

[3.9~3.11절]

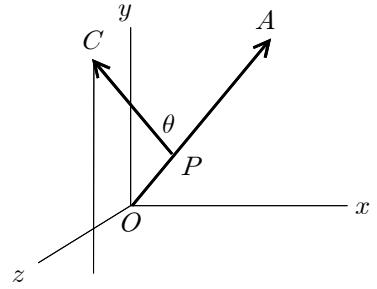
3.42 [스칼라 곱 응용 : 각, 투영]

$$r_{OP} = 6 \text{ cm}, \quad T = 12 \text{ N}$$

(a) <방법 1>

$$\mathbf{r}_{OA} = 12\mathbf{i} + 12\mathbf{j} + (-6)\mathbf{k} \text{ (cm)}$$

$$r_{OA} = \sqrt{(12)^2 + (12)^2 + (-6)^2} \text{ (cm)} = 18.0 \text{ (cm)}$$



$$\mathbf{r}_{OP} = \frac{r_{OP}}{r_{OA}} \mathbf{r}_{OA} = \frac{6}{18} [12\mathbf{i} + 12\mathbf{j} + (-6)\mathbf{k} \text{ (cm)}] = 4\mathbf{i} + 4\mathbf{j} + (-2)\mathbf{k} \text{ (cm)}$$

$$P = (4 \text{ cm}, 4 \text{ cm}, -2 \text{ cm})$$

$$\begin{aligned} \mathbf{r}_{PC} &= \mathbf{r}_{OC} - \mathbf{r}_{OP} = [9\mathbf{i} + 15\mathbf{j} + 12\mathbf{k} \text{ (cm)}] - [4\mathbf{i} + 4\mathbf{j} + (-2)\mathbf{k} \text{ (cm)}] \\ &= 5\mathbf{i} + 11\mathbf{j} + 14\mathbf{k} \text{ (cm)} \end{aligned}$$

$$r_{PC} = \sqrt{(5)^2 + (11)^2 + (14)^2} \text{ (cm)} = 18.493 \text{ (cm)}$$

$$\begin{aligned} \mathbf{r}_{OA} \cdot \mathbf{r}_{PC} &= [12\mathbf{i} + 12\mathbf{j} + (-6)\mathbf{k} \text{ (cm)}] \cdot [5\mathbf{i} + 11\mathbf{j} + 14\mathbf{k} \text{ (cm)}] \\ &= (12)(5) + (12)(11) + (-6)(14) \text{ (cm)}^2 = 108 \text{ cm}^2 \end{aligned}$$

$$\cos\theta = \frac{\mathbf{r}_{OA} \cdot \mathbf{r}_{PC}}{r_{OA} r_{PC}} = \frac{108}{(18.0)(18.493)} = 0.3244$$

$$\theta = \cos^{-1}(0.3244) = 71.07^\circ \quad \Rightarrow \quad \theta = 71.1^\circ$$

<방법 2>

$$\mathbf{r}_{OA} = 12\mathbf{i} + 12\mathbf{j} + (-6)\mathbf{k} \text{ (cm)}, \quad r_{OA} = \sqrt{(12)^2 + (12)^2 + (-6)^2} \text{ (cm)} = 18.0 \text{ (cm)}$$

$$\lambda_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{1}{18.0} [12\mathbf{i} + 12\mathbf{j} + (-6)\mathbf{k}] = \frac{1}{3} [2\mathbf{i} + 2\mathbf{j} + (-1)\mathbf{k}]$$

$$\mathbf{r}_{OP} = \frac{r_{OP}}{r_{OA}} \mathbf{r}_{OA} = \frac{6}{18} [12\mathbf{i} + 12\mathbf{j} + (-6)\mathbf{k} \text{ (cm)}] = 4\mathbf{i} + 4\mathbf{j} + (-2)\mathbf{k} \text{ (cm)}$$

