

{9.11~9.15 }

$$9.117 \quad m = \rho t A = \rho t \left(\frac{1}{2} b h \right) \quad \rho t = \frac{2m}{bh} \quad I^{mass} = \rho t I^{area} = \frac{2m}{bh} I^{area}$$

$$(a) \quad \bar{I}_{AA'}^{area} = 2 \left[\frac{1}{12} (h) \left(\frac{b}{2} \right)^3 \right] = \frac{1}{48} b^3 h$$

$$\bar{I}_{AA'}^{mass} = \frac{2m}{bh} \bar{I}_{AA'}^{area} = \frac{2m}{bh} \frac{1}{48} b^3 h = \frac{1}{24} m b^2$$

$$\bar{I}_{BB'}^{area} = \frac{1}{36} b h^3$$

$$\bar{I}_{BB'}^{mass} = \frac{2m}{bh} \bar{I}_{BB'}^{area} = \frac{2m}{bh} \frac{1}{36} b h^3 = \frac{1}{18} m h^2$$

$$(b) \quad I_{CC'}^{mass} = \bar{I}_{AA'}^{mass} + \bar{I}_{BB'}^{mass} = \frac{1}{24} m b^2 + \frac{1}{18} m h^2 = \frac{m}{72} (3b^2 + 4h^2)$$

$$9.118 \quad (9.117(a)) \quad \bar{I}_{AA'}^{mass} = \frac{1}{24} m b^2, \quad \bar{I}_{BB'}^{mass} = \frac{1}{18} m h^2$$

$$\bar{I}_{DD'}^{mass} = \bar{I}_{AA'}^{mass} + m d^2 = \frac{1}{24} m b^2 + m d^2 = \frac{1}{24} m (b^2 + 24d^2)$$

$$\bar{I}_{EE'}^{mass} = \bar{I}_{BB'}^{mass} + m d^2 = \frac{1}{18} m h^2 + m d^2 = \frac{1}{18} m (h^2 + 18d^2)$$

$$9.119 \quad m = \rho t A = \rho t \left[(2a)(a) + \frac{1}{2} (2a)(a) \right] = 3 \rho t a^2 \quad \rho t = \frac{m}{3a^2}$$

$$I^{mass} = \rho t I^{area} = \frac{m}{3a^2} I^{area}$$

$$(a) \quad I_{x1}^{area} = \frac{1}{3} (2a)(a)^3 = \frac{2}{3} a^4, \quad I_{x2}^{area} = \frac{1}{12} (2a)(a)^3 = \frac{1}{6} a^4$$

$$I_x^{area} = I_{x1}^{area} + I_{x2}^{area} = \frac{2}{3} a^4 + \frac{1}{6} a^4 = \frac{5}{6} a^4$$

$$I_x^{mass} = \frac{m}{3a^2} I_x^{area} = \frac{m}{3a^2} \frac{5}{6} a^4 = \frac{5}{18} m a^2$$

$$(b) \quad I_{z1}^{area} = \frac{1}{3} (a)(2a)^3 = \frac{8}{3} a^4$$

$$I_{z2}^{area} = \frac{1}{36} (a)(2a)^3 + \left[\frac{1}{2} (a)(2a) \right] \left(2a + \frac{2}{3} a \right)^2 = \left(\frac{2}{9} + \frac{64}{9} \right) a^4 = \frac{22}{3} a^4$$

$$I_z^{area} = I_{z1}^{area} + I_{z2}^{area} = \frac{8}{3} a^4 + \frac{22}{3} a^4 = 10 a^4$$

$$I_z^{mass} = \frac{m}{3a^2} I_z^{area} = \frac{m}{3a^2} 10 a^4 = \frac{10}{3} m a^2$$

$$I_y^{mass} = I_x^{mass} + I_z^{mass} = \frac{5}{18} m a^2 + \frac{10}{3} m a^2 = \left(\frac{5}{18} + \frac{10}{3} \right) m a^2 = \frac{65}{18} m a^2$$

$$= 3.61 m a^2$$

9.132

$$\begin{aligned} & \rho \\ & \left(\quad \quad \quad r, \quad \quad \quad h \right) \quad \quad \quad = \frac{1}{2} (\rho \pi r^2 h) r^2 = \frac{1}{2} \rho \pi r^4 h \\ \bar{I}_{AA'} &= \frac{1}{2} \rho \pi a_2^4 h - \frac{1}{2} \rho \pi a_1^4 h = \frac{1}{2} \rho \pi (a_2^4 - a_1^4) h \\ m &= \rho \pi (a_2^2 - a_1^2) h \end{aligned}$$

$$\begin{aligned} \text{(a)} \quad I_{BB'} &= \bar{I}_{AA'} + m a_1^2 = \frac{1}{2} \rho \pi (a_2^4 - a_1^4) h + \rho \pi (a_2^2 - a_1^2) h a_1^2 \\ &= \frac{1}{2} \rho \pi h (a_2^4 + 2a_2^2 a_1^2 - 3a_1^4) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{d}{da_1} I_{BB'} &= \frac{d}{da_1} \left[\frac{1}{2} \rho \pi h (a_2^4 + 2a_2^2 a_1^2 - 3a_1^4) \right] \\ &= \frac{1}{2} \rho \pi h (4a_2^2 a_1 - 12a_1^3) = 2\rho \pi h a_1 (a_2^2 - 3a_1^2) = 0 \end{aligned}$$

$$a_1 (a_2^2 - 3a_1^2) = 0, \quad a_1 = 0 \quad a_2^2 - 3a_1^2 = 0 \quad a_1 = \frac{1}{\sqrt{3}} a_2$$

$$\begin{aligned} \text{(c)} \quad I_{BB'} \Big|_{a_1 = \frac{a_2}{\sqrt{3}}} &= \frac{1}{2} \rho \pi h \left[a_2^4 + 2a_2^2 \left(\frac{a_2}{\sqrt{3}} \right)^2 - 3 \left(\frac{a_2}{\sqrt{3}} \right)^4 \right] \\ &= \frac{1}{2} \rho \pi h a_2^4 \left[1 + \frac{2}{3} - \frac{1}{3} \right] = \frac{2}{3} \rho \pi h a_2^4 \end{aligned}$$