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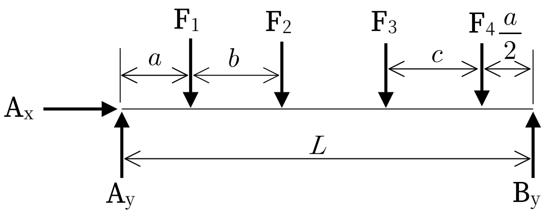
{4.1~4.5절}

4.7　[반력(핀, 롤러), 평형(모멘트, 힘)]

$F_1 = 3.9$ kN, $F_2 = 6.3$ kN,

$F_3 = 7.9$ kN, $F_4 = 7.3$ kN,

$L = 12$ m, $b = 2.6$ m, $c = 2.8$ m



(a) $a = 2.9$ m

$\rightarrow \Sigma F_x = 0$;　$A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$;　$-L\,A_y + (L-a)\,F_1 + (L-a-b)\,F_2 + (c+\frac{1}{2}a)\,F_3 + \frac{1}{2}a\,F_4 = 0$

$$\Rightarrow A_y = \frac{1}{L}\left[(L-a)\,F_1 + (L-a-b)\,F_2 + (c+\frac{1}{2}a)\,F_3 + \frac{1}{2}a\,F_4\right]$$

$$= \frac{1}{12\text{ m}}\{[(12-2.9)\text{ m}](3.9\text{ kN}) + [(12-2.9-2.6)\text{ m}](6.3\text{ kN}) + [(2.8+\frac{2.9}{2})\text{ m}](7.9\text{ kN}) + (\frac{2.9}{2}\text{ m})(7.3\text{ kN})\}$$

$$= \frac{1}{12\text{ m}}(120.6\text{ kN}\cdot\text{m}) = 10.05\text{ kN} \qquad \Rightarrow \qquad \mathbf{A} = 10.05\text{ kN} \uparrow$$

$\uparrow \Sigma F_y = 0$;　$A_y - F_1 - F_2 - F_3 - F_4 + B_y = 0$

$$\Rightarrow B_y = -A_y + F_1 + F_2 + F_3 + F_4$$

$$= -(10.05\text{ kN}) + (3.9\text{ kN}) + (6.3\text{ kN}) + (7.9\text{ kN}) + (7.3\text{ kN}) = 15.35\text{ kN}$$

$$\Rightarrow \qquad \mathbf{B} = 15.35\text{ kN} \uparrow$$

(b) $a = 8.1$ m

$\rightarrow \Sigma F_x = 0$;　$A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$;　$-L\,A_y + (L-a)\,F_1 + (L-a-b)\,F_2 + (c+\frac{1}{2}a)\,F_3 + \frac{1}{2}a\,F_4 = 0$

$$\Rightarrow A_y = \frac{1}{L}\left[(L-a)\,F_1 + (L-a-b)\,F_2 + (c+\frac{1}{2}a)\,F_3 + \frac{1}{2}a\,F_4\right]$$

$$= \frac{1}{12\text{ m}}\{[(12-8.1)\text{ m}](3.9\text{ kN}) + [(12-8.1-2.6)\text{ m}](6.3\text{ kN}) + [(2.8+\frac{8.1}{2})\text{ m}](7.9\text{ kN}) + (\frac{8.1}{2}\text{ m})(7.3\text{ kN})\}$$

$$= \frac{1}{12\text{ m}}(107.08\text{ kN}\cdot\text{m}) = 8.923\text{ kN} \qquad \Rightarrow \qquad \mathbf{A} = 8.92\text{ kN} \uparrow$$

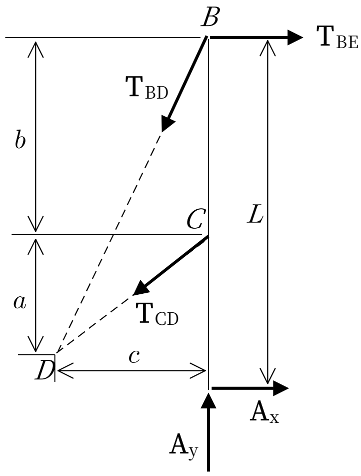
$\uparrow \Sigma F_y = 0$;　$A_y - F_1 - F_2 - F_3 - F_4 + B_y = 0$

$$\Rightarrow B_y = -A_y + F_1 + F_2 + F_3 + F_4$$

$$= -(8.923\text{ kN}) + (3.9\text{ kN}) + (6.3\text{ kN}) + (7.9\text{ kN}) + (7.3\text{ kN}) = 16.477\text{ kN}$$

$$\Rightarrow \qquad \mathbf{B} = 16.48\text{ kN} \uparrow$$

4.15 [반력(거친 면), 평형(모멘트, 힘)]



