<9.11~9.15절>

9.112 $m = \rho\, t\, A = \rho\, t\left(\dfrac{1}{2}\,\pi\, a^2\right)$

$$I^{mass} = \rho\, t\, I^{area} = \frac{2\,m}{\pi\, a^2}\; I^{area}$$

(a) $I_{BB'}^{area} = I_x^{area} - A\,\overline{y}^2$

$$= \frac{1}{8}\,\pi\, a^4 - \left(\frac{1}{2}\,\pi\, a^2\right)\left(\frac{4\,a}{3\,\pi}\right)^2 = \pi\, a^4\left(\frac{1}{8} - \frac{8}{9\,\pi^2}\right)$$

$$= \frac{\pi}{8}\, a^4\left(1 - \frac{64}{9\,\pi^2}\right)$$

$$I_{BB'}^{mass} = \frac{2\,m}{\pi\, a^2}\; I_{BB'}^{area}$$

$$= \frac{2\,m}{\pi\, a^2}\,\frac{\pi}{8}\, a^4\left(1 - \frac{64}{9\,\pi^2}\right) = \frac{1}{4}\, m\, a^2\left(1 - \frac{64}{9\,\pi^2}\right) = 0.0699\; m\, a^2$$
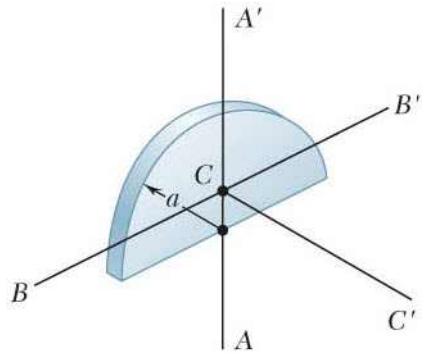
(b) Fig. 9.12에서, $I_{AA'}^{area} = \dfrac{1}{8}\,\pi\, a^4$

$$I_{AA'}^{mass} = \frac{2\,m}{\pi\, a^2}\; I_{AA'}^{area}$$

$$= \frac{2\,m}{\pi\, a^2}\,\frac{1}{8}\,\pi\, a^4 = \frac{1}{4}\, m\, a^2$$

$$\overline{I}_{CC'}^{mass} = I_{AA'}^{mass} + I_{BB'}^{mass}$$

$$= 0.250\; m\, a^2 + 0.0699\; m\, a^2 = 0.320\; m\, a^2$$

9.117 $m = \rho\, t\, A = \rho\, t\left[(2a)(a) + \dfrac{1}{2}\,(2a)\,(a)\right] = 3\,\rho\, t\, a^2 \quad \Rightarrow \quad \rho\, t = \dfrac{m}{3\, a^2}$

$$I^{mass} = \rho\, t\, I^{area} = \frac{m}{3\, a^2}\; I^{area}$$

(a) 사각형 $I_{x1}^{area} = \dfrac{1}{3}(2a)(a)^3 = \dfrac{2}{3}\, a^4$,   삼각형 $I_{x2}^{area} = \dfrac{1}{12}(2a)(a)^3 = \dfrac{1}{6}\, a^4$

$$I_x^{area} = I_{x1}^{area} + I_{x2}^{area} = \frac{2}{3}\, a^4 + \frac{1}{6}\, a^4 = \frac{5}{6}\, a^4$$

$$I_x^{mass} = \frac{m}{3\, a^2}\; I^{area} = \frac{m}{3\, a^2}\,\frac{5}{6}\, a^4 = \frac{5}{18}\, m\, a^2$$

(b) 사각형 $I_{z1}^{area} = \dfrac{1}{3}(a)(2a)^3 = \dfrac{8}{3}\, a^4$

삼각형 $I_{z2}^{area} = \dfrac{1}{36}(a)(2a)^3 + \left[\dfrac{1}{2}(a)(2a)\right]\left(2a + \dfrac{2}{3}a\right)^2 = \left(\dfrac{2}{9} + \dfrac{64}{9}\right)a^4 = \dfrac{22}{3}\, a^4$

$$I_z^{area} = I_{z1}^{area} + I_{z2}^{area} = \frac{8}{3}\, a^4 + \frac{22}{3}\, a^4 = 10\, a^4$$

$$I_z^{mass} = \frac{m}{3\, a^2}\; I^{area} = \frac{m}{3\, a^2}\,10\, a^4 = \frac{10}{3}\, m\, a^2$$

$$I_y^{mass} = I_x^{mass} + I_z^{mass} = \frac{5}{18}\, m\, a^2 + \frac{10}{3}\, m\, a^2 = \left(\frac{5}{18} + \frac{10}{3}\right)m\, a^2 = \frac{65}{18}\, m\, a^2$$

$$= 3.61\; m\, a^2$$

9.141 $\rho = 7{,}850 \text{ kg/m}^3$

$m = \rho V$ ① 원판, ② 원기둥, ③ 원판 구멍

$m_1 = \rho V_1$

$\quad = (7{,}850 \text{ kg/m}^3) \, [\pi \, (0.08 \text{ m})^2 \, (0.04 \text{ m})]$

