

<9.1~9.5절>

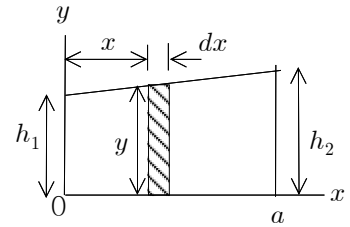
$$9.1\&5 \quad y = h_1 + \frac{h_2 - h_1}{a} x$$

$$dI_y = x^2 dA = x^2 y dx = x^2 \left[h_1 + \frac{h_2 - h_1}{a} x \right] dx$$

$$I_y = \int dI_y = \int_0^a \left[h_1 x^2 + \frac{h_2 - h_1}{a} x^3 \right] dx$$

$$= h_1 \frac{a^3}{3} + \frac{h_2 - h_1}{a} \frac{a^4}{4}$$

$$= \frac{1}{12} a^3 h_1 + \frac{1}{4} a^3 h_2 = \frac{1}{12} a^3 (h_1 + 3h_2)$$



$$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3} \left(h_1 + \frac{h_2 - h_1}{a} x \right)^3 dx$$

$$I_x = \int_0^a \frac{1}{3} \left(h_1 + \frac{h_2 - h_1}{a} x \right)^3 dx = \frac{1}{3} \left[\frac{1}{4} \frac{a}{h_2 - h_1} \left(h_1 + \frac{h_2 - h_1}{a} x \right)^4 \right]_0^a$$

$$= \frac{1}{12} \frac{a}{h_2 - h_1} (h_2^4 - h_1^4) = \frac{1}{12} a (h_1 + h_2) (h_1^2 + h_2^2)$$

$$9.4\&8 \quad y = kx^4 \quad (a, b) \Rightarrow b = ka^4$$

$$\Rightarrow k = \frac{b}{a^4} \Rightarrow y = \frac{b}{a^4} x^4$$

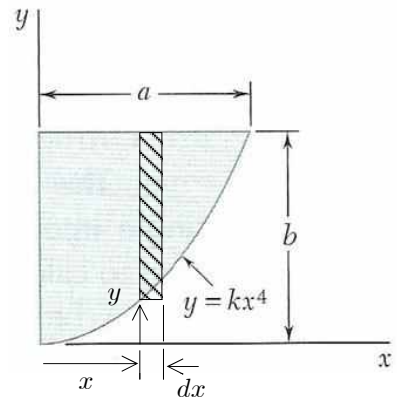
$$dA = (b - y) dx$$

$$dI_y = x^2 dA = x^2 (b - y) dx$$

$$I_y = \int dI_y = \int_0^a x^2 (b - y) dx = \int_0^a x^2 \left(b - \frac{b}{a^4} x^4 \right) dx$$

$$= \int_0^a \left(bx^2 - \frac{b}{a^4} x^6 \right) dx = \left[\frac{b}{3} x^3 - \frac{b}{7a^4} x^7 \right]_0^a$$

$$= \frac{b}{3} a^3 - \frac{b}{7a^4} a^7 = \frac{4}{21} a^3 b$$



$$dI_x = \frac{1}{3} b^3 dx - \frac{1}{3} y^3 dx$$

$$I_x = \int dI_x = \int_0^a \frac{1}{3} b^3 dx - \int_0^a \frac{1}{3} y^3 dx = \frac{b^3}{3} \int_0^a dx - \frac{1}{3} \int_0^a \left(\frac{b}{a^4} x^4 \right)^3 dx$$

$$= \frac{b^3}{3} a - \frac{1}{3} \frac{b^3}{a^{12}} \left(\frac{1}{13} a^{13} \right) = \frac{1}{3} ab^3 \left(1 - \frac{1}{13} \right) = \frac{4}{13} ab^3$$

$$9.21 \quad x = \frac{a}{2} + \frac{1}{2}y$$

$$dA = 2x dy = (a + y) dy$$

면적

$$A = \frac{1}{2} (2a + a)(a) = \frac{3}{2} a^2$$

$$\text{또는 } A = \int_0^a dA = \int_0^a (a + y) dy \\ = (a)(a) + \frac{1}{2} a^2 = \frac{3}{2} a^2$$

I_x

$$dI_x = y^2 dA = y^2 (a + y) dy$$

$$I_x = \int_0^a dI_x = \int_0^a y^2 (a + y) dy = a \left[\frac{1}{3} y^3 \right]_0^a + \left[\frac{1}{4} y^4 \right]_0^a = \frac{7}{12} a^4$$

I_y

$$dI_y = 2 \left(\frac{1}{3} x^3 dy \right) = \frac{2}{3} \left(\frac{a}{2} + \frac{1}{2} y \right)^3 dy = \frac{1}{12} (a + y)^3 dy$$

$$I_y = \frac{1}{12} \int_0^a (a + y)^3 dy = \frac{1}{12} \left[\frac{1}{4} (a + y)^4 \right]_0^a = \frac{1}{48} [(2a)^4 - (a)^4] = \frac{5}{16} a^4$$

$$J_P = I_x + I_y = \frac{7}{12} a^4 + \frac{5}{16} a^4 = \frac{28 + 15}{48} a^4 = \frac{43}{48} a^4$$

$$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{43}{48} a^4}{\frac{3}{2} a^2}} = \sqrt{\frac{43}{72}} a = 0.773 a$$

