

<5.10~5.11절>

5.96 ① 원뿔(cone)

$$V = \frac{1}{3} \pi a^2 h$$

$$\bar{y} = \frac{1}{4} h$$

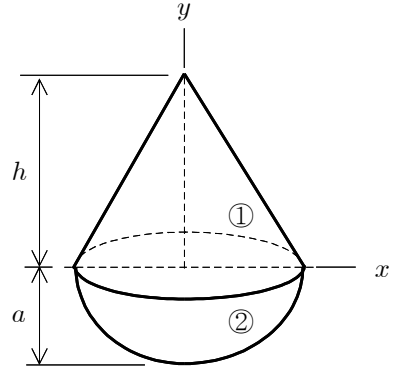
② 반구(hemisphere)

$$V = \frac{1}{2} \left(\frac{4}{3} \pi a^3 \right) = \frac{2}{3} \pi a^3$$

$$\bar{y} = -\frac{3}{8} a$$

$$\Sigma V = \frac{1}{3} \pi a^2 h + \frac{2}{3} \pi a^3 = \frac{1}{3} \pi a^2 (h + 2a)$$

$$\Sigma(\bar{y}V) = \left(\frac{1}{4} h \right) \left(\frac{1}{3} \pi a^2 h \right) + \left(-\frac{3}{8} a \right) \left(\frac{2}{3} \pi a^3 \right) = \frac{1}{12} \pi a^2 (h^2 - 3a^2)$$



(a) $h = \frac{3}{2} a$ 일 때

$$\Sigma V = \frac{1}{3} \pi a^2 \left(\frac{3}{2} a + 2a \right) = \frac{7}{6} \pi a^3$$

$$\Sigma(\bar{y}V) = \frac{1}{12} \pi a^2 \left[\left(\frac{3}{2} a \right)^2 - 3a^2 \right] = -\frac{1}{16} \pi a^4$$

$$\bar{Y} = \frac{\Sigma(\bar{y}V)}{\Sigma V} = \frac{-\frac{1}{16} \pi a^4}{\frac{7}{6} \pi a^3} = -\frac{3}{56} a = -0.05357 a$$

⇒ 도심은 원뿔 밑면에서 아래쪽으로 0.0536 a 거리에 위치함.

(a) $h = 2a$ 일 때

$$\Sigma V = \frac{1}{3} \pi a^2 (2a + 2a) = \frac{4}{3} \pi a^3$$

$$\Sigma(\bar{y}V) = \frac{1}{12} \pi a^2 [(2a)^2 - 3a^2] = \frac{1}{12} \pi a^4$$

$$\bar{Y} = \frac{\Sigma(\bar{y}V)}{\Sigma V} = \frac{\frac{1}{12} \pi a^4}{\frac{4}{3} \pi a^3} = \frac{1}{16} a = 0.0625 a$$

⇒ 도심은 원뿔 밑면에서 위쪽으로 0.0625 a 거리에 위치함.

5.107 박판은 균질[homogeneous]이어서 무게중심이 도심과 일치한다고 가정함.

$$\textcircled{1} A = \frac{1}{2}(0.2 \text{ m})(0.12 \text{ m}) = 0.012 \text{ m}^2$$

$$\bar{x} = 0$$

$$\bar{y} = (0.18 \text{ m}) + \frac{1}{3}(0.12 \text{ m}) = 0.22 \text{ m}$$

$$\bar{z} = \frac{1}{3}(0.2 \text{ m}) = \frac{0.2}{3} \text{ m}$$

$$\textcircled{2} A = \frac{1}{4}[2\pi(0.18 \text{ m})](0.2 \text{ m}) = 0.018\pi \text{ m}^2$$

$$\bar{x} = \frac{2}{\pi}r = \frac{2}{\pi}(0.18 \text{ m}) = \frac{0.36}{\pi} \text{ m}$$

$$\bar{y} = \bar{x} = \frac{0.36}{\pi} \text{ m}$$

$$\bar{z} = \frac{1}{2}(0.2 \text{ m}) = 0.1 \text{ m}$$

$$\textcircled{3} A = (0.2 \text{ m})(0.16 \text{ m}) = 0.032 \text{ m}^2$$

$$\bar{x} = (0.18 \text{ m}) + \frac{1}{2}(0.16 \text{ m}) = 0.26 \text{ m}$$

$$\bar{y} = 0$$

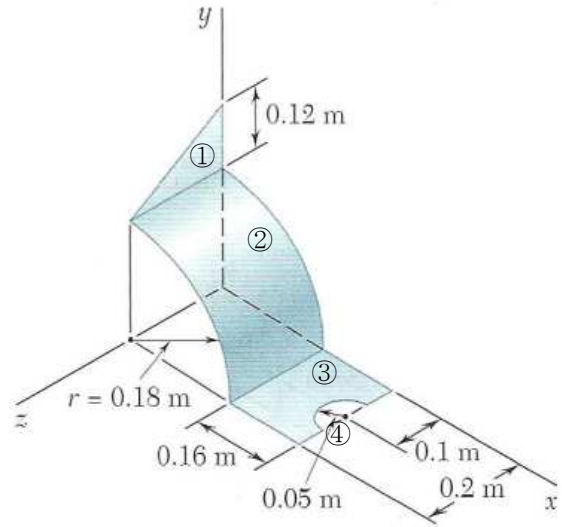
$$\bar{z} = 0.1 \text{ m}$$

$$\textcircled{4} A = -\frac{1}{2}[\pi(0.05 \text{ m})^2] = -0.00125\pi \text{ m}^2$$