$L = 24\ \text{m},\quad a = 8.4\ \text{m},\quad b = 12.6\ \text{m},\quad c = 11.2\ \text{m}$

$T_{BD} = 221\ \text{N},\quad T_{BE} = 161\ \text{N}$

$d_{BD} = \sqrt{(a+b)^2 + c^2}$

$= \sqrt{(8.4 + 12.6\ \text{m})^2 + (11.2\ \text{m})^2} = 23.8\ \text{m}$

$d_{CD} = \sqrt{a^2 + c^2} = \sqrt{(8.4\ \text{m})^2 + (11.2\ \text{m})^2} = 14.0\ \text{m}$

(a) $+\circlearrowleft \Sigma M_A = 0$;

$-L\ T_{BE} + L\,(T_{BD})_x + (L-b)\ (T_{CD})_x = 0$

$\Rightarrow\ (T_{CD})_x = \dfrac{L}{L-b}\ [T_{BE} - T_{BD}\dfrac{c}{d_{BD}}]$

$= \dfrac{24\ \text{m}}{24 - 12.6\ \text{m}}\ [(161\ \text{N}) - (221\ \text{N})\dfrac{11.2}{23.8}] = 120\ \text{N}$

$(T_{CD})_x = T_{CD}\dfrac{c}{d_{cd}}$

$\Rightarrow\ T_{CD} = \dfrac{d_{CD}}{c}(T_{CD})_x = \dfrac{14}{11.2}\ (120\ \text{N}) = 150\ \text{N} \qquad \Rightarrow \qquad T_{CD} = 150.0\ \text{N}$

(b) $\rightarrow \Sigma F_x = 0$; $\quad A_x + T_{BE} - (T_{BD})_x - (T_{CD})_x = 0$

$\Rightarrow\ A_x = -T_{BE} + (T_{BD})_x + (T_{CD})_x$

$= -(161\ \text{N}) + (221\ \text{N})\dfrac{11.2}{23.8} + (120\ \text{N}) = 63.0\ \text{N}$

$\uparrow \Sigma F_y = 0$; $\quad A_y - (T_{BD})_y - (T_{CD})_y = 0$

$\Rightarrow\ A_y = T_{BD}\dfrac{a+b}{d_{BD}} + T_{CD}\dfrac{a}{d_{CD}}$

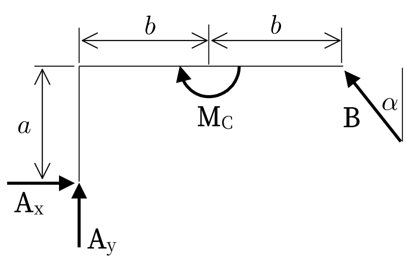
$= (221\ \text{N})\ \dfrac{8.4 + 12.6}{23.8} + (150\ \text{N})\ \dfrac{8.4}{14.0} = 285\ \text{N}$

$A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{(63.0\ \text{N})^2 + (285\ \text{N})^2} = 291.9\ \text{N}$

$\tan\theta = \dfrac{A_y}{A_x} = \dfrac{285}{63.0} = 4.524 \qquad \Rightarrow \qquad \theta = \tan^{-1}(4.524) = 77.5°$

$\Rightarrow$ A = 292 N $\nearrow$77.5°

4.18 [반력(핀, 롤러), 평형(힘, 모멘트)]



$a = 0.3\ \text{m},\quad b = 0.25\ \text{m},\quad M_C = 82.5\ \text{N·m}$

$+\circlearrowleft \Sigma M_A = 0$; $-M_C + a\ (B\ \sin\alpha) + 2b\ (B\ \cos\alpha) = 0$

$\Rightarrow\quad B = \dfrac{M_C}{a\sin\alpha + 2b\cos\alpha}$

$\rightarrow \Sigma F_x = 0$; $\quad A_x - B\ \sin\alpha = 0 \qquad \Rightarrow \qquad A_x = B\ \sin\alpha$

