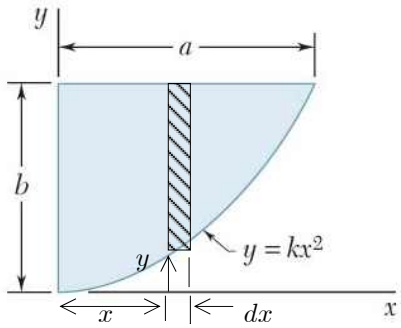


[9.1~9.5절]

9.3&7 $y = k x^2 \qquad (a,\ b) \quad \Rightarrow \quad b = k a^2$

$\Rightarrow \quad k = \dfrac{b}{a^2} \quad \Rightarrow \quad y = \dfrac{b}{a^2} x^2$

$dA = (b-y)\,dx$

$dI_y = x^2\,dA = x^2\,(b-y)\,dx$

$$I_y = \int dI_y = \int_0^a x^2\,(b-y)\,dx = \int_0^a x^2\left(b - \frac{b}{a^2}x^2\right)dx$$

$$= \int_0^a \left(b x^2 - \frac{b}{a^2}x^4\right)dx = \left[\frac{b}{3}x^3 - \frac{b}{5a^2}x^5\right]_0^a = \frac{b}{3}a^3 - \frac{b}{5a^2}a^5 = \frac{2}{15}a^3 b$$

$$dI_x = \frac{1}{3}b^3\,dx - \frac{1}{3}y^3\,dx$$

$$I_x = \int dI_x = \int_0^a \frac{1}{3}b^3\,dx - \int_0^a \frac{1}{3}y^3\,dx = \frac{b^3}{3}\int_0^a dx - \frac{1}{3}\int_0^a \left(\frac{b}{a^2}x^2\right)^3 dx$$

$$= \frac{1}{3}b^3 a - \frac{b^3}{3a^6}\left[\frac{1}{7}x^7\right]_0^a = \frac{1}{3}ab^3 - \frac{1}{21}ab^3 = \frac{2}{7}ab^3$$

9.18 $y_1 = k_1 x^2, \qquad y_2 = k_2 x^{1/2}$

$(a,\ b)$에서 $\quad b = k_1 a^2, \qquad b = k_2 a^{1/2}$

$\Rightarrow \quad k_1 = \dfrac{b}{a^2}, \qquad k_2 = \dfrac{b}{a^{1/2}}$

$y_1 = \dfrac{b}{a^2}x^2 \quad \Rightarrow \quad x_1 = \dfrac{a}{b^{1/2}}y^{1/2}$

$y_2 = \dfrac{b}{a^{1/2}}x^{1/2} \quad \Rightarrow \quad x_2 = \dfrac{a}{b^2}y^2$

$$A = \int_0^a (y_2 - y_1)\,dx = b\int_0^a \left(\frac{1}{a^{1/2}}x^{1/2} - \frac{1}{a^2}x^2\right)dx = b\left[\frac{2}{3}\frac{1}{a^{1/2}}x^{3/2} - \frac{1}{3}\frac{1}{a^2}x^3\right]_0^a$$

