

{9.6~9.7 }

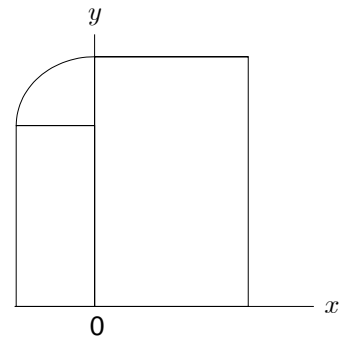
9.35 x

$$I_{x1} = \frac{1}{3} (2a) (4a)^3 = \frac{128}{3} a^4$$

$$I_{x2} = \frac{1}{3} a (3a)^3 = 9a^4$$

$$\begin{aligned} I_{x3} &= \left[\frac{\pi}{16} a^4 - \left(\frac{\pi}{4} a^2 \right) \left(\frac{4}{3\pi} a \right)^2 \right] + \left(\frac{\pi}{4} a^2 \right) \left(\frac{4}{3\pi} a + 3a \right)^2 \\ &= \left[\frac{\pi}{16} - \left(\frac{\pi}{4} \right) \left(\frac{16}{9\pi^2} \right) + \left(\frac{\pi}{4} \right) \left(\frac{16}{9\pi^2} + \frac{8}{\pi} + 9 \right) \right] a^4 \\ &= \left(\frac{\pi}{16} - \frac{4}{9\pi} + \frac{4}{9\pi} + 2 + \frac{9\pi}{4} \right) a^4 = \left(2 + \frac{37\pi}{16} \right) a^4 \end{aligned}$$

$$I_x = I_{x1} + I_{x2} + I_{x3} = \frac{128}{3} a^4 + 9a^4 + \left(2 + \frac{37\pi}{16} \right) a^4 = \left(\frac{161}{3} + \frac{37\pi}{16} \right) a^4 = 60.9 a^4$$



y

$$I_{y1} = \frac{1}{3} (4a) (2a)^3 = \frac{32}{3} a^4$$

$$I_{y2} = \frac{1}{3} (3a) a^3 = a^4$$

$$I_{y3} = \frac{\pi}{16} a^4$$

$$I_y = I_{y1} + I_{y2} + I_{y3} = \frac{32}{3} a^4 + a^4 + \frac{\pi}{16} a^4 = \left(\frac{35}{3} + \frac{\pi}{16} \right) a^4 = 11.86 a^4$$

9.44 1 : (centroid)

$$27+24+24+15 \text{ mm} = 90 \text{ mm}$$

$$A = (40 \text{ mm})(90 \text{ mm}) = 3\,600 \text{ mm}^2$$

$$\bar{x} = \frac{1}{2}(40 \text{ mm}) = 20 \text{ mm}, \quad \bar{y} = \frac{1}{2}(90 \text{ mm}) = 45 \text{ mm}$$

$$A = \frac{1}{2}(24+24 \text{ mm})(30 \text{ mm}) = 720 \text{ mm}^2$$

$$\bar{x} = (40 \text{ mm}) + \frac{1}{3}(30 \text{ mm}) = 50 \text{ mm}$$

$$\bar{y} = (27 \text{ mm}) + (24 \text{ mm}) = 51 \text{ mm}$$

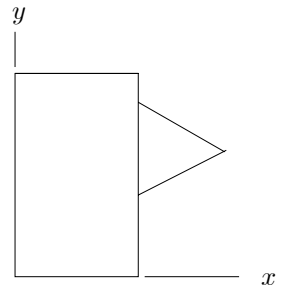
$$A = (3\,600 \text{ mm}^2) + (720 \text{ mm}^2) = 4\,320 \text{ mm}^2$$

$$\bar{x}A = (20 \text{ mm})(3\,600 \text{ mm}^2) + (50 \text{ mm})(720 \text{ mm}^2) = 108\,000 \text{ mm}^3$$

$$\bar{y}A = (45 \text{ mm})(3\,600 \text{ mm}^2) + (51 \text{ mm})(720 \text{ mm}^2) = 198\,720 \text{ mm}^3$$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{108,000 \text{ mm}^3}{4,320 \text{ mm}^2} = 25 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{198,720 \text{ mm}^3}{4,320 \text{ mm}^2} = 46 \text{ mm}$$



2 : \bar{I}_x, \bar{I}_y

$$\begin{aligned} I_{x1} &= \frac{1}{12} b h^3 + A d^2 \\ &= \frac{1}{12} (40 \text{ mm})(90 \text{ mm})^3 + (3\,600 \text{ mm}^2)(46-45 \text{ mm})^2 \\ &= 2\,433\,600 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{x2} &= \frac{1}{36} b h^3 + A d^2 \\ &= \frac{1}{36} (30 \text{ mm})(24 \text{ mm})^3 + \frac{1}{2} (720 \text{ mm}^2)(59-46 \text{ mm})^2 \\ &= 72\,360 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{x3} &= \frac{1}{36} (30 \text{ mm})(24 \text{ mm})^3 + \frac{1}{2} (720 \text{ mm}^2)(46-43 \text{ mm})^2 \\ &= 14\,760 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_x &= I_{x1} + I_{x2} + I_{x3} = (2\,433\,600 \text{ mm}^4) + (72\,360 \text{ mm}^4) + (14\,760 \text{ mm}^4) \\ &= 2\,520\,720 \text{ mm}^4 = 2.52 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{y1} &= \frac{1}{12} b^3 h + A d^2 \\ &= \frac{1}{12} (40 \text{ mm})^3 (90 \text{ mm}) + (3\,600 \text{ mm}^2)(25-20 \text{ mm})^2 = 570\,000 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_{y2+3} &= \frac{1}{36} b h^3 + A d^2 \\ &= \frac{1}{36} (48 \text{ mm})(30 \text{ mm})^3 + (720 \text{ mm}^2)(10+15 \text{ mm})^2 = 486\,000 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} I_y &= I_{y1} + I_{y2+3} = (570\,000 \text{ mm}^4) + (486\,000 \text{ mm}^4) \\ &= 1\,056\,000 \text{ mm}^4 = 1.056 \times 10^6 \text{ mm}^4 \end{aligned}$$

