

<5.10~5.11절>

5.104 ① 직사각판

$$V = (3 + 2 + 2 \text{ cm})(2 + 2 \text{ cm})(0.75 \text{ cm})$$

$$= 21.0 \text{ cm}^3$$

$$\bar{z} = \frac{1}{2}(7 \text{ cm}) = 3.5 \text{ cm}$$

② 반원판

$$V = \frac{1}{2}\pi(2 \text{ cm})^2(0.75 \text{ cm})$$

$$= 4.712 \text{ cm}^3$$

$$\bar{z} = (7 \text{ cm}) + \frac{4}{3\pi}(2 \text{ cm}) = 7.849 \text{ cm}$$

③ 원판 구멍

$$V = -\pi(1.25 \text{ cm})^2(0.75 \text{ cm}) = -3.682 \text{ cm}^3$$

$$\bar{z} = 7.0 \text{ cm}$$

④ 직사각판

$$V = (4 \text{ cm})(2 \text{ cm})(1 \text{ cm}) = 8.0 \text{ cm}^3$$

$$\bar{z} = 2.0 \text{ cm}$$

⑤ 반원판 구멍

$$V = -\frac{1}{2}\pi(1.25 \text{ cm})^2(1 \text{ cm}) = -2.454 \text{ cm}^3$$

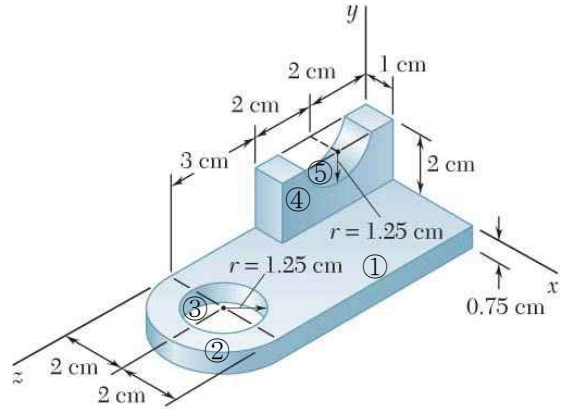
$$\bar{z} = 2.0 \text{ cm}$$

$$\Sigma V = 21.0 + 4.712 + (-3.682) + 8.0 + (-2.454) \text{ cm}^3 = 27.58 \text{ cm}^3$$

$$\Sigma(\bar{z}V) = (3.5)(21.0) + (7.849)(4.712) + (7.0)(-3.682) + (2.0)(8.0)$$

$$+ (2.0)(-2.454) \text{ cm}^4 = 95.80 \text{ cm}^4$$

$$\bar{Z} = \frac{\Sigma(\bar{z}V)}{\Sigma V} = \frac{95.80 \text{ cm}^4}{27.58 \text{ cm}^3} = 3.474 \text{ cm} \quad \Rightarrow \quad \bar{Z} = 3.47 \text{ cm}$$



5.112 y 방향 대칭 구조 $\Rightarrow \bar{Y} = \frac{1}{2}(76 \text{ mm}) = 38 \text{ mm}$

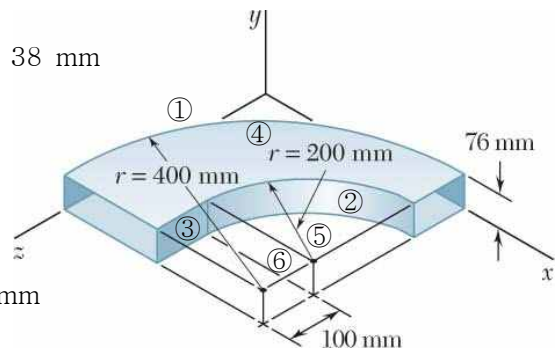
① 바깥 원주 면

$$A = \frac{1}{4}[2\pi(400 \text{ mm})](76 \text{ mm})$$

$$= 47,752 \text{ mm}^2$$

$$\bar{x} = (400 \text{ mm}) - \frac{2}{\pi}(400 \text{ mm}) = 145.35 \text{ mm}$$

$$\bar{z} = \bar{x} = 145.35 \text{ mm}$$



② 안쪽 원주 면

$$A = \frac{1}{4} [2\pi(200 \text{ mm})] (76 \text{ mm}) = 23,876 \text{ mm}^2$$

$$\bar{x} = (400 \text{ mm}) - \frac{2}{\pi}(200 \text{ mm}) = 272.7 \text{ mm}$$

$$\bar{z} = (400 \text{ mm}) - (100 \text{ mm}) - \frac{2}{\pi}(200 \text{ mm}) = (272.7 \text{ mm}) - (100 \text{ mm}) = 172.7 \text{ mm}$$

