \downarrow \\
 \curvearrowright \Sigma M_C : 0 &= -d_1 F_{AB} - d_2 F_{DE} + x F_{BD} \\
 &= -d F + x F_{BD} \\
 x &= d \frac{F}{F_{BD}} = (0.180 \text{ m}) \frac{250 \text{ N}}{900 \text{ N}} = 0.050 \text{ m} \\
 &\Rightarrow x = 50.0 \text{ mm}
 \end{aligned}$$



3.93 [3차원 등가 힘-우력]

$$\begin{aligned}
 F_{AB} &= 1.44 \text{ kN}, \\
 (d_{AB})_x &= -16 \text{ m}, \quad (d_{AB})_y = -32 \text{ m}, \quad (d_{AB})_z = 4 \text{ m} \\
 d_{AB} &= \sqrt{(-16 \text{ m})^2 + (-32 \text{ m})^2 + (4 \text{ m})^2} = 36 \text{ m} \\
 \lambda_{AB} &= \frac{1}{36} (-16 \mathbf{i} - 32 \mathbf{j} + 4 \mathbf{k}) \\
 \mathbf{F}_{AB} &= F_{AB} \lambda_{AB} \\
 &= (1.44 \text{ kN}) \frac{1}{36} (-16 \mathbf{i} - 32 \mathbf{j} + 4 \mathbf{k}) \\
 &= -0.64 \mathbf{i} - 1.28 \mathbf{j} + 0.16 \mathbf{k} \text{ (kN)} \\
 \Rightarrow \mathbf{F}_O &= (-0.640 \text{ kN}) \mathbf{i} + (-1.280 \text{ kN}) \mathbf{j} + (0.1600 \text{ kN}) \mathbf{k} \\
 \mathbf{r}_{A/O} &= (32 \text{ m}) \mathbf{j} \\
 \mathbf{M}_O &= \mathbf{r}_{A/O} \times \mathbf{F}_{AB} \\
 &= [(32 \text{ m}) \mathbf{j}] \times [-0.64 \mathbf{i} - 1.28 \mathbf{j} + 0.16 \mathbf{k} \text{ (kN)}] \\
 &= [(32 \text{ m})(0.16 \text{ kN})] \mathbf{i} + [0] \mathbf{j} + [-(32 \text{ m})(-0.64 \text{ kN})] \mathbf{k} \\
 &= (5.12 \text{ kN}\cdot\text{m}) \mathbf{i} + (20.48 \text{ kN}\cdot\text{m}) \mathbf{k} \\
 \Rightarrow \mathbf{M}_O &= (5.12 \text{ kN}\cdot\text{m}) \mathbf{i} + (20.5 \text{ kN}\cdot\text{m}) \mathbf{k}
 \end{aligned}$$



<3.17~3.20절>

생 략