$\uparrow \Sigma F_y = 0$; $\quad A_y + B\ \cos\alpha = 0 \qquad \Rightarrow \qquad A_y = -B\ \cos\alpha$

(a) $\alpha = 0$

$B = \dfrac{82.5\ \text{N}\cdot\text{m}}{0 + 2(0.25\ \text{m})} = 165\ \text{N},\quad A_x = 0,\quad A_y = -B = -165\ \text{N}$

$\Rightarrow$ A = 165.0 N ↓, B = 165 N ↑

(b) $\alpha = 90°$

$$B = \frac{82.5\ \mathrm{N \cdot m}}{(0.3\ \mathrm{m}) + 0} = 275\ \mathrm{N}, \quad A_x = B = 275\ \mathrm{N}, \quad A_y = 0$$

$\Rightarrow$ **A** = 275 N $\rightarrow$, **B** = 275 N $\leftarrow$

(c) $\alpha = 30°$

$$B = \frac{82.5\ \mathrm{N \cdot m}}{(0.3\ \mathrm{m})\sin 30° + 2(0.25\ \mathrm{m})\cos 30°} = 141.51\ \mathrm{N}$$ $\Rightarrow$ **B** = 141.5 N $\nwarrow$60.0°

$A_x = (141.51\ \mathrm{N})\ \sin 30° = 70.75\ \mathrm{N}$

$A_y = -(141.51\ \mathrm{N})\ \cos 30° = -122.55\ \mathrm{N}$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(70.75\ \mathrm{N})^2 + (-122.55\ \mathrm{N})^2} = 141.51\ \mathrm{N}$$

$$\tan\theta = \frac{A_y}{A_x} = \frac{-122.55}{70.75} = -1.7322 \quad \Rightarrow \quad \theta = \tan^{-1}(-1.7322) = -60.0°$$

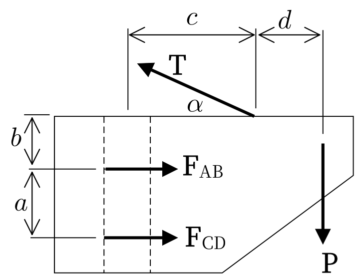
$\Rightarrow$ **A** = 151.5 N $\searrow$60.0°

4.41 [반력(롤러), 평형(힘, 모멘트)]

$a = 0.2\ \mathrm{m}, \quad b = 0.125\ \mathrm{m},$

$c = 0.375\ \mathrm{m}, \quad d = 0.25\ \mathrm{m}, \quad P = 270\ \mathrm{N}$

$\alpha = 20°$



$\uparrow \Sigma F_y = 0$; $T \sin\alpha - P = 0$

$$\Rightarrow \quad T = \frac{P}{\sin\alpha} = \frac{270\ \mathrm{N}}{\sin 20°} = 789.4\ \mathrm{N}$$

$T_x = T \cos\alpha = (789.4\ \mathrm{N})\ \cos 20° = 741.8\ \mathrm{N}$

$T_y = T \sin\alpha = P = 270.0\ \mathrm{N}$

$+\circlearrowleft \Sigma M_B = 0$; $a\ F_D + b\ T_x + (c-h)\ T_y - (c+d-h)\ P = 0$

$$\Rightarrow \quad F_{CD} = \frac{1}{a}\ (-b\ T_x + d\ P)$$

$$= \frac{1}{0.2\ \mathrm{m}}\ [-(0.125\ \mathrm{m})\ (741.8\ \mathrm{N}) + (0.25\ \mathrm{m})\ (270\ \mathrm{N})] = -126.13\ \mathrm{N}$$

F_{CD}는 왼쪽으로 작용함 $\Rightarrow$ **C** = 126.1 N $\leftarrow$, **D** = 0

$\rightarrow \Sigma F_x = 0$; $F_{AB} + F_{CD} - T_x = 0$

$\Rightarrow \quad F_{AB} = -F_{CD} + T_x = -(-126.13\ \mathrm{N}) + (741.8\ \mathrm{N}) = 867.9\ \mathrm{N}$

F_{AB}는 오른쪽으로 작용함 $\Rightarrow$ **A** = 0, **B** = 868 N $\rightarrow$