③ 안쪽 사각 면

$$A = (100 \text{ mm}) (76 \text{ mm}) = 7,600 \text{ mm}^2$$

$$\bar{x} = (400 \text{ mm}) - (200 \text{ mm}) = 200 \text{ mm}$$

$$\bar{z} = (400 \text{ mm}) - \frac{1}{2}(100 \text{ mm}) = 350 \text{ mm}$$

④, ④' 위와 아래의 큰 ¼원 면

$$A = \frac{1}{4} \pi (400 \text{ mm})^2 = 125,664 \text{ mm}^2$$

$$\bar{x} = (400 \text{ mm}) - \frac{4}{3\pi}(400 \text{ mm}) = 230.2 \text{ mm}$$

$$\bar{z} = \bar{x} = 230.2 \text{ mm}$$

⑤, ⑤' 위와 아래의 작은 ¼원 면

$$A = -\frac{1}{4} \pi (200 \text{ mm})^2 = -31,416 \text{ mm}^2$$

$$\bar{x} = (400 \text{ mm}) - \frac{4}{3\pi}(200 \text{ mm}) = 315.1 \text{ mm}$$

$$\bar{z} = (400 \text{ mm}) - (100 \text{ mm}) - \frac{4}{3\pi}(200 \text{ mm}) = (315.1 \text{ mm}) - (100 \text{ mm}) = 215.1 \text{ mm}$$

⑥, ⑥' 위와 아래의 사각 면

$$A = -(200 \text{ mm}) (100 \text{ mm}) = -20,000 \text{ mm}^2$$

$$\bar{x} = (400 \text{ mm}) - \frac{1}{2}(200 \text{ mm}) = 300 \text{ mm}$$

$$\bar{z} = (400 \text{ mm}) - \frac{1}{2}(100 \text{ mm}) = 350 \text{ mm}$$

$$\begin{aligned} \Sigma A &= 47,752 + 23,876 + 7,600 + 2(125,664) + 2(-31,416) + 2(-20,000) \text{ mm}^2 \\ &= 227,724 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \Sigma(\bar{x}A) &= (145.35)(47,752) + (272.7)(23,876) + (200)(7,600) + 2(230.2)(125,664) \\ &\quad + 2(315.1)(-31,416) + 2(300)(-20,000) \text{ mm}^3 = 41,029,081 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \Sigma(\bar{z}A) &= (145.35)(47,752) + (172.7)(23,876) + (350)(7,600) + 2(230.2)(125,664) \\ &\quad + 2(215.1)(-31,416) + 2(350)(-20,000) \text{ mm}^3 = 44,064,681 \text{ mm}^3 \end{aligned}$$

$$\bar{X} = \frac{\Sigma(\bar{x}A)}{\Sigma A} = \frac{41,029,081 \text{ mm}^3}{227,724 \text{ mm}^2} = 180.17 \text{ mm} \quad \Rightarrow \quad \bar{X} = 180.2 \text{ mm}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}A)}{\Sigma A} = \frac{44,064,681 \text{ mm}^3}{227,724 \text{ mm}^2} = 193.50 \text{ mm} \quad \Rightarrow \quad \bar{Z} = 193.5 \text{ mm}$$

center of gravity = (180.2 mm, 38.0 mm, 193.5 mm)