$$\bar{x} = (0.18 + 0.16 \text{ m}) - \frac{4}{3\pi}(0.05 \text{ m}) = 0.3188 \text{ m}$$

$$\bar{y} = 0$$

$$\bar{z} = 0.1 \text{ m}$$



$$\Sigma A = 0.012 + 0.018\pi + 0.032 - 0.00125\pi = 0.09662 \text{ m}^2$$

$$\begin{aligned} \Sigma(\bar{x}A) &= (0)(0.012) + \left(\frac{0.36}{\pi}\right)(0.018\pi) + (0.26)(0.032) + (0.3188)(-0.00125\pi) \\ &= 0.013548 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \Sigma(\bar{y}A) &= (0.22)(0.012) + \left(\frac{0.36}{\pi}\right)(0.018\pi) + (0)(0.032) + (0)(-0.00125\pi) \\ &= 0.009120 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \Sigma(\bar{z}A) &= \left(\frac{0.2}{3}\right)(0.012) + (0.1)(0.018\pi) + (0.1)(0.032) + (0.1)(-0.00125\pi) \\ &= 0.009262 \text{ m}^3 \end{aligned}$$

$$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{0.013548 \text{ m}^3}{0.09662 \text{ m}^2} = 0.1402 \text{ m}$$

$$\bar{Y} = \frac{\Sigma(\bar{y}A)}{\Sigma A} = \frac{0.009120 \text{ m}^3}{0.09662 \text{ m}^2} = 0.0944 \text{ m}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}A)}{\Sigma A} = \frac{0.009262 \text{ m}^3}{0.09662 \text{ m}^2} = 0.0959 \text{ m}$$

$$5.116 \quad \textcircled{1} \quad L = \sqrt{(2.4 \text{ m})^2 + (1.0 \text{ m})^2} = 2.6 \text{ m}$$

$$\bar{x} = \frac{1}{2}(2.4 \text{ m}) = 1.2 \text{ m}$$

$$\bar{y} = \frac{1}{2}(1.0 \text{ m}) = 0.5 \text{ m}$$

$$\bar{z} = 0$$

$$\textcircled{2} \quad L = \frac{1}{4}[2\pi(2.4 \text{ m})] = 1.2\pi \text{ m}$$

$$\bar{x} = \frac{2}{\pi}(2.4 \text{ m}) = \frac{4.8}{\pi} \text{ m}$$

$$\bar{y} = 0$$

$$\bar{z} = \bar{x} = \frac{4.8}{\pi} \text{ m}$$

$$\textcircled{3} \quad L = 2.4 \text{ m}$$

$$\bar{x} = 0$$

$$\bar{y} = 0$$

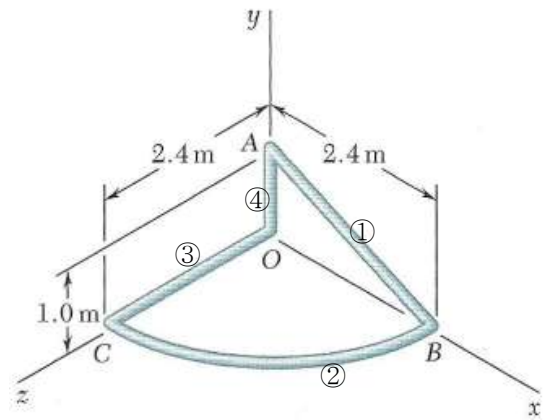
$$\bar{z} = \frac{1}{2}(2.4 \text{ m}) = 1.2 \text{ m}$$

$$\textcircled{4} \quad L = 1.0 \text{ m}$$

$$\bar{x} = 0$$

$$\bar{y} = \frac{1}{2}(1.0 \text{ m}) = 0.5 \text{ m}$$

$$\bar{z} = 0$$



$$\Sigma L = 2.6 + 1.2\pi + 2.4 + 1.0 \text{ m} = 9.7699 \text{ m}$$

$$\Sigma(\bar{x}L) = (1.2)(2.6) + \left(\frac{4.8}{\pi}\right)(1.2\pi) + (0)(2.4) + (0)(1.0) \text{ m}^2 = 8.88 \text{ m}^2$$

$$\Sigma(\bar{y}L) = (0.5)(2.6) + (0)(1.2\pi) + (0)(2.4) + (0.5)(1.0) \text{ m}^2 = 1.80 \text{ m}^2$$

$$\Sigma(\bar{z}L) = (0)(2.6) + \left(\frac{4.8}{\pi}\right)(1.2\pi) + (1.2)(2.4) + (0)(1.0) \text{ m}^2 = 8.64 \text{ m}^2$$

$$\bar{X} = \frac{\Sigma(\bar{x}L)}{\Sigma L} = \frac{8.88 \text{ m}^2}{9.7699 \text{ m}} = 0.9089 \text{ m} \quad \Rightarrow \quad \bar{X} = 0.909 \text{ m}$$

$$\bar{Y} = \frac{\Sigma(\bar{y}L)}{\Sigma L} = \frac{1.80 \text{ m}^2}{9.7699 \text{ m}} = 0.1842 \text{ m} \quad \Rightarrow \quad \bar{Y} = 0.1842 \text{ m}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}L)}{\Sigma L} = \frac{8.64 \text{ m}^2}{9.7699 \text{ m}} = 0.8843 \text{ m} \quad \Rightarrow \quad \bar{Z} = 0.884 \text{ m}$$

center of gravity = (0.909 m, 0.1842 m, 0.884 m)