

<9.1~9.5절>

9.2&6 $y = \frac{k}{x}$

$(a, a) \Rightarrow a = \frac{k}{a} \Rightarrow k = a^2$

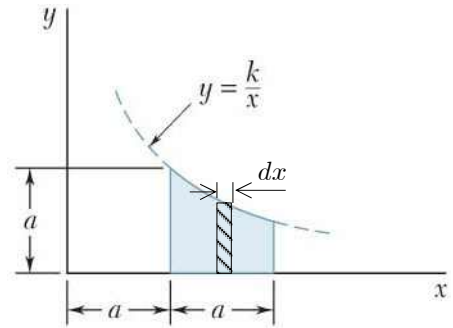
$y = \frac{a^2}{x}$

$dI_y = x^2 dA = x^2 y dx = x^2 \frac{a^2}{x} dx = a^2 x dx$

$I_y = \int dI_y = \int_a^{2a} a^2 x dx = a^2 \left[\frac{1}{2} x^2 \right]_a^{2a} = \frac{a^2}{2} [(2a)^2 - a^2] = \frac{3}{2} a^4$

$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3} \left(\frac{a^2}{x} \right)^3 dx = \frac{a^6}{3} \frac{1}{x^3} dx$

$I_x = \int dI_x = \int_a^{2a} \frac{a^6}{3} \frac{1}{x^3} dx = \frac{a^6}{3} \left[\frac{-1}{2} \frac{1}{x^2} \right]_a^{2a} = \frac{-a^6}{6} \left[\frac{1}{(2a)^2} - \frac{1}{a^2} \right] = \frac{1}{8} a^4$



9.16 $y_1 = k_1 x^2, \quad y_2 = k_2 x^{1/2}$

(a, b) 에서 $b = k_1 a^2, \quad b = k_2 a^{1/2}$

$\Rightarrow k_1 = \frac{b}{a^2}, \quad k_2 = \frac{b}{a^{1/2}}$

$y_1 = \frac{b}{a^2} x^2 \Rightarrow x_1 = \frac{a}{b^{1/2}} y^{1/2}$

$y_2 = \frac{b}{a^{1/2}} x^{1/2} \Rightarrow x_2 = \frac{a}{b^2} y^2$

$A = \int_0^a (y_2 - y_1) dx = b \int_0^a \left(\frac{1}{a^{1/2}} x^{1/2} - \frac{1}{a^2} x^2 \right) dx = b \left[\frac{2}{3} \frac{1}{a^{1/2}} x^{3/2} - \frac{1}{3} \frac{1}{a^2} x^3 \right]_0^a$

$= b \left(\frac{2}{3} a - \frac{1}{3} a \right) = \frac{1}{3} a b$

<방법1>

$dI_x = y^2 dA = y^2 (x_1 - x_2) dy$

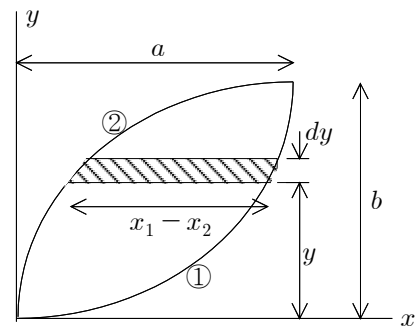
$I_x = \int_0^b y^2 \left(\frac{a}{b^{1/2}} y^{1/2} - \frac{a}{b^2} y^2 \right) dy$

$= \int_0^b \left(\frac{a}{b^{1/2}} y^{5/2} - \frac{a}{b^2} y^4 \right) dy$

$= a \left[\frac{2}{7} \frac{1}{b^{1/2}} y^{7/2} - \frac{1}{5} \frac{1}{b^2} y^5 \right]_0^b$

$= a \left(\frac{2}{7} b^3 - \frac{1}{5} b^3 \right) = \frac{3}{35} a b^3$

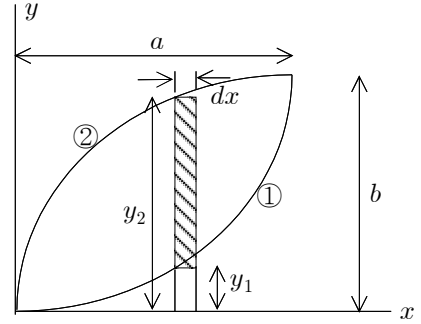
$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{3}{35} a b^3}{\frac{1}{3} a b}} = \sqrt{\frac{9}{35} b^2} = 0.507 b$



<방법2>

$$\begin{aligned}
 dI_x &= \frac{1}{3} y_2^3 dx - \frac{1}{3} y_1^3 dx \\
 &= \frac{1}{3} \left(\frac{b^3}{a^{3/2}} x^{3/2} - \frac{b^3}{a^6} x^6 \right) dx \\
 I_x &= \int_0^a \frac{1}{3} \left(\frac{b^3}{a^{3/2}} x^{3/2} - \frac{b^3}{a^6} x^6 \right) dx \\
 &= \frac{1}{3} \left[\frac{2}{5} \frac{b^3}{a^{3/2}} x^{5/2} - \frac{1}{7} \frac{b^3}{a^6} x^7 \right]_0^a \\
 &= \frac{1}{3} \left(\frac{2}{5} a b^3 - \frac{1}{7} a b^3 \right) = \frac{3}{35} a b^3
 \end{aligned}$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{3}{35} a b^3}{\frac{1}{3} a b}} = \sqrt{\frac{9}{35} b^2} = 0.507 b$$

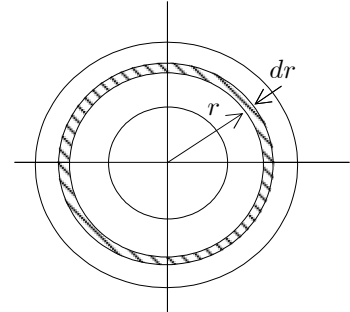


9.23 inner radius R_1 , outer radius R_2

(a) $dA = 2\pi r dr$

$$dJ_O = r^2 dA = r^2 (2\pi r dr) = 2\pi r^3 dr$$

$$\begin{aligned}
 J_O &= \int dJ_O = 2\pi \int_{R_1}^{R_2} r^3 dr = 2\pi \left[\frac{1}{4} r^4 \right]_{R_1}^{R_2} \\
 &= \frac{\pi}{2} (R_2^4 - R_1^4)
 \end{aligned}$$



(b) symmetry $\Rightarrow I_x = I_y$

$$J_O = I_x + I_y = 2 I_x$$

$$I_x = \frac{1}{2} J_O = \frac{\pi}{4} (R_2^4 - R_1^4)$$