5.114 ① $L = 0.8 \text{ m}$

$$\bar{x} = 0$$

$$\bar{y} = \frac{1}{2}(0.8 \text{ m}) = 0.4 \text{ m}$$

$$\bar{z} = 0.6 \text{ m}$$

② $L = 0.6 \text{ m}$

$$\bar{x} = 0$$

$$\bar{y} = 0.8 \text{ m}$$

$$\bar{z} = \frac{1}{2}(0.6 \text{ m}) = 0.3 \text{ m}$$

③ $L = \sqrt{0.6^2 + 0.8^2} \text{ m} = 1.0 \text{ m}$

$$\bar{x} = (0.3 \text{ m}) \sin 60^\circ = 0.2598 \text{ m}$$

$$\bar{y} = \frac{1}{2}(0.8 \text{ m}) = 0.4 \text{ m}$$

$$\bar{z} = (0.3 \text{ m}) \cos 60^\circ = 0.15 \text{ m}$$

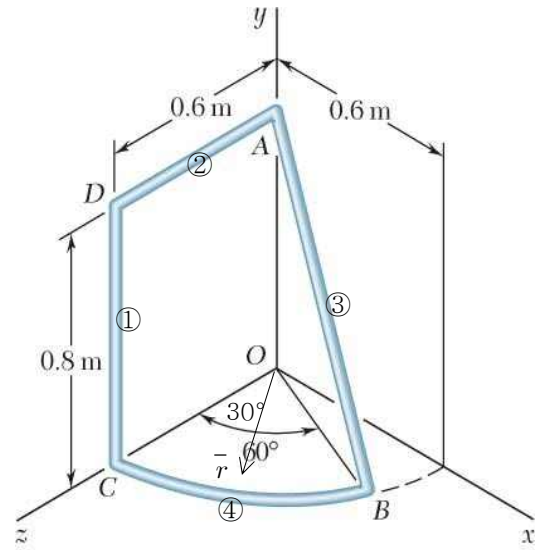
④ $L = \frac{1}{6}[2\pi(0.6 \text{ m})] = 0.6283$

$$\bar{r} = \frac{\sin(\pi/6)}{\pi/6}(0.6 \text{ m}) = 0.5730 \text{ m}$$

$$\bar{x} = \bar{r} \sin 30^\circ = (0.5730 \text{ m}) \sin 30^\circ = 0.2865 \text{ m}$$

$$\bar{y} = 0$$

$$\bar{z} = \bar{r} \cos 30^\circ = (0.5730 \text{ m}) \cos 30^\circ = 0.4962 \text{ m}$$



$$\Sigma L = 0.8 + 0.6 + 1.0 + 0.6283 \text{ m} = 3.0283 \text{ m}$$

$$\Sigma(\bar{x}L) = (0)(0.8) + (0)(0.6) + (0.2598)(1.0) + (0.2865)(0.6283) \text{ m}^2 = 0.4398 \text{ m}^2$$

$$\Sigma(\bar{y}L) = (0.4)(0.8) + (0.8)(0.6) + (0.4)(1.0) + (0)(0.6283) \text{ m}^2 = 1.200 \text{ m}^2$$

$$\Sigma(\bar{z}L) = (0.6)(0.8) + (0.3)(0.6) + (0.15)(1.0) + (0.4962)(0.6283) \text{ m}^2 = 1.1218 \text{ m}^2$$

$$\bar{X} = \frac{\Sigma(\bar{x}L)}{\Sigma L} = \frac{0.4398 \text{ m}^2}{3.0283 \text{ m}} = 0.14523 \text{ m} \quad \Rightarrow \quad \bar{X} = 0.1452 \text{ m}$$

$$\bar{Y} = \frac{\Sigma(\bar{y}L)}{\Sigma L} = \frac{1.200 \text{ m}^2}{3.0283 \text{ m}} = 0.3963 \text{ m} \quad \Rightarrow \quad \bar{Y} = 0.396 \text{ m}$$

$$\bar{Z} = \frac{\Sigma(\bar{z}L)}{\Sigma L} = \frac{1.1218 \text{ m}^2}{3.0283 \text{ m}} = 0.3704 \text{ m} \quad \Rightarrow \quad \bar{Z} = 0.370 \text{ m}$$

center of gravity = (0.1452 m, 0.396 m, 0.370 m)