$$P = (4 \text{ cm}, 4 \text{ cm}, -2 \text{ cm})$$

$$\begin{aligned} \mathbf{r}_{PC} &= \mathbf{r}_{OC} - \mathbf{r}_{OP} = [9\mathbf{i} + 15\mathbf{j} + 12\mathbf{k} \text{ (cm)}] - [4\mathbf{i} + 4\mathbf{j} + (-2)\mathbf{k} \text{ (cm)}] \\ &= 5\mathbf{i} + 11\mathbf{j} + 14\mathbf{k} \text{ (cm)}, \quad r_{PC} = \sqrt{(5)^2 + (11)^2 + (14)^2} \text{ (cm)} = 18.493 \text{ (cm)} \end{aligned}$$

$$\lambda_{PC} = \frac{\mathbf{r}_{PC}}{r_{PC}} = \frac{1}{18.493} [5\mathbf{i} + 11\mathbf{j} + 14\mathbf{k}]$$

$$\cos\theta = \frac{\mathbf{r}_{OA} \cdot \mathbf{r}_{PC}}{r_{OA} r_{PC}} = \frac{\mathbf{r}_{OA}}{r_{OA}} \cdot \frac{\mathbf{r}_{PC}}{r_{PC}} = \lambda_{OA} \cdot \lambda_{PC}$$

$$= \frac{1}{3} [2\mathbf{i} + 2\mathbf{j} + (-1)\mathbf{k}] \cdot \frac{1}{18.493} [5\mathbf{i} + 11\mathbf{j} + 14\mathbf{k}]$$

$$= \frac{1}{(3)(18.493)} [(2)(5) + (2)(11) + (-1)(14)] = 0.3244$$

$$\theta = \cos^{-1}(0.3244) = 71.07^\circ \quad \Rightarrow \quad \theta = 71.1^\circ$$

$$\begin{aligned} (\text{b}) \quad (T_{PC})_{OA} &= \mathbf{T}_{PC} \cdot \lambda_{OA} = T \lambda_{PC} \cdot \lambda_{OA} = T \frac{\mathbf{r}_{PC}}{r_{PC}} \cdot \frac{\mathbf{r}_{OA}}{r_{OA}} = T \frac{\mathbf{r}_{OA} \cdot \mathbf{r}_{PC}}{r_{OA} r_{PC}} \\ &= T \cos\theta = (12 \text{ N}) (0.3244) = 3.893 \text{ N} \quad \Rightarrow \quad (T_{PC})_{OA} = 3.89 \text{ N} \end{aligned}$$

3.48 [좌표 축에 대한 모멘트 $M_z = \mathbf{k} \cdot (\mathbf{r} \times \mathbf{F})$]

$$M_y = 120 \text{ N}\cdot\text{m}, \quad M_z = -460 \text{ N}\cdot\text{m}$$

$$\mathbf{r}_{OB} = (4.8 \text{ m}) \mathbf{j} + a \mathbf{k}$$

$$(d_{BA})_x = 2.2 \text{ m}, \quad (d_{BA})_y = (1.6 - 4.8) \text{ m} = -3.2 \text{ m}$$

$$(d_{BA})_z = -a$$

$$\lambda_{BA} = \frac{1}{d_{BA}} [(2.2 \text{ m}) \mathbf{i} + (-3.2 \text{ m}) \mathbf{j} + (-a) \mathbf{k}]$$

$$\mathbf{T}_{BA} = T_{BA} \lambda_{BA} = \frac{T_{BA}}{d_{BA}} [(2.2 \text{ m}) \mathbf{i} + (-3.2 \text{ m}) \mathbf{j} + (-a) \mathbf{k}]$$

