{9.1~9.5절}

9.3　$x = k(y+c)^2$

$(0,\ b) \quad \Rightarrow \quad 0 = k(b+c)^2 \quad \Rightarrow \quad c = -b \quad \Rightarrow \quad x = k(y-b)^2$

$(a,\ 2b) \quad \Rightarrow \quad a = k(2b-b)^2 \quad \Rightarrow \quad k = \frac{a}{b^2} \quad \Rightarrow \quad x = \frac{a}{b^2}(y-b)^2$

<방법 1>

$$dI_y = \frac{1}{3}x^3\,dy$$

$$I_y = \int_b^{2b}\frac{1}{3}x^3\,dy = \frac{1}{3}\int_b^{2b}\left[\frac{a}{b^2}(y-b)^2\right]^3 dy = \frac{a^3}{3b^6}\int_b^{2b}(y-b)^6\,dy$$

$$= \frac{a^3}{3b^6}\left[\frac{1}{7}(y-b)^7\right]_b^{2b} = \frac{a^3}{21b^6}\left[(2b-b)^7-(b-b)^7\right]_b^{2b} = \frac{1}{21}a^3b$$

<방법 2>

$$I_y = \int x^2\,dA = \int_b^{2b}\int_0^{\frac{a}{b^2}(y-b)^2} x^2\,dx\,dy = \int_b^{2b}\left[\frac{1}{3}x^3\right]_0^{\frac{a}{b^2}(y-b)^2} dy$$

$$= \int_b^{2b}\left[\frac{1}{3}x^3\right]_0^{\frac{a}{b^2}(y-b)^2} dy = \int_b^{2b}\frac{1}{3}\left[\frac{a}{b^2}(y-b)^2\right]^3 dy = \int_b^{2b}\frac{1}{3}\frac{a^3}{b^6}(y-b)^6\,dy$$

$$= \frac{1}{3}\frac{a^3}{b^6}\left[\frac{1}{7}(y-b)^7\right]_b^{2b} = \frac{1}{21}\frac{a^3}{b^6}\left[(2b-b)^7-(b-b)^7\right] = \frac{1}{21}a^3b$$

9.18　$y_1 = k\sqrt{x}$

$(a,\ 2b) \quad \Rightarrow \quad 2b = k\sqrt{a} \quad \Rightarrow \quad k = \frac{2b}{\sqrt{a}} \quad \Rightarrow \quad y_1 = \frac{2b}{\sqrt{a}}\sqrt{x}$

$y_2 = c\,x$

$(a,\ b) \quad \Rightarrow \quad b = c\,a \quad \Rightarrow \quad c = \frac{b}{a} \quad \Rightarrow \quad y_2 = \frac{b}{a}x$

$$A = 2\int_0^a (y_1-y_2)\,dx = 2\int_0^a\left(\frac{2b}{\sqrt{a}}x^{\frac{1}{2}} - \frac{b}{a}x\right)dx = 2b\left[\frac{4}{3\sqrt{a}}x^{\frac{3}{2}} - \frac{1}{2a}x^2\right]_0^a$$

$$= 2b\left(\frac{4}{3\sqrt{a}}a^{\frac{3}{2}} - \frac{1}{2a}a^2\right) = \frac{5}{3}ab$$

$$dI_y = x^2(y_1-y_2)\,dx = x^2\left(\frac{2b}{\sqrt{a}}\sqrt{x} - \frac{b}{a}x\right)dx = \left(\frac{2b}{\sqrt{a}}x^{\frac{5}{2}} - \frac{b}{a}x^3\right)dx$$

$$I_x = 2\int_0^a\left(\frac{2b}{\sqrt{a}}x^{\frac{5}{2}} - \frac{b}{a}x^3\right)dx = 2\left[\frac{2b}{\sqrt{a}}\frac{2}{7}x^{\frac{7}{2}} - \frac{b}{a}\frac{1}{4}x^4\right]_0^a = 2\left(\frac{4b}{7\sqrt{a}}a^{\frac{7}{2}} - \frac{b}{4a}a^4\right)$$

$$= 2\left(\frac{4b}{7}a^3 - \frac{b}{4}a^3\right) = 2\left(\frac{4}{7}-\frac{1}{4}\right)ab^3 = \frac{9}{14}a^3b$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{9}{14}a^3b}{\frac{5}{3}ab}} = \sqrt{\frac{27}{70}}\,a = 0.621\ a$$

9.25 (a) $dA = \dfrac{3\pi}{2} r\, dr$

$$J_O = \int r^2\, dA = \int_{R_1}^{R_2} r^2 \frac{3\pi}{2} r\, dr = \frac{3\pi}{2} \int_{R_1}^{R_2} r^3\, dr = \frac{3\pi}{2} \left[\frac{1}{4} r^4 \right]_{R_1}^{R_2} = \frac{3\pi}{8} \left(R_2^4 - R_1^4 \right)$$

(b) $J_O = I_x + I_y$

$$I_x = I_y = \frac{1}{2} J_O = \frac{1}{2} \frac{3\pi}{8} \left(R_2^4 - R_1^4 \right) = \frac{3\pi}{16} \left(R_2^4 - R_1^4 \right)$$