$$= b\left(\frac{2}{3}a - \frac{1}{3}a\right) = \frac{1}{3}ab$$

<방법1>

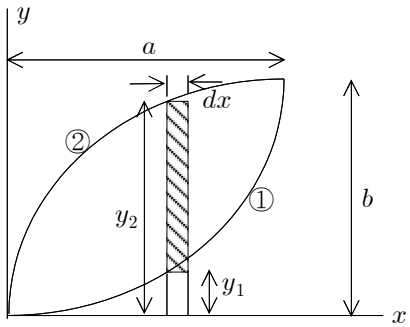
$dI_y = x^2\,dA = x^2\,(y_2 - y_1)\,dx$

$$I_y = \int_0^a x^2\left(\frac{b}{a^{1/2}}x^{1/2} - \frac{b}{a^2}x^2\right)dx$$

$$= \int_0^a \left(\frac{b}{a^{1/2}}x^{5/2} - \frac{b}{a^2}x^4\right)dx$$

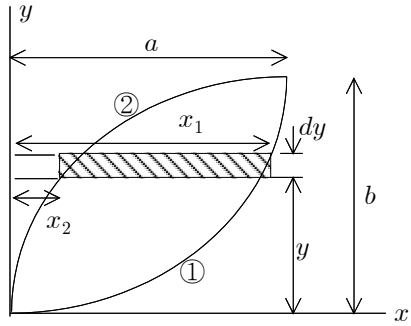
$$= b\left[\frac{1}{a^{1/2}}\frac{2}{7}x^{7/2} - \frac{1}{a^2}\frac{1}{5}x^5\right]_0^a$$

$$= b\left(\frac{2}{7}a^3 - \frac{1}{5}a^3\right) = \frac{3}{35}a^3 b$$



$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{3}{35}a^3 b}{\frac{1}{3}ab}} = \sqrt{\frac{9}{35}a^2} = 0.507\ a$$

<방법2>



$$dI_y = \frac{1}{3}x_1^3\,dy - \frac{1}{3}x_2^3\,dy$$

$$= \frac{1}{3}\left(\frac{a^3}{b^{3/2}}\,y^{3/2} - \frac{a^3}{b^6}y^6\right)dy$$

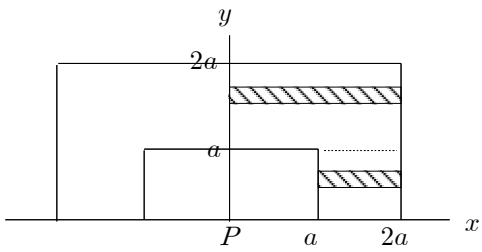
$$I_y = \int_0^b \frac{1}{3}\left(\frac{a^3}{b^{3/2}}\,y^{3/2} - \frac{a^3}{b^6}y^6\right)dy$$

$$= \frac{1}{3}\left[\frac{2}{5}\frac{a^3}{b^{3/2}}\,y^{5/2} - \frac{1}{7}\frac{a^3}{b^6}y^7\right]_0^b$$

$$= \frac{1}{3}\left(\frac{2}{5}a^3\,b - \frac{1}{7}a^3\,b\right) = \frac{3}{35}\,a^3\,b$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{3}{35}a^3\,b}{\frac{1}{3}ab}} = \sqrt{\frac{9}{35}a^2} = 0.507\ a$$

9.21 $A = (4a)(2a) - (2a)(a) = 6\,a^2$



$$dI_x = y^2\,dA = y^2\,(x_2 - x_1)dy$$

$$I_x = 2\left\{\int_0^a y^2(2a-a)dy + \int_a^{2a} y^2\,(2a-0)dy\right\}$$

$$= 2\left\{a\left[\frac{1}{3}y^3\right]_0^a + 2a\left[\frac{1}{3}y^3\right]_a^{2a}\right\}$$

$$= 2\left\{\frac{a}{3}(a^3) + \frac{2a}{3}\left[(2a)^3 - a^3\right]\right\}$$

$$= 10\,a^4$$

<방법1> $I_y = I_x = 10\,a^4$

<방법2> $dI_y = \frac{1}{3}(x_2^3 - x_1^3)dy$

$$I_y = 2\left\{\int_0^a \frac{1}{3}\left[(2a)^3 - a^3\right]dy + \int_a^{2a}\frac{1}{3}\left[(2a)^3 - 0\right]dy\right\}$$

$$= 2\left\{\frac{1}{3}(7a^3)\,a + \frac{1}{3}(8a^3)\,a\right\} = 10\,a^4$$

$$J_P = I_x + I_y = 10\,a^4 + 10\,a^4 = 20\,a^4$$

$$k_P = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{20\,a^4}{6\,a^2}} = \sqrt{\frac{10}{3}}\,a = 1.826\ a$$

<다른 방법>

$$I_x = \frac{1}{3}(4a)(2a)^3 - \frac{1}{3}(2a)(a)^3 = 10\,a^4$$

$$I_y = \frac{1}{12}(2a)(4a)^3 - \frac{1}{12}(a)(2a)^3 = 10\,a^4$$