$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T}_{BA}$$

$$= [(4.8 \text{ m}) \mathbf{j} + a \mathbf{k}] \times \frac{T_{BA}}{d_{BA}} [(2.2 \text{ m}) \mathbf{i} + (-3.2 \text{ m}) \mathbf{j} + (-a) \mathbf{k}]$$

$$= \frac{T_{BA}}{d_{BA}} \{[(4.8 \text{ m})(-a) - a(-3.2 \text{ m})] \mathbf{i} + [a(2.2 \text{ m})] \mathbf{j} + [-(4.8 \text{ m})(2.2 \text{ m})] \mathbf{k}\}$$

$$= \frac{T_{BA}}{d_{BA}} \{[(-1.6 \text{ m})a] \mathbf{i} + [(2.2 \text{ m})a] \mathbf{j} + (-10.56 \text{ m}^2) \mathbf{k}\}$$

$$M_y = \frac{T_{BA}}{d_{BA}} (2.2 \text{ m})a, \quad M_z = \frac{T_{BA}}{d_{BA}} (-10.56 \text{ m}^2)$$

$$\frac{M_y}{M_z} = \frac{(2.2 \text{ m})a}{-10.56 \text{ m}^2} = \frac{120 \text{ N}\cdot\text{m}}{-460 \text{ N}\cdot\text{m}} \Rightarrow a = 1.252 \text{ m}$$

<다른 방법>

\mathbf{T}_{BA} 의 yz 면에 수직인 성분을 T_x 라 하면, $M_y = T_x a, M_z = -T_x (4.8 \text{ m})$

$$\frac{M_y}{M_z} = \frac{a}{-4.8 \text{ m}} = \frac{120 \text{ N}\cdot\text{m}}{-460 \text{ N}\cdot\text{m}} \Rightarrow a = 1.252 \text{ m}$$

3.57 [$\hat{\rightarrow} OL$ 에 대한 모멘트 $M_{OL} = \lambda_{OL} \cdot (\mathbf{r} \times \mathbf{F})$]

$$T_{CF} = 33 \text{ N}$$

$$\mathbf{r}_{C/D} = 0\mathbf{i} + (0.9 - 0.7)\mathbf{j} + (-0.4)\mathbf{k} (\text{m}) = 0.2\mathbf{j} - 0.4\mathbf{k} (\text{m})$$

$$\begin{aligned} \mathbf{T}_{CF} &= T_{CF} \lambda_{CF} = (33 \text{ N}) \frac{(0.6 \text{ m})\mathbf{i} + (-0.9 \text{ m})\mathbf{j} + (-0.6 \text{ m} + 0.4 \text{ m})\mathbf{k}}{\sqrt{(0.6 \text{ m})^2 + (-0.9 \text{ m})^2 + (-0.2 \text{ m})^2}} \\ &= \frac{33 \text{ N}}{1.1} (0.6\mathbf{i} - 0.9\mathbf{j} - 0.2\mathbf{k}) = 18\mathbf{i} - 27\mathbf{j} - 6\mathbf{k} (\text{N}) \end{aligned}$$

$$\mathbf{r}_{C/D} \times \mathbf{T}_{CF} = [0.2\mathbf{j} - 0.4\mathbf{k} (\text{m})] \times [18\mathbf{i} - 27\mathbf{j} - 6\mathbf{k} (\text{N})]$$

$$= [0.2 \times (-6) - (-0.4) \times (-27)] \mathbf{i} + (-0.4 \times 18) \mathbf{j} + (-0.2 \times 18) \mathbf{k} (\text{N}\cdot\text{m})$$

$$= -12\mathbf{i} - 7.2\mathbf{j} - 3.6\mathbf{k} (\text{N}\cdot\text{m})$$

$$\lambda_{DB} = \frac{(1.2 \text{ m})\mathbf{i} - (0.35 \text{ m})\mathbf{j}}{\sqrt{(1.2 \text{ m})^2 + (-0.35 \text{ m})^2}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$M_{DB} = \lambda_{DB} \cdot \mathbf{M}_B = \lambda_{DB} \cdot (\mathbf{r}_{C/D} \times \mathbf{T}_{CF})$$

$$= (0.96\mathbf{i} - 0.28\mathbf{j}) \cdot [-12\mathbf{i} - 7.2\mathbf{j} - 3.6\mathbf{k} (\text{N}\cdot\text{m})]$$

$$= (0.96)(-12) + (-0.28)(-7.2) + 0 = -9.504 (\text{N}\cdot\text{m}) \Rightarrow M_{DB} = -9.50 \text{ N}\cdot\text{m}$$