$\quad = 6.313 \text{ kg}$

$m_2 = \rho V_2 = (7{,}850 \text{ kg/m}^3) \, [\pi \, (0.02 \text{ m})^2 \, (0.06 \text{ m})] = 0.5919 \text{ kg}$

$m_3 = \rho V_3 = (7{,}850 \text{ kg/m}^3) \, [\pi \, (0.02 \text{ m})^2 \, (0.04 \text{ m})] = 0.3946 \text{ kg}$

(a) $I_{x1} = \dfrac{1}{12} \, m_1 \, (3\,r_1^2 + 4\,L_1^2)$

$\qquad = \dfrac{1}{12}(6.313 \text{ kg})[3\,(0.08 \text{ m})^2 + 4\,(0.04 \text{ m})^2] = 13.467 \times 10^{-3} \text{ kg·m}^2$

$I_{x2} = \dfrac{1}{12} \, m_2 \, (3\,r_2^2 + 4\,L_2^2)$

$\qquad = \dfrac{1}{12}(0.5919 \text{ kg})[3\,(0.02 \text{ m})^2 + 4\,(0.06 \text{ m})^2] = 0.7695 \times 10^{-3} \text{ kg·m}^2$

$I_{x3} = \dfrac{1}{12} \, m_3 \, (3\,r_3^2 + 4\,L_3^2)$

$\qquad = \dfrac{1}{12}(0.3946 \text{ kg})[3\,(0.02 \text{ m})^2 + 4\,(0.04 \text{ m})^2] = 0.2499 \times 10^{-3} \text{ kg·m}^2$

$I_x = I_{x1} + I_{x2} - I_{x3}$

$\qquad = [(13.467) + (0.7695) - (0.2499)] \times 10^{-3} \text{ kg·m}^2 = 13.9886 \times 10^{-3} \text{ kg·m}^2$

$\Rightarrow \quad I_x = 13.99 \times 10^{-3} \text{ kg·m}^2$

(b) $I_{y1} = \dfrac{1}{2} \, m_1 \, r_1^2$

$\qquad = \dfrac{1}{2}(6.313 \text{ kg}) \, (0.08 \text{ m})^2 = 20.20 \times 10^{-3} \text{ kg·m}^2$

$I_{y2} = \dfrac{1}{2} \, m_2 \, r_2^2 + m_2 \, d_2^2$

$\qquad = \dfrac{1}{2}(0.5919 \text{ kg}) \, (0.02 \text{ m})^2 + (0.5919 \text{ kg}) \, (0.04 \text{ m})^2 = 1.0654 \times 10^{-3} \text{ kg·m}^2$

$I_{y3} = \dfrac{1}{2} \, m_3 \, r_3^2 + m_3 \, d_3^2$

$\qquad = \dfrac{1}{2}(0.3946 \text{ kg}) \, (0.02 \text{ m})^2 + (0.3946 \text{ kg}) \, (0.04 \text{ m})^2 = 0.7103 \times 10^{-3} \text{ kg·m}^2$

$I_y = I_{y1} + I_{y2} - I_{y3}$

$\qquad = [(20.20) + (1.0654) - (0.7103)] \times 10^{-3} \text{ kg·m}^2 = 20.5551 \times 10^{-3} \text{ kg·m}^2$

$\Rightarrow \quad I_y = 20.6 \times 10^{-3} \text{ kg·m}^2$

(c) $I_{z1} = I_{x1} = 13.467 \times 10^{-3} \text{ kg·m}^2$

$I_{z2} = \dfrac{1}{12} \, m_2 \, (3\,r_2^2 + 4\,L_2^2) + m_2 \, d_2^2$

$\qquad = (0.7695 \times 10^{-3} \text{ kg·m}^2) + (0.5919 \text{ kg}) \, (0.04 \text{ m})^2 = 1.7165 \times 10^{-3} \text{ kg·m}^2$

$I_{z3} = \dfrac{1}{12} \, m_3 \, (3\,r_3^2 + 4\,L_3^2) + m_3 \, d_3^2$

$\qquad = (0.2499 \times 10^{-3} \text{ kg·m}^2) + (0.3946 \text{ kg}) \, (0.04 \text{ m})^2 = 0.88126 \times 10^{-3} \text{ kg·m}^2$

$I_z = I_{z1} + I_{z2} - I_{z3}$

$\qquad = [(13.467) + (1.7165) - (0.88126)] \times 10^{-3} \text{ kg·m}^2 = 14.3022 \times 10^{-3} \text{ kg·m}^2$

$\Rightarrow \quad I_z = 14.30 \times 10^{-3} \text{ kg·m}^2$