<5.1~5.5절>

5.8 ① 직사각형

A = (60 mm)(120 mm) = 7,200 mm^2

$\bar{x}$ = $-\frac{1}{2}$(60 mm) = -30 mm

$\bar{y}$ = 60 mm

② $\frac{1}{4}$원

A = $\frac{1}{4}\pi$(60 mm)^2 = 2827.4 mm^2

$\bar{x}$ = $\frac{4}{3\pi}$(60 mm) = 25.46 mm

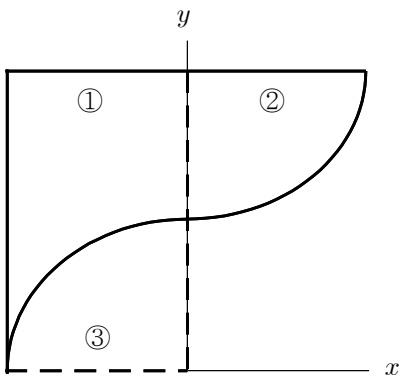
$\bar{y}$ = (120 mm) - $\frac{4}{3\pi}$(60 mm) = 94.54 mm

③ $\frac{1}{4}$원 구멍

A = - (②A) = -2827.4 mm^2

$\bar{x}$ = -(②$\bar{x}$) = -25.46 mm

$\bar{y}$ = (②$\bar{x}$) = 25.46 mm



ΣA = (7,200 mm^2) + (2827.4 mm^2) + (-2827.4 mm^2) = 7,200 mm^2

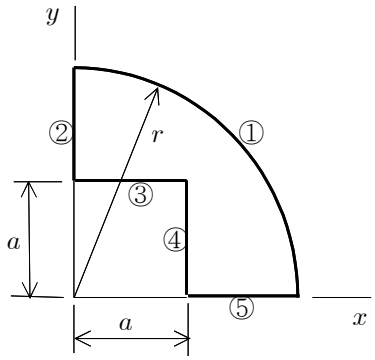
$\Sigma(\bar{x}A)$ = (-30 mm)(7,200 mm^2) + (25.46 mm)(2827.4 mm^2)
+ (-25.46 mm)(-2827.4 mm^2) = -72,029 mm^3

$\Sigma(\bar{y}A)$ = (60 mm)(7,200 mm^2) + (94.54 mm)(2827.4 mm^2)
+ (25.46 mm)(-2827.4 mm^2) = 627,317 mm^3

$\bar{X}$ = $\frac{\Sigma(\bar{x}A)}{\Sigma A}$ = $\frac{-72{,}029\ \text{mm}^3}{7{,}200\ \text{mm}^2}$ = -10.00 mm

$\bar{Y}$ = $\frac{\Sigma(\bar{y}A)}{\Sigma A}$ = $\frac{627{,}317\ \text{mm}^3}{7{,}200\ \text{mm}^2}$ = 87.13 mm ⇒ centroid = (-10.00mm, 87.1 mm)

5.27 r = 10 cm, a = 5 cm, 대칭 $\overline{X} = \overline{Y}$



① 1/4 원호

$L = \frac{1}{4}(2\pi r) = \frac{1}{2}\pi\ (10\ \text{cm}) = 15.708\ \text{cm}$

$\overline{x} = \frac{2}{\pi} r = \frac{2}{\pi}\ (10\ \text{cm}) = 6.366\ \text{cm}$

$\overline{x}\, L = (\frac{2}{\pi} r)(\frac{\pi}{2} r) = r^2$

② $L = r - a = (10\ \text{cm}) - (5\ \text{cm}) = 5\ \text{cm}$

$\overline{x} = 0$

③ $L = a = 5\ \text{cm}$

$\overline{x} = \frac{1}{2} a = \frac{1}{2}\ (5\ \text{cm}) = 2.5\ \text{cm}$

④ $L = 5\ \text{cm}$

$\overline{x} = a = 5\ \text{cm}$

⑤ $L = 5\ \text{cm}$

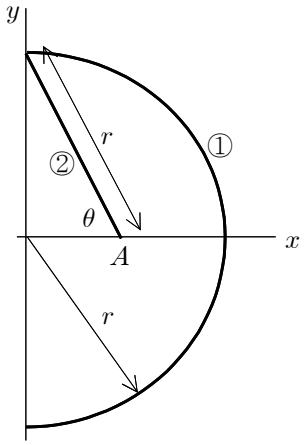
$\overline{x} = \frac{1}{2}(r - a) + a = \frac{1}{2}\ (r + a) = \frac{1}{2}[(10\ \text{cm}) + (5\ \text{cm})] = 7.5\ \text{cm}$

$\Sigma L = 15.708 + 5 + 5 + 5 + 5\ \text{cm} = 35.708\ \text{cm}$

$\Sigma(\overline{x}\, L) = (10)^2 + (5)(0) + (5)(2.5) + (5)(5) + (5)(7.5)\ \text{cm}^2 = 175\ \text{cm}^2$

$\overline{X} = \frac{\Sigma(\overline{x}\, L)}{\Sigma L} = \frac{175\ \text{cm}^2}{35.708\ \text{cm}} = 4.901\ \text{cm} \quad \Rightarrow \quad \overline{X} = \overline{Y} = 4.90\ \text{cm}$

5.30 equilibrium $\Rightarrow$ $\overline{X}$ at A



$\overline{X} = r\ \cos\theta$

① 1/2 원호

$L = \frac{1}{2}(2\pi r) = \pi r$

$\overline{x} = \frac{2}{\pi} r$

② $L = r$

$\overline{x} = \frac{r}{2}\ \cos\theta$

$\Sigma L = \pi r + r = (\pi + 1)\ r$

$\Sigma(\overline{x}\, L) = (\frac{2}{\pi} r)(\pi r) + (\frac{r}{2}\ \cos\theta)\ r = (2 + \frac{1}{2}\cos\theta)\ r^2$

$\overline{X} = \frac{\Sigma(\overline{x}\, L)}{\Sigma L} = \frac{(2 + \frac{1}{2}\cos\theta) r^2}{(\pi + 1) r} = r\ \cos\theta$

$\Rightarrow \quad 2 + \frac{1}{2}\cos\theta = (\pi + 1)\ \cos\theta$

$\Rightarrow \quad \cos\theta = \frac{2}{\pi + \frac{1}{2}} = \frac{4}{2\pi + 1} = 0.5492$

$\theta = \cos^{-1}(0.5492) = 56.69° \quad \Rightarrow \quad \theta = 56.7°$